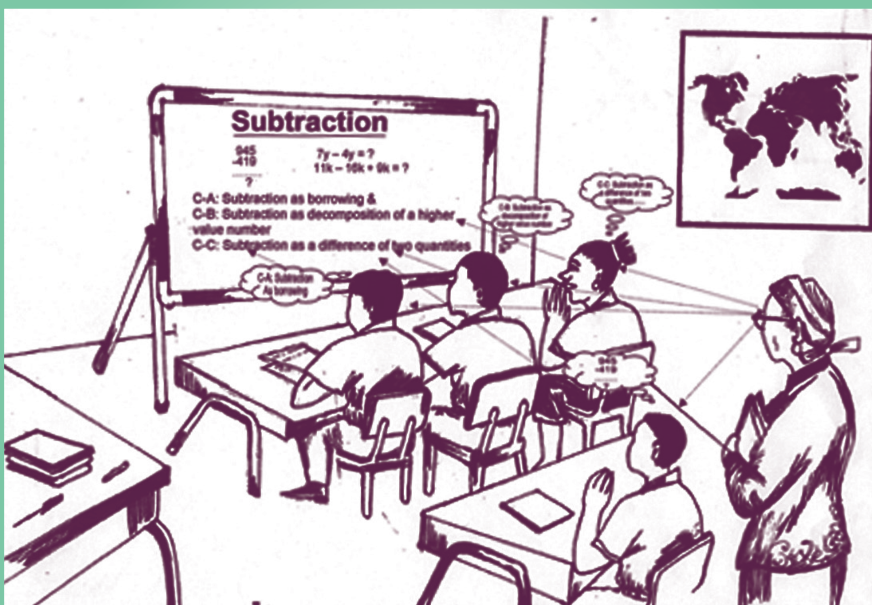
Andrew L. Binde

Conceptions of Mathematics Teacher Education

Thoughts among Teacher Educators in Tanzania



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CONCEPTIONS OF MATHEMATICS TEACHER EDUCATION

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Abstract

This study addresses the question of teacher educators' conceptions of mathematics teacher education (MTE) in teacher colleges in Tanzania, and their thoughts on how to further develop it. The tension between exponents of content as opposed to pedagogy has continued to cause challenging conceptual differences, which also influences what teacher educators conceive as desirable in the development of this domain. This tension is connected to the dissatisfaction of parents and teachers with the failure of school mathematics. From this point of view, the overall aim was to identify and describe teacher educators' various conceptions of MTE.

Inspired by the debate among teacher educators about what the balance should be between subject matter and pedagogical knowledge, it was important to look at the theoretical faces of MTE. The theoretical background involved the review of what is visible in MTE, what is yet to be known and the challenges within the practice. This task revealed meanings, perspectives in MTE, professional development and assessment. To do this, two questions were asked, to which no clear solutions satisfactorily existed. The questions to guide the investigation were, firstly, what are teacher educators' conceptions of MTE, and secondly, what are teacher educators' thoughts on the development of MTE? The two questions led to the choice of phenomenography as the methodological approach. Against the guiding questions, 27 mathematics teacher educators were interviewed in relation to the first question, while 32 responded to an open-ended questionnaire regarding question two. The interview statements as well as the questionnaire responses were coded and analysed (classified). The process of classification generated patterns of qualitatively different ways of seeing MTE.

The results indicate that MTE is conceived as a process of learning through investigation, fostering inspiration, an approach to learning with an emphasis on problem solving, and a focus on pedagogical knowledge and skills in the process of teaching and learning. In addition, the teaching and learning of mathematics is seen as subject didactics with a focus on subject matter and as an organized integration of subject matter, pedagogical knowledge and some school practice; and also as academic content knowledge in which assessment is inherent. The respondents also saw the need to build learner-educator relationships. Finally, they emphasized taking advantage of teacher educators' neighbourhood learning groups, networking and collaboration as sustainable knowledge and skills sharing strategies in professional development. Regarding desirable development, teacher educators' thoughts emphasised enhancing pedagogical knowledge and subject matter, and to be determined by them as opposed to conventional top-down seminars and workshops.

This study has revealed various conceptions and thoughts about MTE based on teacher educators´ diverse history of professional development in mathematics. It has been reasonably substantiated that some teacher educators teach school mathematics in the name of MTE, hardly distinguishing between the role and purpose of the two in developing a mathematics teacher. What teacher educators conceive as MTE and what they do regarding the education of teachers of mathematics revealed variations in terms of seeing the phenomenon of interest. Within limits, desirable thoughts shed light on solutions to phobias, and in the same way low self-esteem and stigmatization call for the building of teacher educator-student teacher relationships.

Key words: phenomenography, conception, thoughts, category of description, mathematics education, mathematics teacher education, variations, and MKT.

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Doing research at this level is an inspiring task, and I am grateful for being part of this challenging opportunity. As a doctoral student in the Teacher Education Project, I had an opportunity to deepen my insights into the domain of practice (mathematics teacher education) thought to be complex and layered in terms of the preparation and development of mathematics teachers.

In the process of creating, shaping and conducting the study, and finally the thesis writing itself, many friends and colleagues helped me to achieve the goal of this study. First of all, I would like to thank my supervisor Professor Ole Björkqvist. As an experienced supervisor, he guided me through the challenging research process. I would also like to thank Professor Sven-Erik Hansén for his encouragement and professional research advice. In the same way, fellow doctoral students Colonel Chambulila, Cosmas Mnyanyi, George Jidamva, and Sebastian Nessoro - teacher educator from the Morogoro teacher college who tirelessly assisted in coding and analysis despite leaving other personal tasks pending. I would like to thank Dr. Wilberforce Meena for the many ideas we shared in the process of studying and Tarja Grahn-Björkqvist from the Faculty of Education at Abo Akademi, Vasa for the technical parts and manuscript layout. I would also like to recognise the assistance of Richard Nzoka and Florida Munisi from the Teacher Education Department MoEVT for their technical support and word processing of study materials when found necessary.

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Abo Akademi University, Vasa, Finland

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Table of Contents

List of Tables

List of Figures

List of Abbreviations

In this study, the following abbreviations have been used. The meanings shown to the right are the same as in the literature consulted in the study structure. A few of these are of my own construction in the interest of meeting the study objectives.

ICT	Information and Communication Technology
MoEC	Ministry of Education and Culture (previous name)
MoEVT	Ministry of Education and Vocational Training (name at the time of the study)
MTE	Mathematics teacher education
MKT	Mathematics knowledge for teaching
NECTA	National Examination Council of Tanzania
PCK	Pedagogical content knowledge
Sida	Swedish International Development Agency
TET	Taasisi ya Elimu Tanzania (Kiswahili translation of The Tanzania Institute of Education')
TIE	Tanzania Institute of Education
OUT	Open University of Tanzania
WEMU	Wizara ya Elimu na Mafunzo ya Ufundi (Kiswahili translation of The Ministry of Education and Vocational Training).

1 Introduction

1.1 Background, motives and aim of the study

This study addresses the question of teacher educators' conceptions of mathematics teacher education (MTE) in teacher colleges in Tanzania. The research task has been inspired by criticisms emerging from teacher educators, consultants' reports, and teachers and curriculum developers that there are signals of a serious conceptual drift between mathematics subject matter and pedagogical knowledge in the present teacher education curriculum (Mwaluko, Makundi, Gaula & Lindugani, 2009; Mrimi, 2005; Mushi, Penny, Sumra, Mhina & Barasa, 2004; Townsend & Townsend, 2009). The conceptual drift almost splits those concerned with teacher education into two opposing distinct groups. Critical ideas have been given in support of mathematics content or subject matter emphasis. Opponents of this view have been seeking ways and means to enhance pedagogical knowledge for teaching. This dichotomy is partly seen in papers calling for relevant education for science and mathematics by Osaki (2009), and in important ways by Mosha, Omari and Katabaro's (2007) critical reflections on gaps in teacher education. Furthermore, and in a more specific way, Wort, Hardman and Mmbando (2008) discuss concerns about teacher education in Tanzania. These concerns will be addressed further.

Before this is done, it seems natural to set working definitions for key terms used in the preceding paragraph. This is also in the interest of scientific principles of investigation on making sense of a given phenomenon within a certain context for the purpose of further reflection. In view of this, mathematics teacher education (MTE) is a domain (area of knowledge or activity) of practice seen to be complex, and layered with distinct sites, for example pre-service, in-service, primary and secondary (Adler, 2005; Adler, Ball, Krainer, Lin, Novotna, 2005). In a very general way, it is a process of learning to become a better teacher. Next is the term *mathematical knowledge for teaching*. Again, this is taken to mean pedagogical content knowledge for teaching mathematics in accordance with the view of Bass (2005), and in a similar way, Shulman (1986). The notion 'pedagogy' as found in the literature refers to the theory of teaching or study of teaching methods rather than its oversimplification as 'methods of teaching' (Fowler & Fowler, 2001: Hornby, 2000; Simpson & Weiner, 1989). In other circles, including Tanzania, the concept 'pedagogy' is taken to mean 'methods of teaching' and could also be associated with literature originating from the US, where it refers to 'instructional methods' or 'methods of formal education' (Webster's Encyclopaedia, 1989; Gove, 1971).

I highly value meanings coming to light from dictionaries, but they sometimes do not tell the state of a phenomenon on the ground. For this reason, Salvatori (2003), for example, views the term pedagogy as a theory-informed alternative to a mechanical conceptualisation of teaching, teaching commitment in ways that are consistent with the theories of reading, writing and thinking. The process of teaching and learning itself has to take into account the interaction of the teacher, learner and the knowledge they produce together. In view of this, and perhaps in the interest of establishing a working definition of the terms for this study, I would say pedagogy refers to the process of knowing and teaching to avoid falling into the same trap of regarding it as methods of teaching. Furthermore, mathematics knowledge for teaching goes beyond or at least above mathematics subject matter alone. Similarly, the notion '*mathematics subject matter knowledge*' and '*mathematics content*' are to be used interchangeably to mean conceptual and procedural knowledge whose features are procedures, definitions, proofs, principles and rules, to mention but a few. This discussion will be taken a step further in Section 2.2 in the discussion about perspectives of MTE.

As already emphasised, discussions about subject matter knowledge and the pedagogical knowledge in teacher education are in sharp contrast. This appears in a study by Wort, Hardman and Mmbando (2008) about the status of in-service teacher education, which has consequences for classroom work in Tanzania. These researchers found that limited classroom interaction is an issue which needs a solution. Behind the scenes, the discussion is connected to dissatisfaction with students' failure in mathematics at elementary and secondary level as the results of mathematics examinations are very often in the spotlight (NECTA, 2008; O-saki, 2005). This is further compounded by some parents' view that to accept that mathematics education fails students in primary, secondary and teacher education is to deny them the potential benefits inherent in learning mathematics (Mmari, 1992). This concern, which seems to evolve from both subject matter and pedagogical knowledge shortcomings, is now rising to record levels, and the finger points to teaching and learning mathematics in teacher colleges. The sharp contrast is noted more often than not, and the concerns relate to MTE. I will try to shed light on the concerns in the following paragraphs.

Criticism of mathematics teaching in teacher colleges in Tanzania is a concern (MoEVT, 2008). In discussions involving mathematics teacher educators, teachers and curriculum developers, one experiences a strong concern as to whether mathematics subject matter should be placed at the forefront. On the other hand, others want mathematics knowledge for teaching or pedagogy, as they call it, to be emphasized. Concerns among individual teacher educators on the content with regard to the mathematics knowledge for teaching (MKT) debate may reflect a much broader issue yet to be uncovered. I have no simple way of presenting the concerns apart from what has been indicated by a recent in-service review report which critically examined the situation of teacher education in general, and within it MTE attracted considerable attention.

The report made an attempt to expose the situation, though at times it went too far. For example, the recognition teachers are not mere transmitters of knowledge, but have the role of ensuring classroom interactions, as well as being unquestionable sources of information, are passé ideas. Similar comments have contributed to the ongoing debate and are shown in the next statement. This statement, though a general one, reflects a system of teacher education that is problem-ridden and talks about the challenges of introducing pedagogical approaches in teaching and learning to other school subjects.

> *"There is much talk of and some evidence of introducing pedagogical philosophies, principles and methodologies in the Tanzania classroom. In the main, the ideologies are very much context bound and require that conditions and resources are available to support the teacher. The gap between the sorts of rhetoric embedded in these ideals and the present levels of teachers and the difficult teaching and learning contexts they find themselves in is very large in many circumstances".* (Wort, Mmbando & Hardman, 2008, p.6).

Wort, Mmbando and Hardman (2008) cast doubts on the emphasis of pedagogical knowledge principles given the present level of teachers' ability, and the difficulties of teaching and learning contexts to make it happen.

The second concern revealing the problematic situation suggests again that pedagogical knowledge and subject content remain the critical elements or determinant factors to some, as the argument goes:

> "*We support the idea that good (quality) teaching methods have a significant positive impact on what students learn. In this respect, teachers' professional knowledge, competencies and skills are some of the most important in-school factors influencing children's learning. What is the view of an effective teacher that can help us make the link between an effective teacher and competences? We rely on the ... five dimensions: knowledge of substantive areas and content; pedagogic skill, including the acquisition and ability to use a repertoire of teaching strategies*". (Wort, Mmbando & Hardman, 2008, p. 30)

It can be seen that in a way the authors are self-contradictory. At one point they seem not to see emphasis on pedagogical knowledge to be working, but at the same time accept pedagogical knowledge as a sustainable solution for quality education. These are two ideas conflicting with each other, proving that teacher education remains a problem. In the final analysis, this has implications for MTE, because the process of preparing a mathematics teacher relates to what happens in schools.

The third concern originates from the Tanzania Institute of Education (TIE) (2007), which raised a number of concerns about the continued dilemma of 'mathematical methods' dictating subject matter, the need to stimulate and provoke thinking, the diffused historical development of mathematics, the need for application of knowledge gained in mathematics, and, of course, MTE, to real-life or teacher work situations, and the weak connection to other disciplines. In addition, there has been a call to action to make mathematics, especially in teacher education, a lively area of research and development. This view is related to some of the latest ideas about strategies for enhancing teacher education through research, is asserted by Jakku-Sihvonen and Niemi (2006).

The fourth concern has been sourced from within the Ministry of Education and Vocational Training (MoEVT) and encompasses a myriad of other concerns focusing on the way mathematics teachers are prepared. MoEVT quickly reacted to the content vs. pedagogy debate as it was in the public's interest to develop a relevant programme that would resolve the dilemmas surrounding mathematics content and pedagogy, and in particular the continued tension between mathematics subject matter and methods – which one to emphasise more. MoEVT's reaction to the problem is contained in a policy letter quoted below:

> *"The conceived diploma in the education syllabus needs to be relevant (fit for purpose) and facilitate the teaching and learning needs of secondary education. The concerns about subject matter and pedagogy need to be fairly resolved. Combining the best of the conventional and modern pedagogical approaches to form an integrated system of teaching and learning which is geared towards competencies in teacher education is of prime importance".* (MoEVT, June 22nd, 2006).

This policy statement was given in the wake of the public's concern about the diploma in education curriculum needing to be reviewed, which could be argued required critical and reflective thinking, not fairness as stated in the policy letter. The interpretation of the statement is not to throw away the present curriculum, but rather to integrate the best of present practice in teaching and learning mathematics in teacher education programmes. However, as the MoEVT prepared itself for a major review of the curriculum, concerns about MTE kept on surfacing in various forms. Some, like the issue of assessment and trend of admissions of students in teacher education, are considered next.

There is concern about the mismatch between the intentions of teacher education and assessment, as reported in a framework for assessing the new diploma in the education syllabus (TIE, 2008; Townsend & Townsend, 2009). It would seem the assessment system did not meet the intentions of the MTE curriculum. It was expected that assessment would support the 'process of one becoming a teacher' by focusing on the mastery of mathematics knowledge for teaching. Experience showed that this did not happen during the assessment process. Instead, elements of assessing content seem to take the upper hand, not the process of becoming a teacher of mathematics. Of course I am aware that assessing mathematics content is part of the process of making a teacher, because strong pedagogical knowledge needs to be based on sound subject matter knowledge.

Finally, there is clear concern about the declining number of admissions of student teachers opting to train as mathematics teachers in teacher colleges. Behind the admission numbers one may experience that very often there are signs of stigmatisation and a mathematics-avoidance syndrome in teacher education. This situation contradicts one of the fundamental reasons for providing mathematics education in the education system. In many circles, it is argued that the mathematics education curriculum is implemented in order to provide individuals with the prerequisites which may help them to cope with life in the various spheres in which they live: education or occupation, private life, social life or life as a citizen (Niss, 2007). The trends in Tanzania appear to defeat this purpose. For example, with reference to Table 1, the number of admissions of student mathematics teachers for primary schools in Tanzania revealed an interesting pattern between 2003 and 2007. The trend is a general decline except for 2005 when a surge was experienced (NECTA, 2007). This was due to enrolment beyond target, but if converted into percentages it does not give a different picture. Again in 2007, Tanzania had candidates for Zanzibar only. The key message in this case is that of declining admissions, which is an issue for preparing teachers to teach mathematics. Table 1 shows student teacher admissions in four key subjects at certificate level as collated from NECTA.

The next level of mathematics teacher preparation is not promising either. Students who completes 'Advanced' level' of secondary education and opt for mathematics as one of their teaching subjects revealed an interesting pattern. Table 2 represents student teacher admissions to MTE at diploma level. The rapidly declining admission of prospective mathematics teachers raises more questions than answers. At diploma level, it has been reported that the pool of potential candidates to train as mathematics teachers has shrunk drastically (MoEVT, 2008).

Table 1. Declining student teacher admissions in mathematics teacher education at certificate level

Teaching subject (certificate)	2003	2004	2005	2006	2007
Mathematics methods	*7,960*	*5,697*	*7,146*	*3,001*	*198*
English teaching methods	12,342	7,686	9,787	4,210	718
Science teaching methods	5,308	3,700	5,426	2,193	196
Kiswahili teaching methods	11,307	7,071	9,299	4,272	522
Total admissions of students	36,917	24,154	31,658	13,676	1,634

Source: Collated from the National Examination Council of Tanzania (2003-2007)

The counts from Table 1 and Table 2 indicate two trends, the first being a unique decline of admissions of student teachers opting to study mathematics at the level of certificate teacher education, and the second an oscillating trend in admissions of student teachers choosing mathematics as their teaching subject in lower secondary schools. With no straightforward explanations for either the decline or the oscillating trend, some serious questions need to be asked. What does it mean to have a decline in the admission of mathematics student teachers? What has gone wrong to produce such a rapid decline? What is the difference between mathematics methods and the mathematics we know? What possible strategies might reverse the situation? A series of questions arises out of the interpretation of Tables 1 and 2, particularly concerning what lies behind the number of students who choose mathematics as their teaching subject. The first three questions may reveal avoidance or something else, with the third reflecting thoughts about a possible solution. Other questions could also be posed, such as why discuss 'mathematics methods' only, while other subjects (English teaching methods, Science teaching methods, and Kiswahili) show a similarly declining trend? One possible explanation is based on the connection between science and mathematics and personal experience. In recognition of this question, Macrae and Nessoro (2006) in important ways have shown some initiatives in enhancing the teaching of mathematics content in secondary education, and this potentially has a bearing on teacher education, not to mention learning mathematics in teacher colleges.

Table 2. Student teacher admissions to mathematics teacher education at diploma level

Teaching subject (diploma)	2003	2004	2005	2006	2007
Mathematics teaching methods	*425*	*398*	*371*	*476*	*568*
English teaching methods	845	794	955	1,138	1,067
Physics teaching methods	319	301	320	355	434
Kiswahili teaching methods	639	630	1,138	1,138	1,182
	2,228	2,123	2,784	3,107	3,251

Source: Collated from the National Examination Council of Tanzania (2003-2007)

In view of the problematic situation of MTE discussed in the previous paragraphs, dialogue within teacher colleges has intensified greatly. Such a situation with its inherent dilemmas would normally generate questions. I visited some teacher colleges during this time as part of the build-up to this study. One question was what is the next step? What is the appropriate programme for preparing mathematics teachers that could be agreed on by most teacher educators? The solution was not easy to come by, and perhaps a comprehensive one is visualised in Mosha, Omari and Katabaro (2007) in the development and eventual implementation of The Teacher Development and Management Strategy (TDMS). The TDMS appears to single out mathematics, English, and the sciences for serious consideration because they are badly affected in terms of what is taught and learnt. Before coming to the motives for this study, I think it important to look how this problem reflects itself at global level as well.

How, then, does this problem manifest itself globally? It tends to have similar features with differences only in focus, and at the global level, the concerns about MTE are equally great. Fou-Lai and Cooney (2001), and in the same way Adler et al (2005), have worked hard to emphasise MTE as a domain of practice apart from the conventional focus on the teaching and learning of mathematics. This is in sharp contrast to the discussions based on relative weight between pedagogical and subject matter knowledge done at the local level in Tanzania. Within mathematics education, from which MTE stems, much research has been done indicating its specific nature. From this viewpoint, mathematics education is considered as a domain with a focus on research as well as teaching and learning (Adda, 1998; Bass, 2005; Ernest, 1998; Gjone, 1998; Johansson, 1998; Lerman, 2001; Lerman, 1998; Lester & Lambdin, 1999; Mura, 1998; Niss, 2007; Presmeg, 1998; Wittmann, 1998 & Sfard, 1998). Although the two domains of mathematics education and MTE are seen as different in terms of focus, they are obviously closely related in important ways because of their underlying common denominators of teaching and learning mathematics and seeking to establish themselves as fields of study. This is more so for MTE, as mathematics education seems to stand on firm ground already. The relationship between mathematics education and MTE, research in the respective domains, and a look at didactics will be discussed further in Section 2.2 of the theoretical framework. It may be imperative to say that, while the discussion of MTE at the local level is anchored around the pedagogy-content divide, at the global level the discussion takes on the dimension of research, as highlighted above.

The second global concern is the number of studies that have been performed, for example in school mathematics and at present in mathematics education. a great deal of these are rarely very specific on the 'process of one becoming' a mathematics teacher to use the words of Garcia, Sanchez, Escudero & Linares, 2006. It is for this reason that Lerman (2001) asserts that research on MTE has only begun to grow substantially in the past ten to twenty years. More importantly, even the theoretical frameworks used do not yet demonstrate coherence and, one may add, have relied on the mathematics education way of seeing things. The tendency towards research is yet to show itself explicitly at the local level and perhaps there is a need to create interest.

Between these various lines of thinking about MTE, both locally and globally, I am of the view that different ideas exist that represent educators' experience of this phenomenon. They seem to have views, thoughts, impressions and perspectives on 'what is' and 'what it means' to become a mathematics teacher. The current state of affairs in terms of teaching and learning mathematics among teacher educators in Tanzania gives only a glimpse of the problem, which is characterised by different views, understanding and thoughts, as shown in the previous discussion. It is the concerns, especially at teacher educator level, about the different ways of seeing MTE and the problems associated with it which motivated me to investigate the problem in greater depth, as elaborated below.

Motives for the study

In the light of the background, the choice of this topic was based on three motives, the first of which was the problematic views of MTE, and in particular the content-pedagogy divide. The second motive was connected to a lack of action, which hinders research-based knowledge regarding MTE. The third motive was my desire to reflect more on personal experience and that of educators in connection with the research topic. To some extent, these motives have been dealt with in the background discussion. In this part, I intend to organise them into four areas, and in the process to highlight the main features and the relevance of the study before articulating its objectives.

The concern about the content-pedagogy divide of MTE has been drawn mainly from within Tanzania, although none of the reports and studies cited from within Tanzania indicated the main features even of what was debated. For example, what does emphasising subject matter or pedagogical content knowledge mean? Further, I would like to raise two more questions regarding the content-pedagogy debate, to guide the discussion of motives of this study and as an extension to the previous question: namely, what has resulted from the continued two-sided debate about content and pedagogy, and what are the main features of the notions being discussed? The answers to these can be multidirectional, but let us stay within the mainstream idea of subject matter and pedagogy in teacher education.

The background has indicated different interpretations and variations in focus, leaving alone contradicting views and impressions of each notion. This echoes a related argument by Shulman (1986), who indicated a concern that the teaching of content has rarely been given serious attention. Westbury, Hopmann and Riquarts (2000) took this debate a step further by indicating that if the teaching of content that is expected to bind everything together is not given attention, the likely result would be the drifting apart of subject matter and pedagogy and the two would become separate fields. The second question raised earlier is equally important, for it demands that the main features of content and pedagogy are brought to light as they relate to MTE. Although it is difficult to have a clear picture of this because of the differences in focus, the next paragraph is a summary of features of sound teacher and teacher educator knowledge of mathematics. Again, I might run the risk of combining teachers and teacher educators because of their many sided similarities and differences. This is apart from the different roles played by teacher educators in teaching about teaching, and learning about teaching. But it is also important to note that teachers, too, have to teach and continuously keep on learning mathematics in order to attain proficiency. To summarise this, in a specific way Adler et al (2005) in their global survey indicate how most teacher education research is conducted by teacher educators studying the teachers with whom they are working. This gives a possibility of shared conceptions. In Rwanda, for example, it is reported that to be a teacher educator you need to have the background of being a master or model teacher in elementary or secondary education (Massenga, Samataba & Chediel, 2009). Thus, model teachers are sometimes regarded as teacher educators in the 'waiting' or in the 'making'.

What are the main features of the knowledge bases of teachers? In a very specific way, Adler et al (2005) also raises a similar question on what teachers need to know, and how to facilitate in order to enhance the quality of instruction. These questions are important in order at least to infer the knowledge base of teacher educators and teachers on the basis of their close relationship. It is also important to note that mathematics knowledge for teaching goes beyond mathematics. On this question, Shulman (1986), for example, emphasised three main features, which are: mastery of subject matter, pedagogical content knowledge (PCK) and general pedagogy. A few years later, and with what one may call the eye of a philosopher of mathematics, Ernest (1986) advocated four features, namely: knowledge of procedures, the history of mathematics, links with other subjects, and knowledge about mathematics. It is possible to see the commonality and the differences between the two. Shulman appeared to emphasise the perspective of general pedagogy, while Ernest's emphasis leans towards mathematics philosophy. Attorps (2006), adapting from the work of Shulman (1986) and Ernest (1986), lists eight features, which are: knowledge of substantive structures, knowledge of content, knowledge of mathematics principles, proofs and rules, knowledge of students' understanding, knowledge of curricula, knowledge of content, and, finally, knowledge of the organisation of learning.

In comparison, Kalder (2007) advocates a deep understanding of the structures of mathematics, how basic mathematics knowledge is developed and how subject matter is mastered. Even with a quick scan of these features it can be seen that mastery of subject matter stands out. In a way, many of the additional features seem to only shape Ernest's earlier view points. Other scholars seem to support pedagogy as an important ingredient in MTE. I have nothing against any preference, but I wish to have a clear picture of the pedagogy-content divide. The discussion about the two notions by teacher educators, TIE and even MoEVT did not deal with the heart of the issues being discussed, but ended up being a superficial tug-of-war between pedagogy and content. My main observation is the lack of a meeting point and the continued pulling apart of fields which otherwise could have been working together. In view of all these, and given the discussions on the pedagogy-subject matter divide at local level, one gets the impression that the notions and the aspects which build them are not sufficiently clear.

The other motive, which arises from the first, stems from the lack of research-based knowledge about MTE, taking into consideration the very specific conditions pertaining to Tanzania. At a global level it has been reported that research on the processes of MTE was just picking up (Adler, 2005), in contrast to research in mathematics focussing on teacher beliefs, knowledge, practices and biographies, and expert-novice comparisons. As MTE picks up momentum in terms of more researchers coming in, mathematics education research has been in focus as researchers continuously work to establish its identity (Fou-Lai & Cooney, 2001; Garcia, Sanchez, Escudero & Linares, 2006; Lerman, 2001). Against this background I found it important to create and shape a study relevant to these conditions. What does the process of becoming a teacher entail? It is difficult to answer, but in a broad sense it refers to the initiation into a certain status. There have been many studies of this nature on mathematics and mathematics education, but less effort in specific fields like MTE, despite being the seat of 'teachers of teachers'.

Furthermore, Adler and Davis (2006), as well as Roschelle et al (2008), respectively worked on two similar themes regarding how much, and what kind of, mathematics for teaching is appropriate for teachers, and what mathematics is worth knowing as a foundation for success in mathematics education. I find this an ideal opportunity to reflect on these themes and find out more for the purpose of being informed. On these grounds, Lerman (2001), for example, likens the status of research in MTE to a blank sheet, or Adler's (2005) black box. To substantiate this, Adler's (2005) discussion of research and MTE in the ten years of the Southern Africa Association of Research in Mathematics, Science and Technology Education (SAARMSTE) is worth noting. The discussion is both general and specific. General in the sense that is focuses on teacher education as a broad field, and specific to mean a careful talk on MTE in relation to international trends guided by what is visible, what remains in shadow and the challenges ahead for MTE. In the interest of what is said to be in shadow, Adler (2005) brings to the surface issues of mathematics in teacher education, and the under-researching of pre-service teacher education with a strong implication for MTE. The 'black box' notion signifies the absence of rigorous research to inform who is doing what in the context of SAARMSTE, and in my view Tanzania is not unique in having a similar situation of absence of notable research in MTE. This is central and an important motive in this study.

The third motive derives from personal interest and my own experience. I started as a lower secondary mathematics teacher in 1976, after which I became a mathematics teacher educator at certificate and diploma level for about ten years. By the time of being involved in this study, I had tasks involving teacher development as part of my duties and responsibilities. I primarily consider myself a mathematics teacher educator because in the final analysis I am constantly in contact with other professionals on matters relating to MTE. It was natural for me to look for an opportunity to critically reflect on my experience through this study situated in the field of pedagogy, and more specifically subject didactics concerned with the selection of content, how to teach the selected content, and reasons behind the processes. As seen in the beginning of this chapter, pedagogy is not restricted to methods of teaching mathematics only, as claimed by some teacher educators. Pedagogy covers, for example, what mathematics teacher educators do, and their work as built around teaching about teaching, and learning about teaching (Loughran, 2006). Important features closely associated with what is debated also have an influence on the understanding of the learners, use of strategies that work, and knowing about the learning environment, its organisation, as well as the curriculum (Attorps, 2006). It is equally important to discuss the features of subject matter which basically refer to conceptual and procedural knowledge. Why is it important to explore both sides of the content-pedagogy divide as voiced by teacher educators? To some extent I am of the view that downplaying either one is likely to pull apart subject matter and pedagogy, and the two, using Westbury's (2000) words, are likely to fall into two separate fields. Therefore, locating their meeting point becomes difficult. The other reason is a matter of logic in that a strong mathematics teacher educators' pedagogical base needs to be built on a sound subject matter base. Therefore, the debated notions of MTE, together with their related features, has served as a source of motivation in this study and are to be discussed in the theoretical framework.

The general aim of the study

The foregoing discussion of the research motives shed light on the contradictory views of the pedagogy-content divide, lack of research-based knowledge and the need to critically reflect on MTE in Tanzania. In view of the background and motives, the aim of this study is to identify teacher educators' conceptions of MTE in Tanzania and to describe the variations in these conceptions. In addition, this study aims at identifying and describing teacher educators' thoughts on the development of MTE, which are based on teacher educators' experiences and are expected to involve different dimensions. The thoughts are expected to form a rich source for a discussion on options for the development of MTE in Tanzania.

In my view, this study has some practical relevance, and for this reason, I take the opportunity to suggest a few possible applications. First, it is my expectation that the results of this study on conceptions of MTE will provide a better understanding of the challenges and various means of teaching and learning mathematics in elementary and secondary education as well as in teacher education. Next, there are possibilities to inform other stakeholders, for example teacher educators, student teachers and curriculum developers. Furthermore, the results of this study might be useful in MTE professional development programmes, especially when designing pre-service and in-service courses for primary and secondary school teachers. Also in important ways, the results of this study will hopefully shed light on possible strategies for solving problems related to ‘shrinking interest’, ‘low esteem’, ‘mathematics-avoidance syndrome’, and negative attitude in mathematics. Finally, apart from the practical applications, the study has the potential to contribute to the theoretical knowledge base for the enhancement of practice in teacher education.

1.2 Thesis outline

Against the background, motives and aim of the study, this section outlines the content of the seven chapters. Chapter 2 is an attempt to position the study within a theoretical framework. In this chapter I strive to develop a sense of MTE as having many faces, shedding light on how teachers and teacher educators have been negotiating the meaning of MTE in a context characterised by changing focuses, trends and purposes, and to analyse the different perspectives of MTE as well as teacher educator professionalism and assessment in MTE.

Chapter 3 deals with the methodological solutions of the research. Guided by the two research questions and based on the nature of the study, a qualitative approach is taken, using phenomenography as the research approach. So as not to lose touch with the phenomenographic principle of studying a phenomenon from a second-order perspective, the study focused on views, ideas, and impressions. Principles that guided the selection of subjects, and the coding and analysis of the research materials are explained.

In Chapter 4, the results are presented and described in terms of categories of descriptions as well as related aspects to define the same. The three category systems under discussion are conceptions of MTE, thoughts on development of MTE, and strategies for sharing knowledge and skills in MTE. A category system is taken to mean a set of categories reflecting the same phenomenon. Where appropriate, a connection between the results and findings in other studies is pointed out.

Chapter 5 is a critical reflection on the methodology. The qualitative-quantitative divide is briefly discussed as a basis for working with numbers in a few cases, together with overall remarks. This chapter also analyses how the issues of validity and reliability of the research results have been addressed. Chapter 6 is a detailed discussion of the research results, focusing on teacher educators’ conceptions of MTE, thoughts on the development of MTE, and strategies for sharing knowledge and skills. An attempt is made to connect the research results and similar new theories in both Chapter 5 and Chapter 6.

Chapter 7 concludes the study, beginning with a reminder of the research tasks and highlighting the research results in term of categories of descriptions. On the basis of the findings and review of literature, a framework for a description of the subject of interest MTE is suggested for pulling together the three category systems and their corresponding categories of descriptions. The chapter also suggests possible applications of the research results and raises searching questions which may serve as the basis of new areas for further research in MTE.

2 The many faces of mathematics teacher education

In the light of the background, motives and aim of the study as articulated in Chapter 1, in this chapter I review the literature relevant to MTE. As a point of departure, a brief description of the teacher education set-up in Tanzania is made before I look into how teacher educators have been negotiating the meaning of MTE over time (chronological order). This process reveals a situation characterised by changing focuses and purposes of MTE in Tanzania. The chronological treatment of educational events as defined by the changing focuses is considered important because of the possibility of unearthing the meaning assigned to the subject under study by those involved in actual teacher preparation. The second task in the literature review focuses on perspectives of MTE in an attempt to make sense of it by taking advantage of the close relationship between mathematics education and related frameworks. The intention has been to scan the literature in order to inform the study on what has been covered in this area of study, and what the gaps are. The third task involves presentation of the important processes following initial teacher education, bearing in mind the question of teacher educator professional development. Professional development has been considered a central issue because MTE in all respects is a process and not a 'once-and-for-all activity' done during initial teacher education only. With that in mind, these themes will be discussed further, one at a time, because they form a cornerstone to my topic of study.

2.1 The changing focus of MTE in Tanzania

I find it important to at least present the context of teacher education on which this discussion on the changing focuses is based. It is for this reason that a brief description of the teacher education set-up in Tanzania is essential. The term 'teacher education' in Tanzania is very often taken for granted. This is exemplified by Kiondo's (2002) report on 'Teacher management and support in Tanzania: An annotated bibliography of teacher education (1985-2000). In such a span of time one expects at least a discussion along this line, but unfortunately it is not provided to a sufficient level. An implied discussion on what is teacher education in Tanzania in terms of types, modes, approaches, policies, content, assessment, duration, certification and accreditation is found in Höjlund, Mtana and Mhando (2001). My task here is to give a general description of the set-up of teacher education in Tanzania. I will therefore briefly highlight the meaning, purpose and types of teacher education in the context of Tanzania. The notion 'types of teacher education' is similar to what Adler (2005) call layers of teacher education. Next is the teacher education entry qualification policy followed by content or the implemented curriculum. It is also important to talk about teacher education training approaches or modalities, assessment and finally certification and accreditation of teacher education in Tanzania.

To begin, Höjlund, Mtana and Mhando (2001) seem to view teacher education as a programme preparation process which leads someone to qualify as a teacher or a better teacher. This view of teacher education is shared by a number of official documents, including the recent Education and Training Policy (forthcoming). The objectives of teacher education do not need repeating in detail, but I will present them in summary. With reference to the education and training policy (forthcoming) and MoEVT (2008) teacher development and management strategy, teacher education objectives in Tanzania are built around the development of pre-service and in-service teacher education (professional development), and strong subject matter mastery and development of sound pedagogical knowledge and skills. There is also a specific emphasis on numerical literacy and language of instruction to those in the process of becoming teachers, the ability to respond to children with special needs, emerging global issues, for example HIV/AIDs, and environmental education. Additionally, a strong sense of work ethics is emphasised.

In view of these objectives, two main types of teacher education are implemented by a system of teacher colleges and universities, namely pre-service and in-service teacher education, elsewhere known as preparation and teacher development (Adler, 2005). Pre-service teacher education consists of three levels. The first level is certificate teacher education, where the entry qualification to a teacher training college is a successful pass at Ordinary level secondary education. The course is full time, lasts for two years and graduates are posted to teach in elementary schools. The next level within pre-service education is the diploma in teacher education, and the entry qualification to a full time course in a teacher training college is a pass at Advanced level secondary education. The course duration is two years, full time, and graduates would normally then teach in secondary schools. Both levels of teacher education are accredited by the NECTA. Furthermore, unlike certificate and diploma teacher education courses, bachelor's degrees are offered, accredited by universities and form the third level of teacher education in Tanzania. Admission is based on a high pass grade at Advanced level of secondary education and student teachers would normally spend at least three years on the course. Upon successful completion they would normally teach in higher classes of secondary education. These levels, plus the possible route between levels, are shown in Figure 2.

In-service teacher education as a process of continuing to learn after qualification is yet to be systematised. Its main aim is to enhance both teachers and teacher educators' professional growth, depending on the need of each group. There are issues around in-service teacher education as most of the programmes are top-down, and very often do not meet the needs of teachers and teacher educators.

What are the modalities or approaches in terms of offering teacher education courses? What is the course content? And what is the procedure of assessment? I will address each of these in turn. Teacher education at certificate and diploma level is conducted by teacher colleges and is meant to prepare teachers for elementary education. Teacher educators (sometimes called tutors) are responsible for facilitation. University teacher education is meant to prepare teachers for Ordinary level secondary education and high schools. Except in times of high demand for teachers, for example during implementation of programmes aiming at basic education for all, teacher education at certificate and diploma level are full time and college-based. However, reforms like the implementation of The Primary Education Development Plan (2002-2006) and the Secondary Education Development Plan (2004-2009) made it necessary to think of a two-tier approach. The two-tier teacher education approach made student teachers spend one full year in the college learning mostly the theoretical part of teacher education. During the second year student teachers are exposed to a long teaching practice for the purpose of putting into practice what they have learnt as theories. The advantage of this approach is mainly quantitative, as twice the number of teachers would be graduating each year. The qualitative part has been questioned in terms of continuous monitoring of student teachers in the schools they are teaching. This is not to mention mentors in schools for the professional guidance of novice teachers.

The issue of teacher education content in Tanzania is at least worth highlighting. At one time, Craig, Kraft and Plessis (1998) emphasised strongly that teacher education can only make a difference depending on the education programme and the support that is put in place. This was said with respect to relevant content. Depending on the levels, teacher education curriculum design and development seem to ensure appropriate teaching subjects and professional content as well as undertaking teaching practice. The structure of the teacher education curriculum across levels appears to be similar except for its depth and how it is conducted. It is possible to structure the curriculum content in three areas. The first area consists of professional studies, which include psychology, guidance and counselling, curriculum and teaching, research measurement and evaluation and foundations of education. The second area constitutes specific teaching subjects, for example mathematics, physics, chemistry, history, geography, information communication technology and development studies, to mention but a few. The third area is teaching practice, and the student teacher is expected to demonstrate the pedagogical knowledge and skills gained in the classroom. The teacher education curriculum is not free from challenges. There are issues regarding the frequent change of time student teachers spend in teacher colleges, compromising subject matter knowledge by favouring teaching methods, fluency and language of instruction, teaching and learning materials, assessing student progress, limited research to inform practice (MoEVT, 2006), to cite only a few issues.

I would like to take on the issue of assessment in teacher education. Across levels, teacher education is assessed in a similar way. At certificate and diploma levels assessment is done mainly in three ways. The first is continuous assessment, which consists of assignments, projects, tests, mid-term and end of year examinations. Next (not in terms of importance) is teaching practice. At the end of the two year course student teachers sit a final examination administered by the National Examination Council of Tanzania. It is the same council which accredits candidates. In all cases, student teachers need to meet certain minimum conditions in order to pass. The minimum conditions are subject to review from time to time. However, a pass in teaching practice and teaching subjects is a necessary condition for qualifying. At bachelors level the procedures might seem different, but the structure take a similar shape as student do course work, tests, examinations and teaching practice. In all cases the weighting between coursework, assignments, and final examinations is the same. Given the objective of this study, it may not be relevant to go beyond this point. Despite this situation, assessment of teacher education is not without shortcomings. There are concerns about the preference for items testing comprehension, application as opposed to analysis, problem-solving, writing project papers and small scale research.

In summary, the context of teacher education (in Tanzania) I have just described touches types or modes of teacher education, approaches, entry qualification, content, assessment and accreditation. According to MoEVT (2009), the teacher education curriculum is conducted in what I would say a system of teacher colleges owned by either the government or private institutions. At the time of writing this report there were respectively 34 government and 43 non-government or privately-owned teacher colleges Only a portion of universities are engaged in running teacher education programmes out of the 32 registered by the Tanzania Commission of Universities (8 government, 5 private). More private universities are opting for teacher education programmes because students opting for teacher education courses receive priority in the Higher Education Loan Board (HESLB). Teacher colleges train teachers for primary (elementary) and lower secondary education and offer in-service courses. Government policy requires teacher colleges to be staffed by teacher educators, with at least a university degree. However, this requirement has remained a distant objective, as, until recently, only sixty five out of one hundred teacher educators are university graduates (MoEVT, 2009). On the other hand, universities prepare teachers for higher classes of Ordinary and Advanced level education. Like any conventional university one sees tutorial assistants, lecturers and professors in the universities´ profiles. Having made a description of the context of teacher education in Tanzania, I will now delve into the changing focuses and purposes of MTE.

Chapter 1 revealed the intense debate on pedagogy vs. subject matter among teacher educators, curriculum developers and those in the community of teacher education over time. This part of the theoretical framework is a broad response to what is important to this thesis, following many questions which arose in the course of the discussion. It covers first the chronological development of MTE from the 1960s to 2000 and beyond. The year 1960 is selected for two main reasons, namely to make the study manageable with regard to time and, in the context of the study, the early 1960s is associated with a number of curricular reforms, which included mathematics, and it is known from experience that these have a major bearing on the preparation of mathematics teachers. The choice of the theme also enabled me to trace and reflect on how the meaning of MTE has been negotiated over time. Viewing mathematics education and for that matter MTE from a historical perspective is not unique to Tanzania. Ernest (1991), for example, argues for two distinct categories of mathematics from a historical point of view. The first category views mathematics as a compilation of objectives and absolute knowledge that is not subject to historical or social influences. This view supports the objectivity of mathematics with respect to teacher education. The second category views mathematics and the advances of scientific management as a process of continuing to learn within a social construct and historical framework and rejects the perpetual nature of absolute knowledge. It is on the basis of this divided thinking that I decided to discuss the changing focuses and emerging issues in order to see the prevalence of perspectives or faces of MTE.

It is challenging to locate a dividing line between teachers and teacher educators in the process of making sense of teacher education, and it sounds logical to extend this to MTE. More often than not, teacher educators had entered the field without formal training ((Lunenberg & Willimse, 2006; MoEVT, 2009). Viewed in this light, I have tried where applicable to use research results related to mathematics teachers in order to argue for teacher educators based on their initial experience as teachers. I have come to learn that 'teacher educators', 'teachers who teach teachers' (Lunenberg & Willimse, 2006; Loughran & Russel, 1997) or 'mwalimu wa walimu' in the national language of Tanzania (Kiswahili), entered the field without any formal training. According to Lunenberg & Willimse (2006), and in my personal experience, teacher educators entered the field on the basis of two criteria. First, they have been model teachers, and secondly they have been experts in some field. In Tanzania, for example, the condition for teaching in teacher colleges has been an adequate experience of teaching in either elementary or secondary school. At present this condition is gradually being eroded and there is flexibility to move either way. This situation gives a possibility for mathematics teacher educators to serve two purposes, given the nature of what they do. In one situation they are teacher educators, but teachers in another, and they carry with them the experience of either side. In addition, some might be aware of the natural relationship between teachers and teacher educators - that of learning from each other, and especially when teacher educators serve as models or are exemplary teachers. Viewed in this light, MTE in Tanzania and elsewhere has survived the changing of focus, trends, shifts and turning points. In this study the notion of trends and shifts is respectively taken to mean a general change of direction, concentration or mode, and change from one position to another (not necessarily a qualitative change), while the term 'turning point' refers to the time when an important change took place (Hornby, 2000; Waite, 2001). These thoughts, especially the first two, are closely related and may generally mean a course, while turning point refers to the time of an important happening.

The history of education in Tanzania seems to confirm that mathematics, and hence the education of mathematics teachers, is as old as teacher education itself (Mmari, 1980; Anger & Haule, 1977). The training of mathematics teachers as it was known then is equivalent to today's MTE, perhaps separated only by history. This makes sense if we do not question the different names and perspectives held by teachers and teacher educators during the changing focuses of MTE. During the preparation of this study, I had a discussion with two long-serving teacher educators. The first is a retired teacher/teacher educator by the name M (a given name), who trained between 1960 and 1962 and served as a middle school teacher and a teacher educator in the mid 1960s before going through other ranks of teacher education. From her library corner, the now retired teacher and teacher educator pulled out an old book stamped 'St Mary's Ndwika', which was used as a teacher education textbook by a number of teacher colleges during that particular time. Out of curiosity, and as a member of staff in that college fifteen years ago, and given my present research task, I immediately scanned the book. The ten important points for teachers to master appeared simple and straight forward probably for student teachers of that time, these were: ‘a good teacher does not remain always seated at his desk’; ‘a good teacher makes few rules and is positive’; and ‘a good teacher never talks when he is facing the blackboard’ (Byrne, 1953), to mention just the first three. Further reflection on the ‘points to master’ would today look like a ‘teacher education gospel’. The ‘tenth point to master’ aroused interest as it relates to this study. It required student teachers to recognise that principles and methods of teaching, subject-matter, and physical resources depend on the needs of the learner, and in that order. The subject matter vs. methods question is again emphasised in the ten points and it would be difficult to challenge the emphasis outright because of its centrality.

The second retired teacher and teacher educator was trained in Morogoro at about the same time as the first upon completion of lower secondary education at Tosamaganga (Standard ten as it was known then). His description of his teacher education experience was straightforward and covered the subjects studied. These were ‘methods of teaching - general’; child psychology; English and methods of teaching English; methods of teaching mathematics; and methods of teaching science apart from the same in history, geography and Kiswahili. The methods part involved aspects like lesson preparation and lesson notes and a four week teaching practice in actual classrooms. In this analysis, it may be important to ask the question of where the content has gone in other disciplines except for English. The response based on this retired teacher/ teacher educator was that for mathematics and science the subject matter was assumed from their previous level of secondary education. But the interplay between the two was very high, such that the cleavage seen today was not an issue. Figure1 is an attempt to represent in a broad way the ideas narrated by the two teacher educators. The central idea was that learning revolves around a high view of the child, subject matter, and what Byrne (1953) calls principles and methods of teaching, subject matter, and finally the physical and material resources.

The knowledge of subject matter vs. methods of teaching formed the core of the ten points, which may be difficult to challenge. The main features indicating what is ′pedagogy′ and ‘subject matter’ was introduced in Chapter 1. In the interests of shared meaning and open-mindedness, pedagogy in a broad sense, when referring to the teacher/educator knowledge base, implies the process of knowing and teaching and takes into account knowledge of students’ understanding, as well as the curriculum and teaching strategies. On the other hand, knowledge of subject matter or content points to knowing algorithms, procedures, principles, proofs, and so on (Kalder, 2007; Attorps, 2006; Ernest, 1989; Shulman, 1986). In addition, mathematics teacher educators have to go beyond the mathematics we know in the sense that they are ‘teachers of teachers’ and to qualify it more in dealing with teaching about teaching, and learning about teaching (Loughran, 2006). Returning to the previous discussion about the points of emphasis in teacher education, Byrne (1953) strongly believed that the order of important needs in the learning process or education is first the child, then the teacher, followed by the principles and methods of teaching. In addition, teachers' subject matter mastery, and buildings and equipment stood out as important needs of the child. The quotation below highlights the emphasis of the different components of teacher education (see Figure 1).

> *"Above all, the good teacher knows that everything in education revolves around the child. He knows that teacher, principles and methods, subject-matter, buildings and equipment must all depend on the needs of the child, and in order of importance shown in the diagram...".* (Byrne, 1953 p.17)

The order of importance is subject to criticism, but it may be inappropriate to open this up for discussion here. I am interested in showing the value and possible roots in the longstanding discussion of content vs. pedagogy in Tanzania and elsewhere.

On the basis of Figure1, it is assumed that the meaning of MTE depended greatly on teacher educators' differences in opinion about the important points to master. Figure 1 shows the order of importance in which principles and methods of teaching have been ranked higher than subject matter. The dotted lines in between what revolves around the learner, in particular the teacher, principles and methods of teaching (in this case mathematics), subject matter, physical and material resources, is a recognition of their interplay. This is to show that they do not work in isolation, but rather augment each other. Methods of teaching mathematics were possibly the focus or way of seeing MTE – the face of MTE during that time. The educative process which was taking place at St. Mary's Ndwika is just one of many similar examples in teachers colleges. It may be relevant to see a comprehensive picture, which is discussed next, with the 1960s as the starting point of the timeframe.

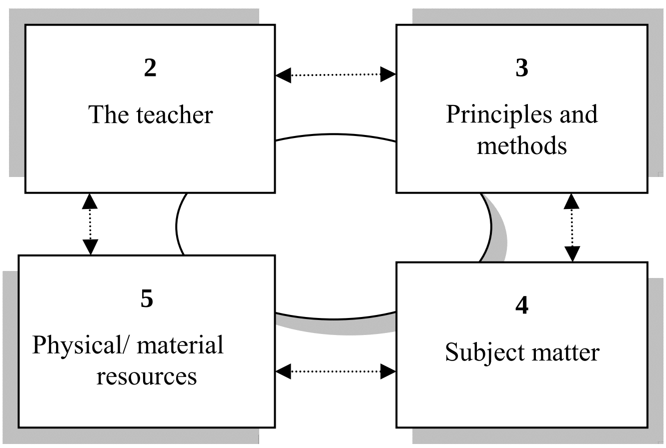

Figure 1. Learning revolves around the child with a focus on methods and subject matter (Adapted from Byrne, 1953 p. 17)

In the 1960s, innovations, interventions and changes not only in MTE but teacher education in general came in numbers. The main reason behind the drastic curriculum changes from 'traditional' to 'modern mathematics' is associated with the Russian launching of the Sputnik in 1957 (Mmari, 1980). In order to differentiate between 'traditional' and 'modern mathematics' one need to look at the role of the teacher or teacher educator in the process of teaching and learning. A sense of teacher-centred approach to teaching and learning is attached to traditional mathematics while modern mathematics was linked to a high consideration of the learner. In this context the term '*traditional mathematics*' is taken to mean the British inherited mathematics curriculum comprising school mathematics and teacher education system. Teachers of mathematics were largely seen as transmitters of knowledge, unquestioned sources, with emphasis on computation and drill work. On the other hand, the term '*modern mathematics*' was seen as a new approach with emphasis on student engagement in the process of teaching and learning not only school mathematics but also appropriate ways of teacher preparation and development. Why a change of focus in teacher education? As already indicated, modern mathematics emerged as a reaction to traditional mathematics, which appeared not to be able to cope with the US space programmes of the 1960s (Howson & Wilson, 1986). Much of what was to be called 'modern mathematics' was exported by the USA to many emerging nations, some of which had no link with the Cold War between the Russians and the USA. One argument is that this happened not because the programmes were superior, but likely based on the capacity of mass publication. Niss (2007) links this trend to the tendency to pay attention to researchers from the US by then. The US was thus very often insular and not open to mathematics answers found in other countries. It was thought that the Americans could effectively counter Russian success through a better mathematics curriculum. Under such unquestioned influence, two projects were initiated in East Africa: one under the influence of the USA and the other of the British, which ultimately engaged teachers and teacher educators in Tanzania in teaching and mathematics teacher preparation respectively (Mmari, 1980).

Under an umbrella project called the American School Mathematics Project, the USA initiated a programme for English-speaking countries, funds for which came largely from USAID and the Ford Foundation. The Entebbe Mathematics Series was a product of this project which was written during the annual summer (an alien definition from northern hemisphere countries) conferences between 1962 and 1968, hence the name Entebbe mathematics, named after the town in Uganda where the first three conferences were held, at which the majority of participants were from Africa, the United States and the United Kingdom. The African participants came from Ghana, Ethiopia, Liberia, Kenya, Malawi, Nigeria, Uganda, and Tanzania. This programme not only produced new materials, to be known as Entebbe mathematics, but was also a turning point in what to teach (content) and how to teach (methods). The British had a similar project in Kenya, and Nairobi hosted writing workshops for English-speaking Africa.

In a situation where mathematics ideas were being shared by teachers and teacher educators of different cultures, the mathematics curriculum in Tanzania was subjected to international culture by aligning itself to the new developments, which were definitely related to teacher education and in particular to teacher educators' tasks. What was specifically new about Entebbe mathematics? According to Sichizya (1997), and Kita (2004), some new topics in school mathematics, which implied the introduction of related teacher preparation and development activities, were set theory, coordinate geometry, probability, vectors, trigonometry, inequalities, matrices, projectiles and plane elevation. Was this relevant to all teachers and teacher educators? I find it difficult to account for the importance of each topic, but perhaps a few. Teachers and teacher educators could find the link between coordinate geometry and geographical positioning (latitude versus longitude), inequalities and linear programming, projectiles and air force technology, to mention a few. Two major implications could be seen immediately, the first being the development of curriculum materials that would match the textbooks that had been developed in Entebbe. The textbooks were produced before the curriculum and syllabus, which may not be a systematic way to process the curriculum. The second issue was that of teacher preparation and in-service teacher education. Teachers met the challenges in the shift of emphasis because transmission was their way of teaching and learning, but adapting to other pedagogical orientations based on scientific inquiry, investigation, and guided-discovery in order to actively engage learners was found to be a difficult area. On the whole, implementation of modern mathematics continued despite the conventional emphasis on commercial arithmetic and computational skills that were built around teacher-centeredness, and which were among the first faces of MTE.

During the 1970s, MTE was still influenced by international borrowing of mathematics ideas. Topics like set theory still dominated the school mathematics curriculum, and had to be related to teaching and learning mathematics in teacher colleges and vice versa. The terms 'modern' or 'new' mathematics are now history and there is little interest in turning to the past among teachers and teacher educators (Sichizya, 1997; Kita, 2004). However, traces of what to emphasize in the teacher knowledge base are still there, especially the pedagogical knowledge associated with modern mathematics (Mmari, 1980). It has been said before that Tanzania, like many other countries, adopted the new mathematics as a result of international influence. In a strict sense, this was another paradigm shift which teachers, teacher educators and student teachers had to negotiate. The conventional way of seeing mathematics as one body of knowledge defined by concepts, rules, algorithms and theories prepared for transmission continued to be seen as the acceptable meaning by some teachers and teacher educators. As argued in the opening chapter, mathematics could be seen as an activity of human interest which is process-focused, analytical, problem-driven, and investigative, and above all one which promotes creativity. Teachers and teacher educators were therefore subjected to two operating perspectives, between which they had to negotiate.

Both the traditional and modern mathematics era experienced a severe shortage of teachers, and a lack of appropriate curriculum materials (Kita, 2004; Mmari, 1992). This is not to mention mathematics teacher educators for preparation and development. For this reason, Mmari (1980), for example, cites irrelevant and out-of-context mathematics problems, such as asking students to calculate the speed of an underground bullet train from point X to Y or questions related to working hours of a coal miner in Newcastle, UK. This was not relevant in the Tanzanian context. To make matters worse, the questions were new to students in schools, student teachers and teacher educators. Student teachers were expected to analyse the secondary mathematics syllabus as part of their MTE programme. TIE influenced the production of most mathematics curriculum materials as well as teacher preparation, as it was mandated to do so by Act No. 13 of 1975 (TIE, 2008). There was little flexibility for mathematics teacher educators to develop their own teaching and learning materials. Recent discussion about multi-textbook policy in Tanzania would have appeared completely out of place at that time.

As an example I experienced modern mathematics from a background of arithmetic, algebra and geometry in middle school. The sudden change to set theory, probability, coordinate geometry, trigonometry, vector analysis and more, accompanied by the lack of capacity of teacher educators to take up new roles, might have contributed to some of the reasons for its failure (Sichizya, 1997; Kita, 2004: Mtandika, 2003). What were the options for addressing this immediate setback? Globally, a basic mathematics curriculum was initiated and developed as a reaction against emerging teaching and learning approaches which were yet to prove viable. In Tanzania the need to contextualise the approach to teaching and learning mathematics and reaction against continued international borrowing was the starting point. This, in turn, called for an appropriate teacher education approach to match the new situation. Mathematics as a discipline continued to be taught separately from educational studies and students were expected to demonstrate a combination of the two in actual classroom teaching. Surprisingly, it was the same USA which spearheaded the back-to-basics idea, as reported by Schoenfeld (2002). Basic mathematics was therefore designed and promoted after the spectacular failure of modern mathematics (Sichizya, 1997). I would also think that strong traditions elsewhere, the resistance to and challenges of adapting to the constructivist-inspired approaches seem to have triggered the turnaround from modern to basic mathematics. The guiding principle in the constructivist approach is the emphasis on the learners' previous knowledge structure and the individual's role in knowledge construction. Teacher education systems used to the unquestionable status of the teacher could find this very challenging. The challenges regarding constructivist approaches in teaching and learning mathematics are highlighted by Treagust, Duit, and Fraser (1996) and Sahlberg (2003). In my view, the difference between modern and basic mathematics is pedagogical and in the role assigned to the teacher or teacher educator and the learner in the teaching and learning process. For example, in a typical problem-solving lesson the teacher educator may present a problem and leave the learner to work independently towards the solution.

In the case of Tanzania, this decade witnessed the introduction of reading, writing and numeracy, especially to teachers who were to teach in the lower grades (MTUU, 1984). MTE was still emphasised and divided into algebra, geometry and arithmetic. Teachers and teacher educators continued to regard computational skills and fixed concepts as almost set into the final form as mathematics – another face of MTE with a difference. Teacher education encouraged this orientation by emphasising the memorisation of formulae, and singing of multiplication tables, as student teachers who emphasised and demonstrated mastery of these skills were considered to be prospective mathematics teachers. Even today there is a circular letter regarding the emphasis on multiplication tables (Mpama, 2002).

MTE in the 1980s up to the start of the 1990s continued to be taught separately as solid mathematics on the one side, and educational studies on the other. Like the previous decade, shortages of teachers were the norm, with mathematics and the natural sciences being hardest hit, which is also reported in the Presidential Commission for Reform of Education in Tanzania, commonly known as the Makweta Commission (1982). During this time, some interventions were made to address not only the shortage of teachers in mathematics and science, but also to enhance mathematics and science education. These interventions deserve recognition, not because of their specific achievements, but because of the role they played in the development of MTE as a process. One example was the science and MTE diploma course at Monduli and Mkwawa teacher college (now Mkwawa University College of Education). This intervention emphasised mathematics content, while professional studies, psychology, guidance and counselling, foundations of education, research measurement and evaluation, taught separately, were expected to focus on the practice of teachers in the classroom. It was assumed that student teachers would link the various aspects of mathematics content with their professional studies. This raised concerns among teacher educators because subject content and pedagogical knowledge were separated within the same teacher education curriculum, and their criticisms in this regard gave rise to other interventions in the late 1990s.

Other measures included a salary increase for mathematics/science teachers as a way to attract and retain them. One may question if the stimulus-response approach was an ideal and sustainable solution. In addition, some measures to promote the teaching and learning of mathematics were proposed and enforced by NECTA for candidates failing mathematics, which involved a penalty in the ranking order of candidates failing mathematics. This had serious implications for MTE, including converting mathematics teachers into drill masters and causing students to fear the subject. In the process, modern mathematics started to give way to basic mathematics with a focus on analytical power, computational skills, learner-centred approach, creativity, and learning through investigation. Back-to-basics was a change of focus and another face of mathematics, and therefore MTE, because teacher education had to relate to what was happening in schools.

The 1990s and 2000s saw the beginning of a new era which was backed by the intentions of teacher educators and curriculum developers to make MTE more inspirational and interesting, promoting active participation and focusing on problem-solving (MoEVT, 1997; TET, 2000). This triggered the curriculum review of the mid-1990s ready for implementation in July, 2000 (TET, 2000). Two important initiatives happened during this time. First, an attempt was made to integrate mathematics content and methods of teaching. Within a short period of implementation, the content of mathematics in the new curriculum was questioned for lacking subject matter, as was the quality of integration (TIE, 2007). During this time, implementation of the MTE curriculum faced a dilemma, and other developments led to a U-turn, which meant abandoning the methods-oriented diploma curriculum in favour of a content-oriented one, spearheaded by the influence of universities in Tanzania, which happened because of the intention of linking non-degree to degree programmes in teacher education (OUT, 2005). However, as the reviewed mathematics curriculum for the diploma was rolled out, methods-oriented ideas persisted and teachers and teacher educators continued to negotiate the terms of meaning. No wonder that a lot of questions about MTE required answering. For example, what were the reasons supporting integration between subject matter and methods? Are the reasons no longer valid? These are difficult questions to tackle at the moment as they require a study of their own. Despite this situation, it has been proposed that mathematics should be 'compulsory' for all student teachers training as primary school teachers and as one of the options for those training to teach at lower secondary level (MoEVT, 2008).

A second initiative during the 1990s to 2000 in MTE was the Korogwe mathematics upgrading programme (named after the host teacher college). The programme was designed to enhance elementary school teachers' pedagogical approaches (Close & Chediel, 1998), and was built around activity-based MTE. The purpose of the intervention was to actively engage learners and inspire them through activities. Although the programme targeted serving teachers, there was a spill-over effect on pre-service, in that it was expected that student teachers would be motivated and the activities would give meaning to the mathematics they had been learning. This was a shift from the conventional educator-centred to a student-centred approach in the process of preparing mathematics teachers. Like the Mkwawa and Monduli interventions discussed in the preceding paragraph, its impact on MTE was not effectively realised and perhaps opened the door for further reflection. Thus, computational skills and the teacher-centred approach, with a small emphasis on analytical power and creativity, remained the dominant modes and faces of MTE.

The period after 2000 until today triggered some of the present differences in how MTE is seen because of the sudden disappearance of subject content in mathematics (see Figure 2 and Figure 3). Teacher educators, teachers and inspectors gradually noticed that mathematics does not end with computational skills. A shift from teaching to learning was considered better solution in learning mathematics. The entire MTE curriculum, like other fields, was reviewed to give pedagogy or methods of teaching and learning higher recognition. Thus, methods of teaching mathematics were brought to the forefront. I often think that the decision by the government to make the methods of teaching a priority in the curriculum resulted in the drifting apart of subject content and methods of teaching mathematics, and fuelled the current debate. The structure of teacher education appeared to link certificate and diploma courses that were detached from university programmes involving teacher education. I need to be clear on this issue. I am not glorifying either of the two levels, as each level might have problems of its own. Figure 2 represents the structure of teacher education at the time of the study with a focus on mathematics teacher education.

Figure 2 also gives details of the major components of MTE at different levels of teacher education programmes as at the time of this study. The dotted lines also indicate areas of current debate. Furthermore, Figure 2 indicates the missing link between teachers at certificate and diploma level and what is happening at the universities, as well as the value of the methods of teaching and learning at the initial stages of teacher education and the emphasis of subject matter at institutions of higher learning. These are perhaps two different worlds of the same teacher education system. The relationship between the four components (teaching practice, professional studies, mathematics teaching methods and optional studies) can be seen from the role of each component and how it can be used in the actual classroom situation.

Teaching practice is a task where the student teacher is expected to demonstrate mastery of mathematics knowledge for teaching and skills and, at the same time, apply what has been learnt in methods of teaching, professional studies, and connection with other fields of study. I am aware of the difficulties of making these meet entirely, and it is a big debate (Hudson, Burberger, Kansanen & Seel, 1999).

There were also parallel and overlapping projects during this time, for example, ‘active learning’, ‘child-centred learning’, ‘diagnostic teaching’, ‘collaborative learning’, and many more. Despite the differences in focus, they raised the same cautionary flag. The message was to involve the child in the process of teaching and learning. In brief, it was the start of initiatives focusing on the constructivist approach, investigation, inspirational learning, and more. For the purpose of this study I will consider the Morogoro-based teacher educator project and a latecomer - inspirational mathematics - for various reasons, as it appeared to be more comprehensive and covered a large number of teacher educators. The inspirational mathematics project, as the name suggests, was expected to revive MTE, which had been challenged by a mathematics-avoidance syndrome, low self-esteem, negative attitude, and phobia among student teachers, which led to a decline in the number of admissions of student teachers opting to train as mathematics teachers.

Records indicate that the Morogoro-based teacher educator project emerged strongly around the year 2000 (Babyegeya, 2005; OUT, 2005). Though not directly related to teaching and learning mathematics, its effects were far-reaching as its teaching approaches were expected to be adopted in the teaching of specific subjects and the programme appeared to focus on a constructivist approach. As is also argued elsewhere by Treagust, Duit and Fraser (1996), a constructivist approach to teaching and learning was expected to address the problem of viewing subject content transmission as a faithful copy of the world outside. The intervention, grounded in a constructivist approach, emphasised students’ own constructions of meaning based on what they already know, the support of teacher educators and interaction between students. Though nothing like constructivist mathematics existed (Sahlberg, 2003), the constructivist perspective attracted the interest of teachers and teacher educators and many tried it and even labelled themselves constructivist. They indicated that they were often anxious to cover all the content because of pressure from examinations and often handed out the final statements and proofs (content) to students, and since students were not prepared for the long process of knowledge construction, they resorted to their usual way of learning - memorisation.

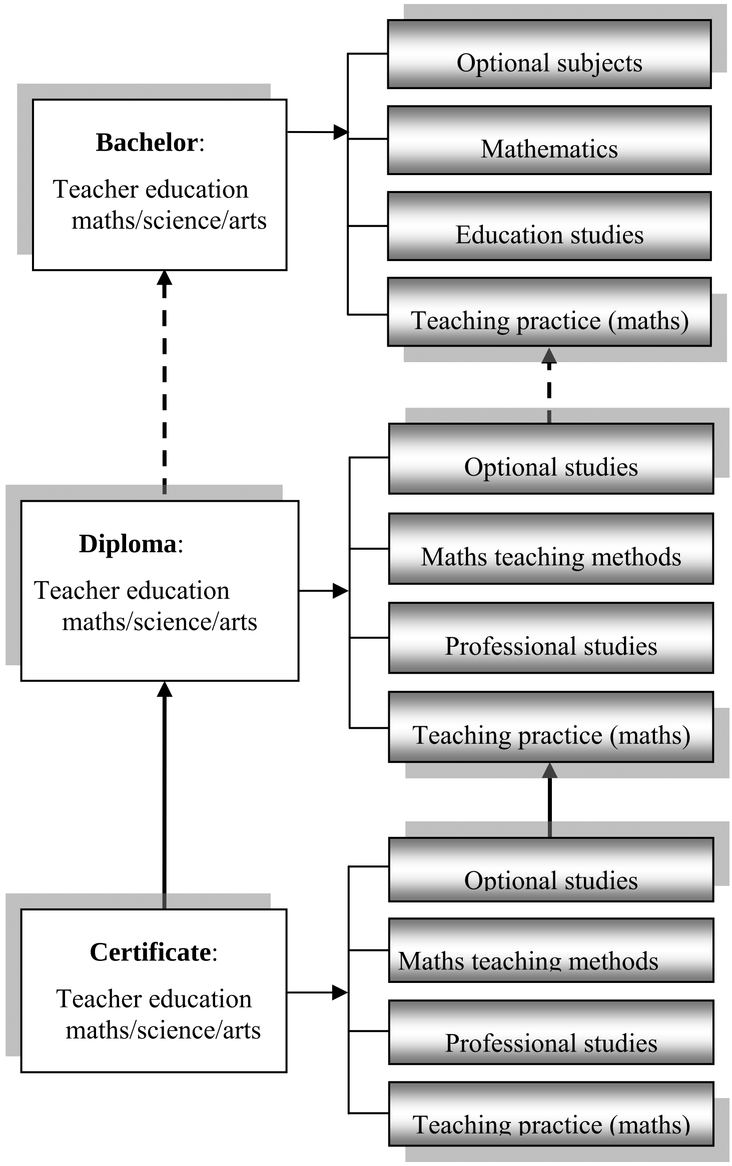

Figure 2. Major components of mathematics teacher education and the missing link

If the constructivist approach was to be practised fully, then teacher educators' beliefs about transmission approaches to teaching and absorption had to be seriously challenged. This again led to subject matter preference rather than methods of teaching and learning. Like other interventions seen before, it sowed the seeds of its own failure because of the demands of examinations to complete the syllabus and the teachers' preference for transmission of knowledge rather than knowledge construction.

The second important intervention after the year 2000 was the inspirational mathematics project for teacher educators undertaken in Tanzania in collaboration with Finset (Finnish support to education in Tanzania). This intervention focused on investigation, practical work and some elements of mathematical modelling. Berry (2003) summarises it by using the generic term problem solving. The intention of inspirational mathematics was to create an interest in mathematics education among prospective teachers and teacher educators. It was expected that this would enable them to shift from educator-centred to learner-centred approaches. Though inspirational mathematics was the preferred idea, it was not mainstreamed in MTE to ensure sustainability as with the previous interventions. In a real sense, I would say teacher educators had their own theories and to enter into learning mathematics through investigation was not in their line of thinking. Changing this attitude required a much more comprehensive programme.

To sum up this section, MTE has steadily been changing aspect and is seen as a practice characterised by the changing of focuses from being seen purely as commercial arithmetic in the 1960s to modern mathematics up to the 1980s, then back to a period of what I may call self-realisation when basic mathematics came to the fore. Along time, computational skills, analytical power, investigation and problem solving have been emphasised as summarised in Figure 3. In between the timeframe demarcation, various interventions were carried out, for example with Mkwawa and Monduli as designated teacher colleges to train mathematics and science teachers. Promising recent reform directions for teaching and learning mathematics and science is an issue of discussion (Osaki, Hosea, Ottevanger, 2004; Seka, 2004). Equally important was the Morogoro-based teacher education project, and the inspirational mathematics project. Financial support for the Morogoro and inspirational mathematics projects came respectively from Sida (Sweden) and Finset (Finnish support to education in Tanzania).

The shaded part of the time line indicates transition periods between focuses on commercial arithmetic, traditional, modern and basic mathematics. The un-shaded part represents the differences in emphasis between mathematics content and methods of teaching of the same. Each focus represents a major curriculum decision of the political and social context of the day (MoEC, 1995), and coincidentally occurred in between the demarcated decades: hence the basis of choosing the scale. Major events which had general implication for teacher education include the education ordinances of 1962 and 1978, The Presidential Commission on Education - commonly known as the Makweta Commission (1982), and the Education and Training Policy (MoEVT, 1995). These events regulated and influenced the changing focuses, shifts of thinking and turning points, which in turn influenced teacher educators' views regarding MTE after qualification and while in service. The Makweta Commission, for example, recommended that serious consideration be given in mathematics and science teacher education to quality and quantity.

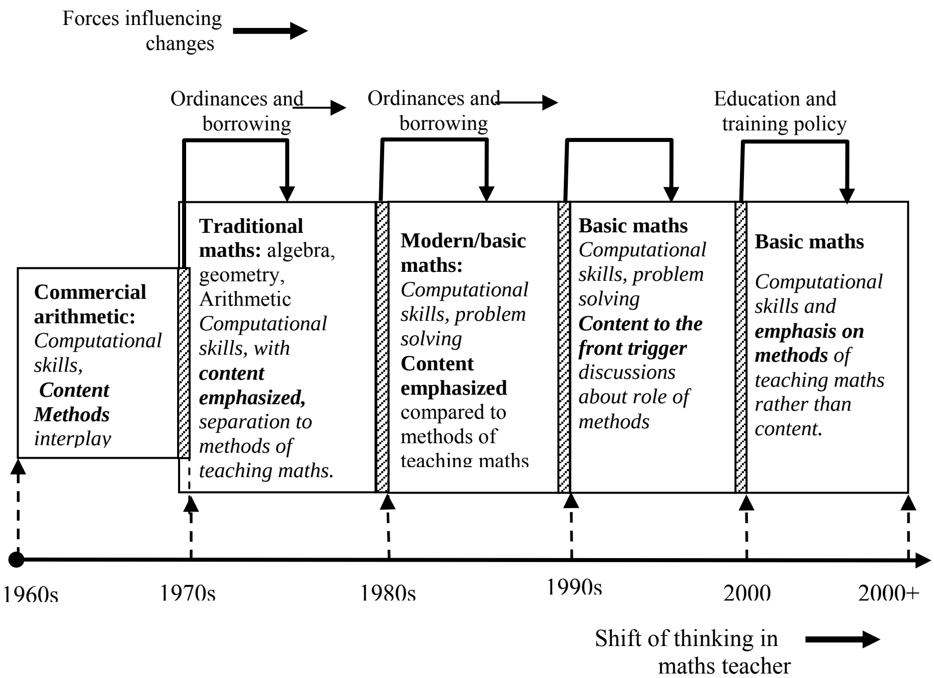

Figure 3. Changing focuses and turning points in mathematics teacher education in Tanzania

At this point in time it has been brought to light how teachers and teacher educators had to personally negotiate the meaning of MTE given the changing focuses, and how intervention could not provide a sustainable solution. There is little proof as to whether the changing focuses, major shifts and turning points were based on knowledge derived from research.

2.2 Perspectives of mathematics teacher education

The preceding section of the theoretical framework has indicated the changing focuses of MTE since the 1960s in Tanzania. Along time questions have been raised regarding the different interpretations given to the subject of interest. This part of the chapter will deal with the perspectives of MTE and its relationship with mathematics education as found from a number of studies which are shaped for clarity. This study is about teacher educators′ conceptions of MTE and thoughts on development of the same. By studying perspectives in MTE, it will help to illuminate even more the phenomenon I am studying. The term perspective refers to mental view, seeing all the relevant data in a meaningful relationship without exaggerating their importance, the capacity to view things in a certain connection (Hornby, 2000; Simpson & Weiner, 1989; Gove; 1971). I have nothing against adopting this point of view except adding to it that the notion perspective may also be taken to mean a way of seeing the same phenomenon, but in different ways.

The task of identifying and comparing the theoretical views of MTE and finally coming up with perspectives has not been easy. It involved reading different studies, pulling together their key findings, and shaping and comparing them to reveal meanings without detaching them from the main findings. It is important to note that the perspectives of interest are not standpoints, as researchers were not found to be asserting the same perspective over time, nor are the ones I studied the only authors of the perspectives to be discussed, although at a particular time they concentrated on a particular perspective. The next point to note is that some key words or phrases kept on appearing in different literature with the purpose of making sense of MTE. Examples of some of these phrases are ‘a composite of many influences’, and ‘the process of becoming’. Even more important is the fact that the perspectives are not the preserve of mathematics teacher educators, as there have been cases of mathematicians working in collaboration with mathematics teacher educators, and general teacher education. With that in mind, I find it natural to discuss the relationship between mathematics education and MTE before the perspectives of MTE for the purpose of a logical order. This will be followed by an account of each perspective, one at a time.

Very often I had thought of consulting a rich description of the relationships between mathematics education, research in mathematics education, MTE and research in MTE. It has been challenging to accomplish this task, since terms which are frequently used tend to establish own informal meanings. My ambition to have the terms discussed under ‘one roof’ made it necessary to consult Adler et al (2005) on MTE as an emerging research field, Niss (2007) on trends in research on mathematics teaching and learning, and Ball (2009) on the emphasis on mathematics subject matter. Furthermore, I looked at a number of publications presenting focused discussions or perspectives of fields or domains (Lin and Cooney, 2001; Lerman 2001). In addition, I also consulted Sierpinska and Kilpatrick (1998), and then pulled together the main features of the two domains. To launch a task seeking the relationships between mathematics education and MTE, and, on the other hand, research features in the respective domains, made it necessary to develop some criteria which could serve this purpose. The proposed criteria (though not exhaustive) which could capture important features in the two domains and research in the respective fields are: features related to making sense of the domains; features related to research in the domains; what appears to be common in both or similarities; and, finally, challenges regarding the two domains. In order to remain focused on the topic of study and at the same time make sense of the relationships between the two domains, Table 3 is presented to serve this purpose. The features brought to light in Table 3 may not tell it all, but are only meant to give a glimpse of the relationships and serve as the basis for further reflection to follow next.

Where is MTE within mathematics education? A review of the features seems to strongly indicate MTE falling within mathematics education as a domain of practice which is complex and layered into mathematics teacher education and development. Taken together, this is the general working definition as I continue a discussion about perspectives in MTE. This way of describing MTE as initially argued by Adler et al (2005) is close to seeing it as a domain of many faces.

Table 3. Features related to making sense of mathematics education, MTE and research in the domains.

Criteria of making sense of the relationship	**Mathematics Education**	**Mathematics teacher education**
1. Features related to making sense of the domain.	1. A domain of scientific research and a process with details of what scholarly groups do, craft, guided by procedures to be followed, and by criteria for acceptable work. 2. An academic activity developing towards a position of attaining a degree of status as a scientific field. 3. Seen as a field covering the practices of mathematics teaching and learning in all levels, in and outside the education system in which it is embedded. 4. A field of study in its own right, falling within the field of education, and having interdisciplinary relationships to a set of foundation disciplines, for example psychology, anthropology, and bearing a special relationship with mathematics itself concerned with teaching and learning of mathematics. 5. Seen as research in the didactics of mathematics rather than simply the didactics of mathematics.	1. A domain of practice which is complex and layered in preparation and development of mathematics teachers typically restricted to distinct sites, for example primary and secondary. 2. Education of mathematics teachers and their mathematics know-how for teaching and seen as a focus on mathematics knowledge for teaching.
2. Features related to making sense of research in the domain.	1. The object of research in maths education mostly falls within the teaching of mathematics, learning of mathematics, teaching–learning settings, didactical settings, the relationship between teaching, learning and mathematical knowledge. 2. The main aim is to develop mathematics education as a recognized academic field of research with its own structure, for example maths subject matter, approaches to the teaching and learning of mathematics, classroom interactions, student understanding of concepts. 3 The aim seems to suggest that the concern of maths education is to develop the knowledge of maths teachers, mathematics teacher educators or researchers in mathematics.	1. Although the object is sometimes not clearly defined, the ultimate aim is enhancement of mathematics teaching and learning in schools. 2. The aim is enhancement of the learning of teachers, and the practice of teacher education itself. 3. Research is multifaceted and may focus on getting teacher perspectives (beliefs, mathematics pedagogy, and teacher knowledge). 4. Research in this domain is often done in the context of teacher education and the focus is teaching-learning through professional development. 5. Research in this domain has teacher education as the focus (looking at interaction, and so teacher education is taken as the object). 6. Research in this domain is used as a means of teacher development, for example what is done in action research – teachers researching their own

Criteria of making sense of the relationship	Mathematics Education	Mathematics teacher education
	4. Each types of research result may be related to practical use, theoretical orientations, or both. Results may energise practice (innovative patterns), reflect specific learning difficulties, relationships of factors influencing learning, and demolishers of illusions (correcting misconceptions), and tell about the consequences of methodological innovations, theoretical perspectives like descriptions, classification or interpretations of phenomena, models. 5. Research questions or problems are formulated within objects of research of the domain.	practice as a means of changing their classroom practice.
3. Communalities	1. The implication of research results is pragmatic but also goes beyond to cover theoretical perspectives. 2. Mathematics education has teaching and learning as one of the main objects of study. 3. It deals with the process of becoming a mathematics teacher with the support of wide-ranging research.	1. The implication of research results is also pragmatic, but mainly intended to enhance practice or bring change in the quality of instruction. 2. MTE has mathematics teaching and learning as its primary concern and so its ultimate objective. 3. It deals with the process of becoming a teacher, where conceptual and technical tools are developed in the broad sense of mathematics education.
4. Challenges	1. There are still debates on the shared conceptions of the research domains, for example on what it means for mathematics educators to do research and a mathematician doing research. 2. It is an emerging field of research and as such has varying object with what mathematicians do when researching	1. It is a newly emerging field of study and close to a black box; scaling-up is a challenge in this research area. 2. Many of the studies in this area are done with the context and perspectives of teacher education and so have yet to establish its identity. 3. The field needs a better theoretical framework of teacher learning. 4. Research in this area is dominated by teacher educators studying their own contexts. It seems to be done without capacity building in research as part of the teacher education process.

To extend the discussion of MTE as a many-faceted , some researchers view it *as a composite of many influences* (Fou-Lai & Cooney, 2001; Sfard, 1998), while others view it as *the 'process of becoming' a teacher* through initial, induction and in-service teacher education (Garcia, Sanchez, Escudero, & Linares, 2006). Yet others envisage this domain as a *blend of subject content, subject methods and some school-based practices in mathematics.* Exponents of these perspectives include, but are not restricted to, Shulman (1986), Lester and Lambdin (1999). Finally, and to be specific, MTE could also be viewed as focusing mainly on the two tasks of teaching about teaching, *and learning about teaching*, which have evolved from Loughran's (2006) search for pedagogy for teacher education. Although this perspective originates from general teacher education, it seems to have a direct bearing on MTE. As said at the outset, these are mainstream ideas or faces and not to be taken as standpoints, nor do they have a one-to-one relationship with each of the researchers cited.

In the first perspective, MTE is viewed as a composite of many factors, and it is one of the most general perspectives. The notion of composite is taken to mean a combination of factors influencing the subject of interest. In a discussion on making sense of MTE, Fou-Lai and Cooney (2001), Jaworski (1998) and Jaworski, Wood and Dawson (1999) in critical international perspectives are of the view that there is growing interest in research in this domain, but currently it remains as a composite of many influences and purposes. In the same vein, Lerman (2001) reports that the amount of research in this area has grown substantially in the past ten years with considerable acknowledgement of the role of the teacher in children's learning of mathematics. However, the limited theoretical perspectives and empirical approaches are enough evidence of how much there is to say and how much there will still be to learn in relation to MTE. Further evidence of the composite nature of MTE is a discussion by Adler (2005), where MTE is characterised as complex and layered or a field of distinct sites. The so-called distinct sites include pre-service and in-service teacher education, which respectively matches the notions 'preparation' and 'professional development'. In the context of Adler's discussion, the MTE layers in question are primary and secondary education. The ultimate concern is mathematics learning in schools. This forms one of the strengths of teacher education - a direct relationship between teacher educators' work with what mathematics teachers do in schools. This adds to the complex or composite nature of MTE as viewed by the different researchers.

How researchers see mathematics education as well as MTE is a question worth noting. This is based on the assumption that the two terms are closely related, and at the same time each is regarded as a research domain (Adler, 2005). Mathematics education, for example, is rigorously discussed by many with the assumption that for any situated learning there is a possibility of emphasising different aspects (Bishop, 1998; CBMS, 2001; Ernest, 1991; Gjone, 1998; Lester & Lambdin, 1999; Mura, 1998; Niss, 1998; Presmeg, 1998; Sierpinska & Kilpatrick, 1998; Sfard, 1998; Wittmann, 1998). I will discuss a few of these in order to show the composite nature not only of MTE but also of mathematics education because of their close relationship. Mura (1998), for example, reporting from Canadian experience, argues that mathematics education is loaded with various meanings. Both MTE and mathematics education are concerned with the teaching of how students are guided in the teaching of mathematics, and also with the learning of mathematics, or how persons come to know and understand mathematics. In addition, the same author argues that mathematics education in particular is a theoretical process directed towards researching into teaching and learning, how students come to know and understand mathematics, the study of teaching contexts, pedagogical issues, and so on. Further, mathematics education is an art, as well as an applied science, and a practical process directed towards searching for ways of improving the teaching and learning of mathematics. A domain of various meanings or aspects may summarise Mura's (1998) way of explaining mathematics education. The role of both mathematics education and MTE is not to transmit facts, formulae, rules and theorems as much as it is in the absolute view of mathematics. Instead, it has to awaken learners' personal drive for insights into the patterns, the beauty (creativity) and the personal relevance of mathematics. Although the explanation has largely been about mathematics education, it is also applicable to MTE.

A study by Sfard (1998) gives another striking perspective, pointing end to the composite nature of both mathematics education and MTE. Mathematics education as an educative process which teachers undergo is a field with many aspects at present undergoing rapid growth. Sfard (1998) stresses that there is only one focus in mathematics, and anything different from what mathematicians deal with cannot be called mathematics. In my view, this sends a very strong message to exponents of MTE. The author seems to confirm the existence of a conceptual gap between the two communities of mathematicians and mathematics educators. This provides more evidence of the composite nature of MTE. But what is the distinction between MTE and mathematics education? What exactly contributes to the composite nature of MTE? Both questions are dealt with next. In the same vein, Wittmann (1998) carried out an outstanding study on mathematics education. Upon further reflection, I find it has similar features to MTE. Wittmann argues that the search for the identity of mathematics education has generated ideas and perspectives which are quite interesting to think about, and views the domain as a design science and an applied scientific concept constructed to serve a certain purpose. Building on this, I would say that both mathematics education and MTE focus on the investigation and development of mathematics teaching at all levels, including premises, goals and contexts. It requires crossing boundaries between disciplines, for example, mathematics, general didactics, sociology, pedagogy, psychology, and the history of science. Because of this composite nature depicting the many faces of both mathematics education and MTE, it is difficult to arrive at shared conceptions of both. Presmeg (1998), just like Wittmann (1998), seems to acknowledge a challenging situation for those attempting to study mathematics education this way. What exists is a pattern of conflicting and yet inspiring ideas. All in all, the meaning of mathematics education and MTE has inherently inseparable aspects linking them together.

Rather than reviewing the various overlapping definitions of mathematics education as given by the researchers in the previous paragraph, I have decided to construct one. There is no straightforward reason for doing this, but mine is an attempt to tell it as a working definition of fewer words than some which run close to a hundred and fifty words. This takes me to Niss's (1998) characterization of mathematics education, and any gap to be found is mine. With that in mind, mathematics education is regarded as a research domain whose aim is to identify, characterize, understand processes involved in the teaching and learning of mathematics in the various levels of education. Within this broad outline the main task is to provide descriptions of phenomena, explain their causal relationships by designing or using a variety of frameworks and methods from other fields, for example psychology and sociology in a range of mathematics activities. Viewed in this way, it is possible to see the place of MTE in the broad picture of mathematics education. Further, and on the basis of the definition, it may be fair to claim that the ultimate aim is providing a description of what is happening in mathematics classrooms, an explanation of why this is to enhance students´ learning of mathematics.

How is MTE related to mathematics education, and what makes MTE a composite of some factors? I find it important to respond to these two questions because in the process it will help to identify the main factors which contribute to the composite nature of MTE. A good starting point is to differentiate the meaning of MTE from the mathematics we know. For now I have three interrelated concepts, the first of which is mathematics as a discipline that we experience in schools. Then there is mathematics education as a domain with a focus on teaching and learning as well as researching into the same. The third is the subject under discussion, which this study embraces, and all are conceived in different ways. I am aware that to establish an acceptable meaning, not in the sense of a definition of mathematics as a discipline, is difficult, again because of the different perspectives. For this reason, to simply give a meaning in the sense of a sensible and comprehensible word or phrase may help to differentiate the concepts.

To many educated people, mathematics is characterised by accurate results and infallible procedures, whose basic elements are arithmetic, algebra and geometry, with various terms and theorems. In a similar vein, Ernest (1998) brings to our attention a number of ways teachers conceive mathematics. First is the problem-driven concept of mathematics, which sees it as a continually expanding field of human creation and invention. It is a process of inquiry in order to know, and it adds to the sum of knowledge. Its results remain open to revision through research into teaching and learning about mathematics. Secondly, mathematics is a unified body of knowledge set into a final form. This way of understanding mathematics entails viewing it as a solid and specialised field where themes are systematically tied together and not open for revision. Seen in this way, mathematics is reduced to a body of knowledge, with concepts, rules, algorithms and theories set into a final form or absolute knowledge. I would say that this view contradicts the fact that the daily experiences of people are full of quantification. Think of an ‘intelligent customer’ in a supermarket, a landlord repairing a house, a parent thinking of children and the school year: they all need to estimate the finance they need. Thirdly, it is also conceived that mathematics is an accumulation of facts, rules and computational skills for application. Knowing mathematics means demonstrating computational skills and proving theorems without necessarily backing them up with sound reasons in the process. This instrumentalist conception regards the rules and procedures in mathematics as a bag of tools. The variations in understanding what mathematics is account for the sharp differences in how mathematics is taught and learnt (Skemp, 1978), and as such results in at least two types of mathematics in the classroom: mathematics for understanding as opposed to mathematics for performance. I would say that the latter is very often emphasised on the pretext of standards.

I now turn to mathematics education and MTE in that order. Ernest (1998), for example, reports that mathematics education is a discipline whose foundations are the philosophy of mathematics itself, the nature of learning, and the nature of teaching, linked with the aims of education and the study of teaching and learning at all levels of education. In addition, Sfard (1998) argues for other views of mathematics education as firstly a field of many aspects, including teaching and learning. Secondly, it is a field undergoing rapid growth in the direction of research. Thirdly, there is only one focus in mathematics, and anything different from what mathematicians deal with cannot be called mathematics.

The relatedness of MTE and mathematics education can be used to build the case for their differences. On this basis, I also suggest using a combined model to explore the meaning of MTE. Sierpinska and Kilpatrick (1998) produced one of the high profile studies which established the significance of mathematics education and with the help of a working group developed a framework for it. In the profile of studies Johansson (1998) and the working group show vividly the inseparable domains (see Figure 4). In view of this recognition, the point of departure in mathematics education is to see it as a field of study covering the practice of mathematics teaching and learning in and outside the formal system of education. It is a field of study in its own right and a discipline which lies within the field of education, and has interdisciplinary relations with, for example, psychology, anthropology and philosophy. Despite the relatedness between mathematics education and MTE, it remains challenging to locate a dividing line between the two, apart from the latter seemingly sub-summed. It is important to emphasize that mathematics education has a special relationship with mathematics itself (Johansson, 1998), and, I would add, MTE. To strongly support this view, Bass (2005) argues that mathematics has a long and recognized tradition of involvement in mathematics education. Figure 4 is an adapted framework for explaining MTE in relation to mathematics education and mathematics. It is further argued by Niss (2007) that the widening perspectives of research in mathematics education have necessitated the adaptation of investigational frameworks. The investigational frameworks themselves become numerous and sometimes complex. On these grounds, the subject of interest is seen as a composite of many influences (see Figure 4).

At this stage, it is possible to see what mathematics education and MTE have in common, in that teaching and learning as well as researching into the same are the underlying factors that bind them together. A point to note about researchers, whether in mathematics education or MTE, is the emphasis they give to the role of research in mathematics education as well as MTE, for the important reasons of enabling teachers and teacher educators to discover new insights, and to give them better strategies for teaching and learning mathematics, which in turn help to open their minds and enable them to make better pedagogical decisions (Jakku-Sihvonen & Niemi, 2006).

It is logical to assume that within the process of making pedagogical decisions one is discussing what content to teach, how and why. The whole issue again gravitates around content vs. pedagogy as seen in Chapter 1. With the picture of mathematics, mathematics education and MTE in mind, this is an opportune time to summarise the main factors contributing to the composite nature of MTE before I proceed to the next perspective.

In view of the previous discussion and the adapted framework, the main factors contributing to the composite nature are now interpreted. They include mathematics teacher educators' different experiences in teaching and learning strategies, the curriculum and selection of learning materials, teacher education, the strong relationship with mathematics, assessment, as well as mathematics education, which in a way have all been discussed. Studies also indicate that the complexity is a result of teacher educators as learners having increasingly diverse mathematical histories (Adler, Ball, Krainer, Lin, Novotna, 2005). This implies that having a shared meaning of MTE is challenging because of the combination of factors. These in turn add to the composite nature of the subject under study. I choose to discuss a few to illustrate my point.

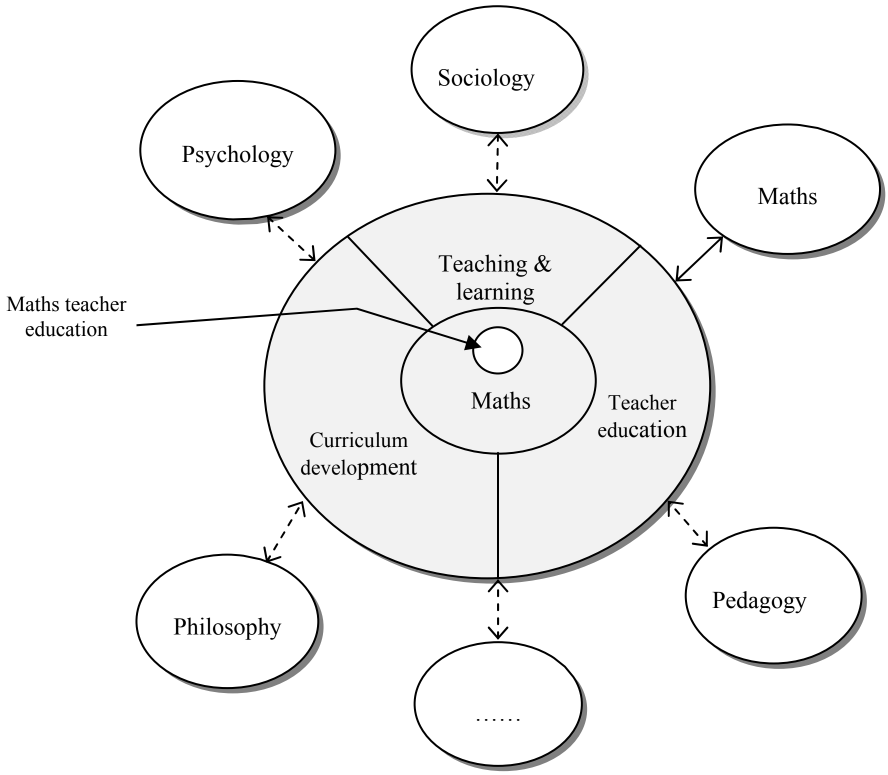

Figure 4. A framework explaining mathematics teacher education as a composite of influences (Adapted from Sierpinska & Kilpatrick, 1998 p. 29)

The process of curriculum making and how it contributes to the composite nature of MTE is now explained. The MTE curriculum exists in various forms, for example as understood by those who design it (curriculum developers), and by teacher educators, and as experienced by the learners themselves. The bottom line here is that different people may understand it in different ways because of the differences in contexts. O-Saki (2000), for example, in his description of a similar situation in Tanzania, sees the curriculum as all the learning experiences provided in the education system from pre-school, through primary, secondary, tertiary to non-formal education systems. The author goes further to differentiate between the components of the curriculum by bringing to light the 'official curriculum', which refers to the prescribed curriculum, including the subjects, for example mathematics. The second level is the ongoing curriculum or implemented curriculum, and this refers to what is actually taught by teachers. The third level is the received curriculum, which is taken to mean what is actually experienced by learners. Taken together, it is plausible to sense a possibility of diminishing value from what was actually intended by the official curriculum down to what is actually experienced by students. This means that there are possibly different interpretations of MTE, which adds to its composite nature, because, in the case of Tanzania those who prescribe (curriculum developers), implement (teacher educators), and supervise (education inspectors) have a role in shaping the curriculum for training mathematics teachers, and hence its conceived meaning.

Furthermore, teacher education as a field contributes to the composite nature of MTE in important ways. Lewin and Stuart (2003) and O-Saki (2005), for example, view teacher education as experiences throughout the training programmes as taught by teacher educators, as organised both on and off campus and as learnt by student teachers. Teacher education as a process is presented in the form of theories of education which are built on the cornerstone disciplines of psychology, curriculum and teaching, foundations, measurement and evaluation. The major function of professional studies is to provide a basic understanding of how children develop and learn and how all this can be combined during field experience, commonly known as teaching practice. The purpose of teaching practice is sometimes seen as testing the link between theory and practice, as a way of perfecting professional skills, and as an opportunity to bring together the three components of teacher education in one room. How this is correctly carried out in MTE leads to many more questions because of the different viewpoints of teacher education as a field. In the end, MTE finds itself as a composite or an amalgamated field of study. Detailed study of the term 'teacher education' reveals even more interesting viewpoints which may also have bearing on MTE. Cheng et al (2001), for example, writing from a South-East Asian viewpoint, regard teacher education as a field of study and as a professionally educative process. The latter, to my understanding, implies specialised knowledge which takes into account the teaching and learning of specific subject content, for example in MTE. In addition, the author views teacher education as an enterprise which encompasses all aspects of teacher preparation and development. The author proposes the possibility of developing a 'new teacher education' in the South-East Asia region in order to share the many and rich viewpoints, ideas, experiences, experimental results and reform outcomes. This gives another glimpse of differences in the focus of teacher education and further adds to the composite nature of MTE.

To this point, the commonality between MTE and mathematics education and how they complement and influence each other has been discussed before, as well as their natural relationship to mathematics. The broad fields of psychology, sociology, philosophy and pedagogy are considered as part of teacher education. This concludes the discussion of MTE as a composite of many influences.

In the second perspective, MTE is viewed as 'the process of becoming' a teacher, and for this specific study, I mean becoming a mathematics teacher. To substantiate this perspective, Garcia et al (2006) argue that the process of becoming a primary school teacher, for example, may be understood as the process of being introduced into the community of practising teachers. In this way, learning to teach is seen as the beginning of the use of conceptual and technical tools in carrying out professional tasks, whereby the term conceptual tools refers to concepts and constructs which have been generated from research in teaching mathematics. Along the same lines, the term technical tools refers to tools used in the 'practice' and may include teaching materials, software, techniques for managing discussions, procedures and answers to problems. I find this to be quite a different way of understanding not only teacher education but also MTE.

This view can be traced back to Zeichner (1983), who argued for four paradigms (patterns) of seeing the process of teacher preparation. Though Zeichner uses the word 'paradigms' of teacher education, I prefer to use the word 'pattern' or simply 'system' in the interest of a better analysis of the composite nature of MTE without a reduction of its original meaning. My argument is based on Webster's Encyclopaedia unabridged dictionary of the English language (1989) as well as Simpson and Weiner (1989), Gove (1971), Wehmeir and Ashby (2000), where the term 'paradigm' has a common feature and is taken to mean 'a pattern', 'a set of forms', 'a case taken to be a representative', a typical example, and 'a model'. On this basis, behaviourist teacher education as one of the patterns is regarded as the process of developing a teacher where the emphasis is on the hands-on skills of teaching. Although this is derived from general teacher education, it can be applied in the case of, for example, the preparation and presentation of mathematics lessons which may focus on basic operations. Behaviourism in terms of teacher training may be more interested in drill-oriented outcomes and final answers, and less in the process of reasoning and the connection between concepts. The second pattern is personalised teacher education, which concentrates on developing an individual teacher based on his/her interests or needs. The third pattern is related to apprenticeship and for this reason is referred to as traditional craft teacher education, which, in a strict sense, is like shaping a person into becoming a teacher. The fourth pattern is inquiry-oriented teacher education, in this case MTE. This way of thinking about the educative process in teacher education devotes time to and puts emphasis on developing a questioning mind, and constantly reflecting on the teaching and learning process. It develops a culture of thinking and making a personal assessment of the entire activity, and so MTE in many ways has to relate to teacher education in general. What Zeichner (1983) calls 'paradigms of teacher education' can be seen as 'patterns' or systems of teacher education, and MTE is part and parcel of the process. In the words of Schon (2005) and Mason, (1998), it is a way of developing people towards a system of specialised knowledge. I would say it is a process of initiating prospective teachers through a process into the career of teaching during and after qualification. This concludes the discussion of MTE as 'a process of becoming' a mathematics teacher.

In the third perspective, which is one of the most common*, MTE is viewed as a combination or* a *blend of pedagogical knowledge* and subject matter knowledge (Attorps, 2006; Bass, 2005; Bullough Jr., 2001; Lester & Lambdin, 1999; Niss, 2007; Shulman, 1986; Shulman, 1987). In a discussion about competencies for mathematics teachers and how they should be developed, Niss (2007) reminds us of the long standing conception of teachers of mathematics as persons who know concepts, facts, results, rules, methods on the one hand, and how to put the lesson across on the other. The necessary competencies for a teacher were thus a demonstration of subject matter knowledge and general pedagogy. Shulman (1983), for example, argues for pedagogical content knowledge (PCK) in the sense that there is a particular form of content which embodies the aspects most relevant to teaching, and as an extension of this argument, a method relevant to the teaching of a specific subject. Three years later, Shulman (1987) expanded this argument in an attempt to make a case for teacher professionalism, whereas years later, Bass (2005) makes a case for mathematical knowledge for teaching. In the same way, Lester and Lambdin (1999) argue for a combination of content, pedagogical knowledge and some school-based practice, and they view MTE as a practical way of guiding teachers' actions. In a way, this is taken to the level of subject didactics, where the focus is on what content to teach, how and why.

The researchers cited in the previous paragraphs seem to support the blend of subject and pedagogical content as a general trend. Within this view, Niss (2007) argues for a non-separation of subject matter and pedagogy. To elaborate this point, mathematics knowledge for teaching is used as a case in hand, and is reported to constitute important knowledge beyond the theoretical mathematics that we know. That is to say, apart from mathematical competencies, teachers should possess specific knowledge such as didactical competencies, curriculum competencies, teaching competencies, knowledge of learners' competencies, assessment competencies, collaboration competencies and professional development competencies. It is further argued that initial teacher education can make a difference in teaching and learning, but what happens in terms of professional development after qualification is even more critical. In another discussion, reflecting the same perspective of a blend between pedagogy and subject content, Cooney et al, (1996) perceive it as pedagogy, or knowledge and skills about teaching and learning, as a way of creating and guiding learning environments for problem-solving and reasoning and as a connection within mathematics and between mathematics and other disciplines. Mathematics education is also understood from the way that teaching and learning strategies are emphasised in the classroom. Cooney et al (1996) seem to focus more on the concept of PCK and in important ways substantiate the MTE perspective built around the blending of subject and pedagogical content.

The previous paragraphs have brought in the notion 'subject didactics' in relation to 'pedagogical content knowledge'. This is an interesting subject, but I only want to highlight their difference, though they are closely related. In recognition of this challenging situation, Van Dijk and Kattmann (2007), for example, hesitate to be drawn into a discussion involving a comparison of the

American tradition of instructional strategies in the sense of pedagogical knowledge and didactics. It may be both theoretically and practically difficult to make a distinction between the two traditions. Perhaps the safe line of thinking about their differences is to look at the role of teacher and teaching from the teacher education point of view (Van Dijk & Kattmann, 2007). From the outside didactics is a tradition in most of northern and central Europe, while pedagogical content knowledge is a common term in the American tradition. On this basis, what are the differences in classroom actions? As a response to this important question, Westbury, Hopmann and Requarts (2000) suggest looking at the role of teachers following these two traditions. It is reported by the same researchers that German didactics as a tradition is more teacher-centred than the American system-centred pedagogical knowledge.

Further, in the didactic approach the role of the teacher emerges as an active maker of the classroom curriculum. This is felt when, for example, the teacher educator make decisions on what to teach, and how and why to make certain decisions. There is a certain degree of autonomy, though the state may prescribe the curriculum content. This is not vividly seen in the American tradition, where the relationship between scholars and practitioners is for the latter to use knowledge generated at the top (hierarchical). The same authors also relate teacher educators and teachers in terms of the latter being seen as passive conduits who merely implement the prescribed curriculum. Stated in very general terms, this is argued as the main difference between subject didactics and pedagogical content knowledge. I found it important to say something on subject didactics and pedagogical content knowledge in order to minimise possible misinterpretation, but at the same time to point out that Shulman's (1986) discussion of teacher strategic knowledge for teachers is closely related to the two traditions put in focus here. However, the central discussion of MTE as a blend between content and pedagogical knowledge is carried further in the next few paragraphs.

It may be difficult to unearth fundamental differences among researchers conceiving MTE as a blend between content and pedagogical knowledge. Some are writing from a general teacher education point of view, but still their ideas are applicable to MTE. In my view, a skilful blend of content and pedagogical knowledge seems to be argued for by many who favour this perspective. The important aspects of PCK are knowledge of subject content, in this case mathematics, knowledge of instructional strategies (methods of teaching and learning), curricular knowledge, and knowledge of students' understanding as well as knowledge of the purpose of teaching the subject matter (education). PCK is affirmed as

> *"the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and ability of learners, and presented for instruction".* (Shulman, 1987, p.8)

This quotation characterises PCK and it can be seen that content, pedagogy, instructional strategies and students are emphasised. One may ask how this can be seen in the classroom. Shulman (1987) puts it in a practical way by saying that PCK refers to the knowledge of multiple ways of presenting the content to students. In a real classroom situation this is seen by the teacher educators' use of illustrations, examples and explanations, and by demonstrating an understanding of what makes a topic easy or challenging. Many other researchers identified other aspects; for example, Van der Valk and Broekman (1999) identified five aspects of PCK: pupils' prior knowledge, pupils' problems, relevant presentations, strategies and student activities. One would think assessment is aligned in the relevant aspects of PCK, for example student activities, and it is suggested that the list is not exhaustive.

The issue of assessment is worth examining because PCK emphasises the knowledge of students' understanding, together with the relationship between teaching and learning which is at the centre of PCK. Characterising the PCK of an individual teacher or student teacher involves assessment. The relationship between assessment and professional development is to be discussed later in this chapter (as part of Section 2.3).

In answer to the question of investigating mathematics knowledge for teaching, Baker and Chick (2006) from Melbourne University designed a study involving two primary mathematics teachers at different levels of PCK. The two teachers (Andy, 20 years of experience, and Clara, 6 years of experience - given names) were asked to complete a questionnaire, and were interviewed about their responses. Andy was teaching Grade 5 and Clara Grade 6. The study contained 17 items and only 5 are reported here in the interest of space and relevance. For the same reason I will pick only 2 out of five items to exemplify the importance of PCK, and use only one item to demonstrate the responses. The items chosen requested teachers to demonstrate their PCK, which was analysed against eight criteria, or aspects, as Shulman (1986) labels them. The criteria were teaching strategies, student thinking, and cognitive demand of task, profound understanding of mathematics, and deconstructing content to a key component. Other aspects were procedural knowledge, methods of solution, and finally, goals for learning. To illustrate the content-pedagogy perspective of MTE, I will choose just two of the eight criteria in the interest of similar experience to Tanzania, and these are teaching strategies and knowledge of student thinking. I will use a similar method when illustrating subject matter, but with a different example (see Table 4).

How did the teachers demonstrate possession of mathematical knowledge for teaching in relation to teaching strategy and student thinking? I choose the fraction item because of its similarity in MTE classes in Tanzania and the power of the questions to explicitly show the interplay between content and pedagogy in the perspective I am pursuing in relation to the teachers' responses. Demonstrating knowledge of teaching strategy and student thinking by both student teachers follows next. I have tried to shorten Baker and Chick's (2006) analysis of the two teachers' demonstration of PCK without losing touch of the details.

This part of the study uses 'teaching strategies' as criteria to demonstrate the PCK of the two teachers (Baker & Chick, 2006). Both Andy and Clara demonstrated knowledge of teaching strategies. Clare's responses were more detailed than those of Andy, who suggested alternative strategies. Taking the fraction item as an example, Andy's explanation was based on the number line to show the sum of 7/10 and 4/10, or the use of a long rectangle and pie chart, but his explanation lacked detail compared to Clara. He attempted to stress the importance of students´ understanding the concept without elaborating the concept being talked about.

Table 4. Items used to demonstrate teachers' PCK or mathematical knowledge for teaching (Baker & Chick, 2006).

Category of item	Items to demonstrate mathematical knowledge for teaching
Subtraction item	You notice students working on these two subtraction problems: 438 − 172 = 346 5819 − 2673 = 3266 What would you do to help this student?
Fraction item	A student submits this question and solution as part of his/her homework: 7/10 + 2/5 = 7/10 + 2/10 = 11/20 a) What does this student understand? What do he /she not understand? b) How do you quickly convince the student that the answer is incorrect c) What does the student need to learn before he/she can complete questions of this type, how would you help to him/her achieve this understanding?

In the end he recommended a lot of examples to practice with. With the same fraction item, Clara suggested using diagrams (though not specified), readily available materials, such as a pie chart, number line, cutting up a piece of paper, together with detailed suggestions.

She started by demonstrating to the students that when you add 4/10 to 7/10 the result is more than a whole, so that the student could see that the given answer was incorrect. To enhance this, she suggested comparing 11/20 with the correct answer 11/10. She went on to show the student that 3/10 must be added to 7/10 to make a whole. Clara's emphasis was on helping the student to understand the effect of the denominator on the size of the pieces. On the surface, both explanations appear similar, but further reflection would reveal that Andy and Clara's approaches actually differ in quality. Andy concentrated on shaping the correct answer (starting with the incorrect answer), rather than the correctness of his method, Clara's first explanation seemed to raise a question in the students' minds, after which she attempted to develop conceptual understanding.

In the same vein as the preceding paragraph, 'student thinking' is used as a criterion to demonstrate understanding of PCK. The clearest difference between Andy and Clara in this connection is reflected in Clara's ability to discuss students' thinking far more often than Andy. In addition, Clara demonstrated better overall knowledge of student thinking, as with the ideas she was using to correctly guide students, for example the sum of 7/10 and 4/10 lead to more than a whole. This was a better starting point, and her understanding of the fraction addition item is evidence of her tendency to be mathematically specific and detailed. Clara was able to see that the reason for the students' error was the lack of understanding concerning 7 out of 10 and 4 out of 10, because of the misconception of adding the denominators. In the course of the interview, that student error also demonstrated lack of understanding that two-fifths is four-tenths. She then discussed errors she had seen in the students' thinking about fractions, and attributed the origin of these errors to the way the students had been taught and the experiences they had had in the classroom.

This discussion on the blend of content and pedagogical knowledge has shown that this is one of the perspectives of MTE, but emphasising PCK overlooks the point advocated by many that sound pedagogical decisions are based on sound content knowledge (Wu, 2005), which is the area I would like to focus on in order to consolidate this MTE perspective before dealing with the fourth one. Much of what constitutes subject content has been highlighted by different authors. Suffice it to say that subject content means knowledge of mathematics or content (conceptual understanding), representations, as it appears in school curricula, including various representations for concepts, procedures, principles, proofs, and rules (Wu, 2005: Bass, 2005; Turnuklu & Yesildere, 2007; Ernest, 1998; Sfard, 1998). How subject content combined with PCK emerged as a strong perspective of MTE is worth investigating in order to see the basis of its recognition or position. Using a practical example from a teacher educator's point of view might help to confirm the high regard for subject content as one of the MTE aspects in the wider perspective. Menon's study (2009) demonstrating the interplay between subject matter and PCK enhances the view of MTE as a skilful combination of content and pedagogical knowledge is now analysed. For the purpose of clarity and detail, I have made some necessary changes without departing from the original meaning.

The study sample consisted of sixty-four pre-service teachers from a mathematics methods class for middle schools, who were given 3 mathematics problems, which were: first, multiply a three-digit number by a two-digit number; second, divide a whole number by a fraction; and third, compare the volume of two cylinders made in different ways from the same rectangular sheet. They were then asked to a) solve them and explain their solution, b) classify them as easy, medium or difficult, giving a reason for their classification, and c) explain how they would teach/help children to solve them. The responses were classified into three categories, namely subject matter knowledge, traditional pedagogical knowledge, and reflective knowledge. According to Menon (2009), the notion 'traditional' is defined as 'the way it is usually done by teachers', and reflection is used to mean teachers' self-assessment or critique of the actual content of the subject they are teaching, apart from how it is taught. For the purpose of clarity, I will deal with the third problem, and the problem of interest is stated below.

a) Suppose you are given a rectangular sheet of paper, with its length L, which is twice its width W. By rolling the length of the paper into a circle, you get a cylinder V-L with the *width* of the paper as its height (W). Alternatively, by rolling the width of the paper into a circle, you get a cylinder V-W with the *length* of the paper as its height (L). Explain whether the volume of the cylinders V-L and V-W are the same or unequal.

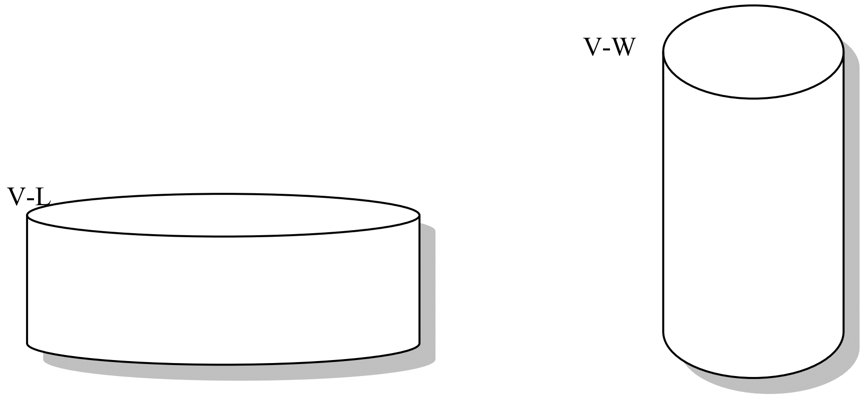

Figure 5. Demonstration of subject matter knowledge using cylinders made from the same material (Menon, 2009)

b) Describe how would you teach/help a child solve this problem about the cylinders.

The responses from the student teachers are summarised in Table 5, followed by their analysis. From Table 5, it is possible to see that about 95% of them had an incorrect answer – they indicated the same volume, or did not attempt the problem. It is reported that of the 3 out of 64 who got it correct, 2 used the formula for the volume of a cylinder (pi, radius squared times height). Only one stated clearly that the radius squared contributes more to the volume of the cylinder than the height. This follows from the circumference, $L= 2\pi r_1$ or $W=2\pi r_2$, and this gives, $r_1 = L/2\pi$, $r_2 = W/2\pi$. Since L>W, the volume of the cylinder V-L is greater than that of V-W. This forms part of the logical explanation from the pre-service teachers.

What is the interpretation of the results? They indicate that students' subject matter knowledge is weak, even at the traditional (the way they have been doing it or experience) and learned-knowledge stage. As regards the results of question b), they relied much on pedagogical knowledge, as the 55% of them who responded relied on a practical, hands-on approach to the solution, by rolling papers into cylinders and actually pouring in water or sand to compare the volume of V-L and V-W.

The mathematical comparison is better seen from V-W= $\pi(r_2)^2L$, which make V-W=$W^2 L/4\pi$ and V-L= $L^2 W/4\pi$ after simplification. Where L and W reflect radius, the volume V-L is greater than V-W as it is the radius which finally determines the difference between the two volumes. On the basis of the cylinder problem, it can be seen why content and pedagogical knowledge feature as strong perspectives of MTE.

Table 5. Student teachers' demonstration of subject matter knowledge of mathematics (adapted from Menon, 2009)

Question	Correct answer	Incorrect answer/ Not done	Procedural explanation	Logical explanation
a)	3 (5%)	51/10 (80% /16%)	2 (3%)	1
	Practical explanation	Incorrect explanation	Procedural explanation	Logical explanation
b)	35 (55%)	0/29 (0% /45%)	0	0

These examples illuminating the interplay between subject matter knowledge and pedagogical content knowledge concludes the discussion of the third perspective and opens up for the fourth perspective.

In the fourth perspective, MTE is viewed as learning about teaching. That is, taking mathematics teacher educators as a case, they have two main tasks: one is to teach about teaching mathematics and the second is learning about teaching mathematics. While the first task may be regarded as exclusively the work of teacher educators, the second task also engages mathematics student teachers. Thus, they are in a process of learning about teaching, and this continues even after qualification. For this reason, Loughran's (2006) view of the pedagogy of teacher education brings a sense of teaching about teaching and learning about teaching. In addition, Loughran emphasised that this is an important view yet to be explored and the research knowledge gained to be put to good use. Though its main focus is on what teacher educators do, it also reflects a lot on the process of learning how to teach mathematics. In other words, it is a way of gaining expert knowledge of a particular field as in the case of MTE. To emphasise the point of learning to teach, Loughran and Russel (1997) and as also cited in Loughran (2006), argues that:

> *"Becoming a teacher educator (or teacher of teachers) has the potential (not always realised) to generate a second level thought about teaching, one that focuses not on content but on how to teach …This new perspective constitutes making the 'pedagogy turn', thinking long and hard about how we teach and the messages conveyed by how we teach….. I have come to believe that learning to teach is far more complex than we have ever acknowledge".* (Loughran & Russel, 1997, p. 44)

This quotation illuminates the perspective that teacher education, and for that matter MTE, is more than focusing just on content. Of course, not only Loughran's (2006) ideas emphasise 'teaching about teaching' that provides another vision of teacher education, but also so do Adler et al´s (2005) ideas on the double role of teacher educators. They teach and investigate in order to provide solutions regarding challenges in teaching and learning mathematics. 'Teaching about teaching' and 'learning about teaching' is a reminder of a well-known fact that the work of a teacher educator and that of a teacher can be described as teaching, and that of a student teacher as learning (Uljens, 1997). The concentration therefore is on the relation between the student and some content which is visible as studying, doing something in order to achieve the aims and goals in the curriculum – the mathematics teacher education curriculum. Kansanen and Meri (2009) expand this point to say that the results of studying may be learning and other consequences of the instructional process. To use Kansanen and Meri´s argument from the didactical relation between the student and content, it may be said that in order to learn about teaching the student teacher is expected to do something, study mathematics in order to qualify as a mathematics teacher. The role of the teacher educator/teacher is to guide the studying part, as to control it is demanding and complicated. It is complicated taking into consideration that every student teacher is supposed to think and decide on him/herself on how to cope with the process of learning about teaching mathematics. It is therefore imperative to concentrate on the relation between the student and content or on studying, as this is the core of the teachers' profession. In view of the perspective on 'teaching about teaching' and 'learning about teaching', there is an inevitable relationship between what happens as MTE in teacher colleges and its application in schools. I will briefly discuss this in view of the relationship between MTE and teaching mathematics in schools.

Teacher education as a domain stands on its own, but there is a natural relationship with what happens in schools given the central role of the former in developing mathematics teachers for schools. What makes this relationship, and how? I would like to discuss this in the context of Tanzania because of the experience I have in working with teacher educators. Foremost, developing mathematics teachers through MTE and finally making them serve in schools forms the first line of cooperation. This makes it absolutely necessary for teacher colleges to relate the MTE curriculum and how to teach the school mathematics curriculum. In order to achieve this objective the institution responsible for MTE curriculum making would study the needs of both teacher educators who prepare teachers on the one side, and the challenges of teachers in schools, and use this as the basis of the intended MTE curriculum. The second line of relationship demands that student teachers analyse school mathematics syllabuses and locate their strengths and shortcomings before they even qualify as teachers. This kind of task is part of a compulsory continuous assessment not restricted to mathematics only. The third line of cooperation is how mathematics teacher educators are recruited. Before recent reforms in elementary and secondary education were enacted, teacher educators were sourced from at least university graduates with experience in teaching either in primary or secondary schools. This condition, though loosely considered, now made teacher educators relate their experiences in teaching mathematics in the lower levels and MTE. In an actual sense it is a constant reflection of the relationship between the concept of ‘teaching about teaching’, ‘learning about teaching’ and what happens in the mathematics classroom.

As a conclusion of the fourth perspective, teaching about teaching and learning about teaching serves as a reminder to ‘teachers of teachers’ that they are a part of the MTE perspectives under discussion, and must show what both teacher educators and the students do. It is therefore not a question of ‘other teachers’ but an equally important perspective sourced from them.

To sum up, it has been possible to establish four perspectives of MTE. My argument is that the perspectives are not to be seen as standpoints of researchers who reflect a certain line of thinking, nor is there a one-to-one relationship between researchers and the perspectives generated as a result of reading different literature on both mathematics education and MTE. However, as one studies literature, certain key notions keep on appearing. First, MTE is a composite of many influences, and some of the factors which add to this composite nature are variations in teaching and learning approaches by teacher educators. Second, teacher education as a field means different things to different mathematics teacher educators, as well as to those who prepare the MTE curriculum. It is also important to note the strong relationship between MTE and mathematics education, in that they influence each other because of their underlying commonalities. Both are related to the mathematics we know, whose influence is significant, and it is sometimes difficult to establish a dividing line. They all have their roots in the classroom and teacher education circles. Furthermore, MTE may be seen as ‘the process of becoming’ a mathematics teacher, while others see MTE as a skilful blend of content and subject matter with which prospective teachers are engaged, and finally it is seen as involving two main tasks - teaching about teaching and learning about teaching.

2.3 Thoughts in relation to MTE professional development

In this section I intend to discuss MTE with a focus on the concept of professional development as a process of continuing to learn after qualification. In the course of this discussion I will try to shed light on the meaning of key terms, and the motives behind teacher educators’ drive towards professional development. A discussion follows on important considerations or principles needed in order to support professional development and possible areas of focus, within which the role of assessment will be looked at. Finally, a selected number of promising professional development strategies or models will be discussed. The reasons for doing this emanate from the fact that MTE is a process focusing on ongoing professional growth and the support given to mathematics teacher educators, and the tasks outlined above are important processes along a continuum of teaching, learning and studying about MTE.

Setting the stage for professional development

I start the discussion about professional development with an attempt to establish the meaning of some key words, namely 'thoughts', and 'development' as a start, which are often discussed by teacher educators, who may, for example, decide to meet in order to discuss 'mathematics club development activities', or 'thoughts about their role in the Mathematics Association of Tanzania'. Consider statements like 'concluding thoughts', 'many thoughts arose during the discussion of teaching and learning mathematics for problem solving,' 'students gave up all thought of attempting the challenging mathematics puzzle', 'the team investigating teacher educators had few thoughts on the subject'. These statements show that the term 'thought' refers to a system of ideas carefully considered, views, perspectives, impressions, judgements, assessments, conceptions and strategies, which are context-based (Fowler & Fowler, 1995; Hornby, 2000; Thompson, 1992; Waite, 2001). I am of the view that this will help in laying the foundation for discussing professional development which is to be examined next.

Sometimes familiar terms like 'development' create a life of their own as a series of understandings, views, perspectives, and thoughts emerging within a given timeframe which have proved to be context-specific. In such a situation, more often than not, the meanings of the terms are taken for granted, as exemplified by terms like 'teacher development', 'professional development', teacher competencies, teacher proficiency (Björkqvist, 1982), which are frequently used without even asking what the key word 'development' means. It may be difficult to locate the beginning and end of teacher educator/teacher education as a growth process, but it makes sense to suggest that it starts with initial teacher education and continues thereafter. Loughran (2006), in his discussion on the possibility of developing a pedagogy of teacher education, uses the term 'development' to mean neither the beginning nor the end, but a sense of coming to or pushing ahead towards a more advanced state. Translated into teacher educators' work, it means that if they are developing then they are growing in understanding, moving forward, and purposefully building on that which is currently known. In any case, developing suggests value in extending that which is already known, being able to do, and questioning that which is overlooked or taken for granted with an open mind.

To emphasise the term development and the difficulties in locating a beginning and end, Loucks-Horsley et al (2003) argue that once the process of teacher development (applicable to teacher educators) is in progress, it does not stop evolving, for important reasons. Those who have been in the classroom might have witnessed that teacher educators are inspired by learning from mistakes, and learning from misconceptions becomes a driving force for readjustment. Adler et al (2005) argue for teacher-learner support in the broad sense of MTE and characterise this situation as one of increasingly diverse mathematical histories, limited school mathematics learned by prospective elementary teachers entering teacher education, and general under-preparedness. To facilitate mathematical proficiency in as many students as possible, they have to continue learning because teacher preparation and development contexts, as well as programmes, keep on changing. Parallel to this, and to be precise, gaps in MTE programmes, as well as between teacher educators and student teachers, keep occurring, and hence the need to review curricula. These specific reasons make it necessary to put in place professional development programmes for teacher educators, and they are therefore compelled to find alternative routes for professional development in MTE.

If the terms ‘thoughts’ and ‘teacher development’ are taken together, then ‘thoughts for continuous professional development’ of teacher educators would mean ideas for ‘getting started’ or ‘ideas to define the way forward’ in order to support present teacher educators and those of the future. This is related to Loucks-Horsley et al´s (2003) idea that, at least for the moment, thoughts on the development of science and mathematics teachers would be based on the relevance of present practice. I would think this idea holds for mathematics and science teacher educators except for their different needs regarding teacher preparation and development. It is also important to note the traditional relationships between mathematics teachers and mathematics teacher educators (Bass, 2005), which can also be seen in Johansson’s (1998) adapted framework in Figure 4. Indeed, ideas about the development of MTE seem to be rooted in teacher educator professional development.

Professional development as a process of continued learning

I will now address the concept of professional development in the context of teacher education. In view of what has been discussed in the preceding paragraph, I find it important to delve into the world of professional development. Mathematics teacher educators’ professional development is a concept taken to mean the process of continuing to learn (Kelly & Praft, 2007) and one may add ´before and after qualification’. In another discussion about making sense of MTE, Fou-Lai and Cooney (2001) use the words teacher education, teacher development, and teacher change in a manner which may help us to visualise professional development as a process. There is a point in emphasising this as development along a continuum rather than a ‘once-and-for-all activity’. In the researchers’ view, teacher education seems to be generally used when talking about pre-service programmes, while teacher development and teacher change is frequently used in relation to in-service programmes. However, ‘teacher change’ seems appropriate when a specific change is to be effected (Fou-Lai & Cooney, 2001)). This expression would suggest some stages in the process of teacher educator professional development, which is in line with Loughran’s (2006) ideas about developing pedagogy for teacher education. In this sense, professional development is a continuum starting with initial teacher education, in some cases followed by induction, and then a series of in-service teacher education programmes, depending on the needs and available support system. One might question whether these make a difference in the classroom or not. This question is not the concern of this study, and may warrant a study of its own in a given context, but the interest is to have an understanding of the process and important considerations for achieving the objectives of a professional development programme. Figure 6 illustrates the important stages of an integrated teacher educator professional development programme. Professional development, irrespective of specific disciplines, is an important means of enhancing teacher educators’ pedagogical decisions, in the same and learning more from closely related experiences in changing the views and practices of those in the career of teaching (Kelly & Praft, 2007; Fou-Lai & Cooney, 2001; Loughran, 2006; Lunenberg, Korthagen & Swennen, 2007; Jakku-Sihvonen, & Niemi, 2006). Making use of some of the ideas from the authors cited, Figure 6 illustrates the process of teacher educators as learners after qualification.

Figure 6 starts with the recognition that professional development derives from both formal and informal sources. Informal professional growth takes place through experience, in clubs, in discussion with peers, and so on. For a detailed discussion of this, Tanzania is used as an example. The first stage represents the pedagogical and content knowledge gained during initial teacher education, which is commonly referred to as pre-service teacher education, the main feature of which is the knowledge and skills gained before and during formal and informal MTE. Informal MTE refers to experience, discussions in clubs, and sharing ideas among teacher educators (peers), to mention but a few.

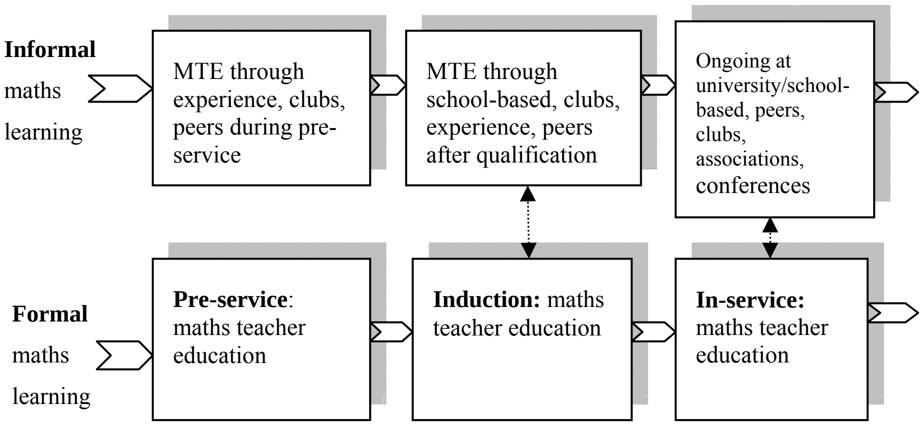

Figure 6. Teacher educators´ process of continuing to learn after qualification

The second stage of professional development is sometimes referred to as induction, which is basically a continuation of the first stage, and is expected to introduce the newly qualified teacher to the new career of teaching. In Tanzania this stage is almost non-existent as a system except for a few initiatives by individual schools and colleges. Ideally, this is enhanced through teacher educators’ mentorship or broadly through MTE development activities (both formal and informal experiences). This second stage is of prime importance as it marks the beginning of expertise, as teacher educators face the reality of teaching. For many, it is a shock, as working conditions do not help them to grow professionally, despite their expectations (Wort, Mmbando & Hardman, 2008; Mosha, Omari & Katabaro, 2007). Those teaching in primary and secondary schools might struggle using teaching methods in overcrowded classrooms, not experienced in teacher colleges. Newly recruited mathematics teacher educators are challenged by changing from teaching school mathematics to teaching and learning about teaching mathematics, as well as finding relevant teaching and learning materials. The gap between professional studies and subject matter teaching is also a challenge, but in this case, learning can take place through taking part in in-service forums, colleagues teaching in nearby schools and colleges, as well as associations and clubs dealing with mathematics.

Theoretically, the third stage represents planned in-service courses for further development of MTE, and is expected at least to lead to expert knowledge in mathematics teaching, and may be related to what Loughran (2006) calls episteme, which refers to expert knowledge in a specific area, derived scientifically. This stage, like the previous two, stems from professional development activities, and is further enriched by local and international exposure of teacher educators in MTE. Mathematics teacher educators' interaction with organisations, associations, clubs and commissions with a focus on mathematics teaching knowledge supports the process of developing expert mathematics teacher educators, which continues throughout their lives. This briefly represents the conceived framework generated from the literature. A comparison of the theoretical view and the reality in Tanzania indicates some serious gaps. My experience in working with teacher educators, as well as my role in MoEVT, leads me to conclude that in-service programmes for MTE in particular are scarce and, if any the ideas are of a top-down nature, they give teacher educators little choice over the content. In addition, the programmes are not always appropriate for teacher educators; they are very often highly structured, and outside the context of teacher educators' work. Above all, there is little one can say in terms of sustainability. Against this background, I now turn to a few studies on professional development in order to shed light and see the important considerations in professional development.

One such study which has dealt with teacher professional development was by Björklund (2008), and although it is directed at language teachers, it can also be used to discuss teacher educator professional development in a broad sense, which is regarded as a process of constantly continuing to learn (Björklund, 2008). Professional development seen in this way is said to focus first on adaptation and second on development. In case of teacher educators, for example, there is a tendency to simply react and adapt to the learning environment without seriously reflecting on the purpose of the learning activity. I choose to deal with the latter as it fits well with the topic under discussion, though both complement other. It is important to note its focus on development or growth in the capacity of teachers for self-management, constantly acting on and shaping the teaching and learning environment according to their needs. This development or growth is thought to consist of five working principles, which are tied together (Björklund, 2008).

The suggested working principles of the approach in professional development-focused as argued by Björklund (2008) consist of the aim of the activity, the character of the tasks, the climate of the organisation, integration between experiential learning and planned professional development, and finally a reflection on the entire process. With regard to aims, teacher educators' ambitions (motives) are crucial for the success of learning, and the tasks given need to provide opportunities to acquire new knowledge and skills. In addition, a supportive development-learning culture is a priority, as it allows initiative and tolerance of different and sometimes conflicting views. It is important to take note of the disadvantage of being excessively goal-oriented and paying little attention to an evaluation of the programme, which is important for taking corrective measures. The principle of planned professional development is of interest in Tanzania on the grounds that it supports the idea of teacher educators taking responsibility for their own professional development. The ground rules argued for are, first, the problem needs to be defined in relation to developmental needs; second, the problem has to reflect the experience of the individual teacher educators and be practice-oriented; third, the programme has to bring together a variety of interested parties in the institutions and experts in the field; and fourth, the programme is prepared with the cooperation of the beneficiaries.

Interestingly, the principles and ground rules highlighted in the preceding paragraph are what Loucks-Horsley et al (2003) summarise in a framework indicating the key tasks of initiating a mathematics/science professional programme, setting objectives, and implementing and assessing a given programme. This will be discussed later, but it can be said that the Loucks-Horsley et al (2003) framework for designing and finally developing science/mathematics continued learning is more comprehensive. The principles outlined in the preceding paragraph are said to be prerequisites of professional development, and both the Björklund (2008) and Loucks-Horsley et al (2003) principles and rules or important considerations are to a large extent relevant to teacher educators as a group. Furthermore, Kohonen's (2006) analysis of the motives behind teachers' professional growth can be extended to teacher educators as a matter of general principle while at the same time recognising teachers as individuals. To that end, the researcher advocates what is referred to as the 'inner capacities' of professional development, namely the ability to recognise the significance of professional interaction for growth, developing an open and critical mind towards professional work, having a personal regard for the process of learning, and developing a reflective attitude as a basic habit of mind. Even more important is conscious risk-taking, as well as learning to live with uncertainty. These carefully thought-out features of professional development seem to be forward-looking, and are of interest. To extend this discussion, the principles and what to take into consideration when designing a professional development programme for teacher educators are elaborated in the following paragraphs.

In line with Figure 7, Loucks-Horsley et al (2003) designed a framework for the professional development of science and mathematics educators, which reminds me of the process I went through several times when working with consultants engaged by the MoEVT. It was only recently when conducting this study, and especially after reading the literature on important considerations in teacher educators' professional development, that I understood the process I had been going through with experts called in to advise on how to enhance the teaching and learning of mathematics in teacher colleges. It was rare for the experts to explain the process or the important considerations. They would just prescribe to me and my colleagues. One of the projects I remember having taken part in, similar to Loucks-Horsley's et al (2003) framework, was about 'the introduction of ICT in teacher colleges in Tanzania'. The intention of the project was to support teacher educators in using ICT as a tool to facilitate teaching and learning (Sida & MoEVT, 2005). This discussion about professional development and related frameworks is relevant in the sense that this study is about teacher educators' conceptions of MTE. With that in mind, MTE is not only an issue of mathematics teacher preparation but also a process of continuing to learn after qualification. Seen in this way, it is worth spending time to reflect more about professional development. In other words, MTE is expected to be a continuous process which starts and continues after initial teacher education. The framework is also considered because of the possibility of using it as a guide during the design of a professional development programme.

Professional development starts with teacher educators' vision

With focus on the framework in Figure 7, the starting-point in the discussion of professional development of mathematics teacher educators suggests taking into account a number of factors. These are considerations which Björklund (2008) calls principles. The framework is a six-stage process, starting with the development of a vision, followed by setting student and teacher educators' objectives, programme objectives, developing and implementing a programme, and finally continuous assessment of the programme. The framework takes into account teacher educators' beliefs, the context and the priorities of MTE, and how to implement them, and these feed into the six stages of the framework itself.

The vision of a professional development is the 'distant objective' of the process of continuing to learn after qualification. Teacher educators would therefore constantly work towards it in order to achieve what is desired. The next step is setting the objectives which have to reflect professional needs. It is important to note that teacher educators have their own theories (knowledge and beliefs) about MTE which may help to feed into learning goals. Discussions could be built around this theme in order to internalise the overall objectives and match personal professional needs. Further, and in the light of the context, issues from both student teachers and teacher educators need to be charted in order to establish their learning goals, which in turn allows for the next step, the setting of goals for the MTE professional development programme. Based on my personal experience of working with teacher educators in mathematics upgrading programmes and elementary teachers' qualification programme, I am of the view that programme goals remain a lifeline for the programme so developed. Then follows an important stage regarding the implementation of whatever has been decided on as the area of professional development in MTE. A number of professional development strategies or models could be applicable at this stage, some of which are to be discussed here, and depending on the needs, some could be used for different categories of mathematics teacher educators. In the course of implementation it is important to keep assessing a given professional development programme in order to determine the level of achievement and establish the basis for programme enhancement.

It should not be assumed that the framework for teacher educator professional development (Figure 7) follows a strictly logical sequence. It does to some extent, but some of the activities may occur simultaneously, depending on the model applied and other considerations. For example, newly qualified teacher educators who are just reporting to the teacher colleges they have been posted to could achieve better results from mentoring, modelling, or lesson study, while experienced teachers could benefit more from action research and community of learners. It is important also to note that assessment of a professional development programme does not stop at determining the success or failure of what was intended. It may go as far as assessing student teachers' achievements in MTE.

Professional development: What to emphasise?

In the beginning of this section I tried to draw attention to the meaning of professional development as a process of continuing to learn after qualification. In addition, the motives behind teacher educators' professional development were brought to light, and its theoretical stages. To show how ideas about teacher educator professional development could be consolidated, a framework could be constructed to incorporate the vision, objectives, plan, and finally implementation of professional development activities. Building on these, what are the areas of focus, and why? What are the strategies for taking the priorities of professional development into classrooms? I understand there is a difference between what teacher educators need and the need of school teachers in professional development. But the issue of their common ground for example teaching and learning mathematics, connections and transferability should not be left to the periphery on the pretext of their different needs. At times and where applicable their similarity has been the basis of a common consideration.

As one reads different literature on, for example, what mathematics teacher educators do in in-service courses, seminars, and workshops, and so on, one obtains categories of performance-enhancing tasks which form part of the answers to the questions raised above. In important ways, the first two questions have been answered in the discussion of content and pedagogical knowledge. In many professional development seminars one finds 'content and pedagogical knowledge, and assessment' as priority areas in MTE, mathematics education and school mathematics (Bregman, 2009; Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003; Dalgarno & Colgan, 2007; Stigler & Hierbert, 1999; Loughran, 2006; Williams, 1998; Carr, 1998).

While content and pedagogical knowledge have been discussed (Attorps, 2006; Bass, 2005; Bullough Jr., 2001; Lester Lambdin, 1999; Shulman, 1986; Shulman, 1987), assessment which enhances teaching and learning has not, and so this important area will be dealt with following the discussion on how to take professional ideas into the classroom. The justification of professional development has been dealt with before, but it is necessary to adapt to new situations, and fill the gaps of both teacher educators and student teachers.

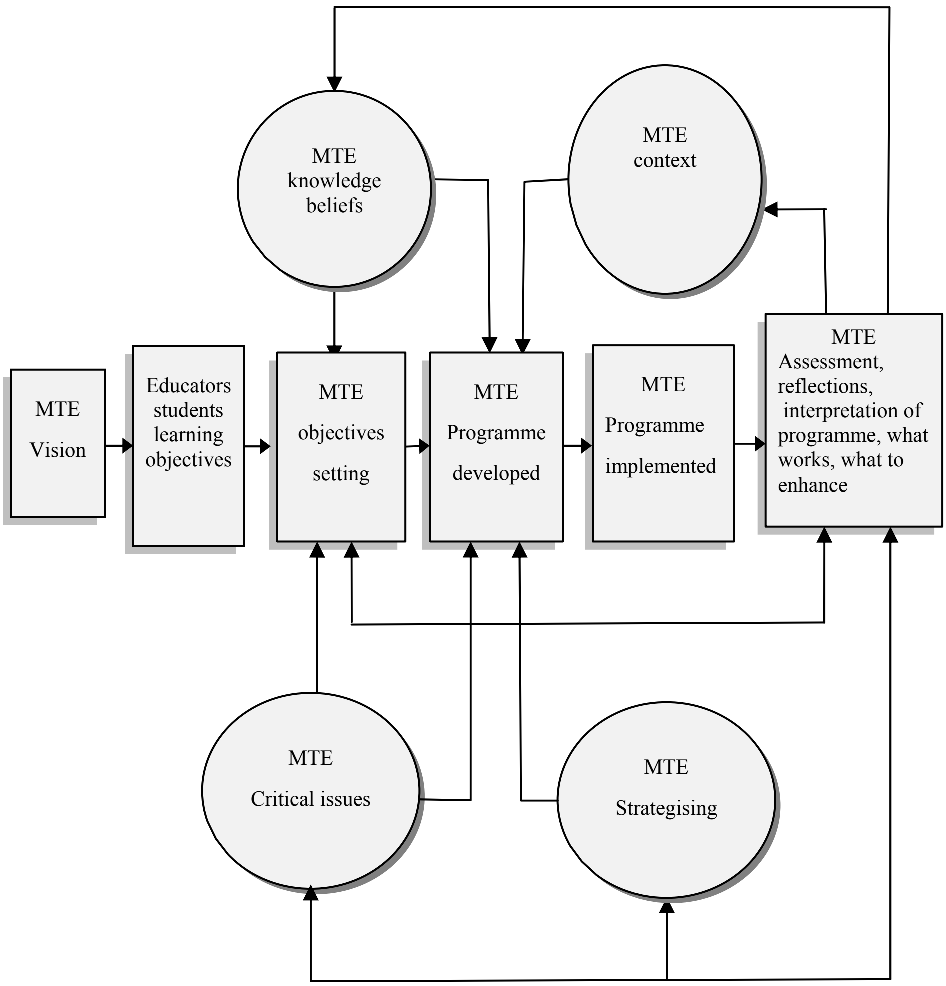

Figure 7. Framework for development of mathematics teacher educators (Adapted from Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003)

For these reasons, teacher educators are said to undergo a process of professional development from carrying out tasks in the way they are used to (traditionally), to developing practical wisdom by which they modify their teaching of the subject. This comes to light as a result of working with learners and a deliberate reflection on the actual content of the subject they are teaching, and not only on how it is taught (Menon, 2009). With that in mind, and the three questions raised earlier, a discussion about critical strategies for carrying out the objectives of professional development tasks seems necessary. This will be followed by a discussion of assessment as an integral part of professional development.

Promising strategies for professional development

I start with a list of 'promising strategies' or models, which serve as the process by which teacher educators gain knowledge, and these are strategies which have been proved to work. The term promising is used to mean a degree of assurance, some greater possibility of working. Table 6 could be thought of as a summary of eight teacher educators' professional development models. A few researchers are selected to represent a wide range of those who have worked on teachers´ and teacher educators' professional development models. Dalgarno and Colgan (2007), for example, have worked on communities of learners and lesson studies in supporting novice elementary mathematics teachers, which focused on providing innovative forms of PCK. This is of interest in this study because of its focus on inspiring not only teachers but it is also applicable to teacher educators with limited experience. In light of Loucks-Horsley et al (2003), a wide range of professional development models in mathematics and science with connections and transferability of instructional strategies between disciplines are discussed. In summary, the researchers provide a discussion of eighteen strategies, which is difficult to accommodate here. For this reason, I suggest dealing with *community of learners, lesson study, demonstration lessons, and coaching, mentoring, modelling. As a point of emphasis, it is also important to add mathematical knowledge for teaching* not as a professional development, but as a focus on possible content. The selection of these models rests on my experience of for example, mentoring and dealing with some communities of teacher educators and teachers as learners. The other reason is the desire to have a collection of these strategies, given that there is no 'one-size-fits-all'. Tanzania, for example, has different teacher education conditions compared to the conditions in which the framework was designed. Table 6 is a summary of adapted teaching and learning strategies or teacher educator development models originating from the researchers cited in the preceding paragraphs. A description of each of the models comes first, followed by a discussion on their potential challenges.

A *community of learners* would normally involve a group of teacher educators who share common interests, knowledge, behaviours, language and practice. In a related study involving teachers, a community of learners is seen as a strategy that allows teachers to mentor and support each other (Dalgarno & Colgan, 2007; Dooner, Mandzuk & Clifton, 2008; Sztajn, Hackenberg, White & Allexsaht-Snoder, 2007). According to the researchers, inquiry is at the heart of the community of learners. This approach to professional learning offers an important opportunity and a distinct form of professional development. The community of learners referred to could also be a group of mathematics teacher educators acting on an ongoing basis to develop their common interests. In practical terms, it involves sharing individual resources and engaging in a critical dialogue, and this approach, in my view, makes it possible for them to take responsibility for their own professional development. It may seem important to ask what the ground rules for the approach to succeed are. As the researchers seem to indicate, there are no rigid rules. The common ones which could be adapted to teacher educators are, first, that teacher educators need to appreciate the demands inherent in the collaborative process of communities of learners. Second, individual teacher educators need to define their actions since they join the communities with different expectations. This is important if they want to create a shared practice. Third, out of necessity, mathematics teacher educators using this strategy are expected to coordinate their activities.

Some of the immediate benefits of the community of learners as a professional development strategy are that more often than not they come from the same area and context. Teacher educators can also directly apply the mathematics knowledge they gain to the particular needs of a college. As has been argued by the researchers, there is an ongoing interplay between the notion of community and its demand for a shared perspective.

Table 6. Teacher educators´/teachers´ professional development models

Dalgarno, Colgan (2007)	Loucks-Horsley et al (2003)	Lunenberg, Korthagen, Swennen (2007)
• Community of learners • Lesson study	• Lesson study • Demonstration lessons • Coaching • Mentoring	• Modelling

The downside of it is that although teacher educators could share space, time and other resources, they may not share the same vision, aspirations or intentions. The starting point of this approach to professional learning could be vague to teachers, as well as teacher educators, as they are very often deceived by what would seem to be shared beliefs, interdependence and meaningful relationships. The state of affairs is challenging, surprisingly ambiguous, and it is no wonder that it is reduced to everybody just trying to get along (Dooner, 2008).

Next, but not in terms of importance, is *lesson study*. There is now a shift to ground professional development in the actual classroom situation (Lee, 2008; Glencoe, 2008). The lesson study approach to professional development appears to be preferred by those in the teaching profession (teachers, teacher educators). With focus on teacher educator development, the principle guiding the conduct involves that the lesson to be improved should be based on research, jointly planned among teacher educators, observed, and followed by reflection on the lesson. The cycle is repeated in that order. In a typical classroom situation the basic lesson study process could be for a group of teacher educators to choose a theme (challenging and college-wide), to set a goal focusing on the student teachers, and prepare a lesson in order to achieve the set aim, and teach and observe collectively. Finally, a group of the same teacher educators would discuss with the purpose of improving the lesson. The entire process is documented and the cycle repeated. It starts and ends in the classroom.

A study comparing the use of the lesson study in teaching and learning (the Japanese experience) and USA research-oriented professional development has been discussed by Crockett (2007), and previously by Stigler and Hiebert (1999). The difference is that the Japanese approach is systematic, and at the local level it is linked to specific nationwide school goals. The USA approach, whether with teachers or teacher educators represents terminal research interventions through innovations and projects. It may start outside the classroom in the form of seminars on pedagogical knowledge and end in the classroom. Experience indicates that the beneficiaries (teachers, teacher educators) may not necessarily come from the same school or teacher education institution and the approach need not be linked to school improvement. What are the benefits and challenges of the lesson study approach to professional development? Drawing from the research information given above, and with some reflection, lesson study as a strategy for professional development is beneficial in important ways. First, it is put into the hands of mathematics teacher educators themselves, and second, learning is seen from the learners' perspective. Even more importantly, the central role of teacher educators in mathematics, for example, is respected. On the other hand, the lesson study approach if applicable to teacher educator development is not free from challenges. A number of pitfalls include pressure on teacher educators, demand for regular attendance, planning, observation, focused discussions, as well as being time-consuming. Furthermore, it may be difficult to work with inexperienced teacher educators and it is a long-term process.

A *demonstration lesson* is another important professional development tool (Harcourt-Heath, 2003; Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003). What is a demonstration lesson in this context? When is it applicable and why? Though a relatively difficult question, the meaning is likely to focus much more on the two terms, i.e., 'demonstration' and 'lesson', and when taken together it refers first to an exemplary model of teaching by an experienced teacher educator (within the context of this study). While the presentation is going on, teacher educators, as well as student teachers, could be observing the lesson with an open mind in order to discuss it.

In an attempt to add sense to this motivating model, Harcourt-Heath (2003) describes a professional development centre in Norfolk in the UK providing demonstration lessons to teachers, giving support to newly qualified teachers and mathematics education subject leaders. On this basis, demonstration lessons in MTE, for example, would rest on a few principles, including a model lesson by the teacher educator, demonstration by oneself and others within teacher educator working groups, reflection on the demonstrations, and observation-based discussions before and after the presentations. Even more important is that the observations and discussions are shaped by clear objectives. Demonstration lessons as a professional development strategy could be applicable in a number of situations. For example, in the case of Tanzania, a demonstration lesson based on the new MTE competency-based curriculum could be prepared, taught, discussed, and reflected on by other teacher educators. The lesson could come from some topic considered challenging in MTE. Experience indicates that the main reason for demonstration lessons is for teacher educators to benefit from practice-based learning. There is much to learn from pre- and post-observation through assessing how one teacher educator approached a given mathematics education topic and the reasons for emphasising certain activities in an MTE lesson. It should also be noted that a demonstration lesson is not free from constraints. First, it is possible to rely too much on the master teacher educator and as such the creativity of others is constrained. Second, the success of pre- and post-observation may depend on the level of collegiality and willingness to share among teacher educators and to learn from one another, as otherwise there may be a lack of constructive criticism.

Coaching, like other models which have been discussed so far, is a practice-based professional development model and provides opportunities for teacher educators to teach about teaching and learn about teaching mathematics. Coaching in particular, according to Loucks-Horsley et al (2003), is a one-on-one learning environment for teacher educators to improve MTE. A number of studies support, as does my own experience in working with teacher educators, that coaching is a professional development strategy providing teacher educators with professional learning opportunities for improving mathematics teaching, by reflecting on their own practice (Loucks-Horsley, 2003; Put, Warren & Herrington, 2004). It is further argued that coaching is a shift from the traditional supervisory learning opportunities to more collaborative peer guidance. It involves interactions and on this basis the strategy provides mutual benefits for the person coaching and the teacher/teacher educator being coached. In my view, this model, also applicable to teacher educator development, is often characterised by facilitation of learning rather than evaluation of practice.

With all this elaboration, one could still ask why and where is coaching applicable? The why part seems to be addressed in the discussion that coaching provides learning opportunities in one-on-one interactions, with mutual benefits for the coach and the person coached. But it is also important when teacher educators are learning a new curriculum or an innovation. For example, when introducing problem-solving in a new mathematics education curriculum, the teacher educator may have several options on how to facilitate learning, depending on the level of teacher educators. Many advocates of coaching seem to agree that direct demonstration through interaction could benefit teachers, and in the same way for teacher educators given the natural relationship between teachers and teacher educators. This approach is relevant when teacher educators do not understand the reason, for example, for emphasising problem-solving (for justification, motivation, recreation and practice (see Stanic and Kilpatrick, 1989). On the other hand, when teacher educators have a fair knowledge and skill in coaching, then a collaborative style may be more effective for professional development in MTE. As Loucks-Horsley et al (2003) argue, a collaborative style of coaching is one where the one doing the coaching and the one being supported engage in a collegial exchange of ideas, planning together and solving problems. Though seemingly an effective professional development strategy, coaching has its own shortcomings. Consider, for example, when teacher educators' willingness to open up their lessons for observation and scrutiny is an issue. Secondly, an approach is needed that is non-threatening, in order to build relationships before any classroom observation. Many schools may not have scheduled time for the arrangement and discussion of classroom observation unless there is some flexibility and voluntary work is offered by the teacher educators. I am of the view that, despite all these challenges, the advantages seem to outweigh the constraints. One could think of the benefits of the personal guidance that teacher educators receive, especially during the early years of their practice.

Mentoring as a professional development model is basically a tool for supporting novices or new teachers (Jakku-Sihvonen & Niemi 2006; Loucks-Horsley, 2003). The same argument can be extended to mathematics teacher educators. Its underlying features include more experienced teachers or teacher educators talking about experiences and problems in an atmosphere of trust and peer support, giving the chance to learn from the understanding of other new teachers and helping newly recruited teachers share experiences of day-to-day classroom challenges. Very often pedagogical and didactical issues are brought up for discussion. If carried out as intended, the immediate benefits of mentorship are likely to include gaining knowledge and skills, correcting misconceptions, overcoming isolation and giving the opportunity for individuals to open up. The intention of mentoring as a professional development model seems good, bearing in mind the one-to-one teacher relationship. Willingness by the newly recruited teacher to be mentored is an important factor, which may not be common to all novice teachers. My own experience is that an individual novice teacher has to take the initiative, as it is rarely arranged by schools or colleges, but is often reduced to private consultation.

Modelling is another professional development strategy for mathematics teacher educators. A report by Lunenberg, Korthagen and Swennen (2007) indicates the importance of teacher educator modelling as a strategy focusing on teacher educators, but it is rarely done, although it has been cited as both the body of knowledge on teacher education and the actual practice of teacher educators. Modelling by teacher educators and learning about what they do in MTE should be thought-provoking when seen as a combination of traditional professional models and more productive pedagogies. Lunenberg, Korthagen and Swenen (2007) view modelling as teacher educators' practice of intentionally displaying a certain kind of teaching behaviour, with the aim of promoting student teachers' professional learning. Modelling by teacher educators is also seen as a way of improving education, since when students are introduced to new practices in teacher education they become socialised in new ways of educational thinking. On the basis of the examples given, they are able to shape their own way of teaching and learning mathematics. In addition, modelling can also improve the teaching of teacher educators (Loughran, 2006). All in all, it concerns the shaping of teaching about teaching, i.e., establishing an appropriate pedagogy for those whose job is preparing mathematics teachers and learning about teaching. This means that in the course of doing the job of a teacher educator, learning by both parties takes place - teacher educators on one side and student teachers as the other beneficiaries. The downside may be similar to that of coaching and mentorship, where willingness or relationship plays an important part.

There is also an emphasis on *mathematical knowledge for teaching (MKT*) as an area of possible content for teacher educator professional development (Bass, 2005). Though initially meant for teachers, it is still relevant for teacher educators on the grounds of connections and transferability of knowledge and skills to enhance teaching and learning mathematics. This may also be regarded as a common issue of discussion in many of the strategies discussed to this point. What is mathematical knowledge for teaching? According to Bass (2005):

> *“.. the term ‘mathematical knowledge for teaching’... represents the mathematical knowledge, skills, habits of mind, and sensibilities that are entailed by the actual work of teaching.... ‘work of teaching’ we mean the daily tasks in which teachers engage, and the responsibility they have to teach mathematics,.... for example: planning lessons, designing and modifying tasks, communicating with parents about their children’s work, progress, introducing concepts, writing,.. Assessing tests... These comprise the specialised tasks which teachers need to know”.* (Bass, 2005 p.429)

With that in mind, a better way of understanding is to think about it in relation to Shulman (1987), who contends that ‘teaching is essentially a learned profession’. It follows then that learning to teach is a lifelong development process that involves the continued deepening of knowledge and skills. The underlying assumption is that teachers as well as teacher educators are learners on their own professional journey.

In order to be effective, and as a way of expanding Bass’s (2005) ideas, three areas of knowledge are important for those engaged in mathematics education. These are a deep knowledge of the subject content, pedagogical knowledge, entailing methods and strategies; and then PCK for teaching a specific discipline. This involves a special emphasis on methods and strategies that have been proved to work in relation to teaching and learning the particular content area in mathematics. All in all, MKT goes beyond conceptual and procedural knowledge of mathematics to include in essence pedagogical content knowledge.

Reflections on professional development strategies

At one time, Schoenfeld (2002) said that to fail children in mathematics (I would add, to fail someone in MTE) or to let mathematics fail them is to close an important means of access to society’s resources. To let professional development fail teacher educators is to close practice-based effective teaching and learning opportunities. This is my starting point in a short discussion about the downside of longstanding professional development strategies. Take, for example, traditional and popular workshops, the training of trainers and guest speaker series. Many of these general strategies have inherent problems and therefore provide neither the content nor the opportunities which teachers view as essential for their professional growth (Dalgarno & Colgan, 2007; Nessoro, 2005).The present practice-based learning opportunities for teachers, and I would also add teacher educators, have arguably relied primarily on transmitting ideas of teaching and learning through top-down, hierarchical structures, and Tanzania is no exception. Moreover, activities for developing MTE have been carried out outside the context of teacher educators’ actual work. Furthermore, the nature of MTE practice-based learning opportunities gives teacher educators little control over what to emphasise, and very often they are held at an inappropriate time in the teachers’ college calendar. Reflecting more on the kind of teacher development opportunities in Tanzania, one can really work hard to find a sustainable or a strong and collaborative professional development programme similar to the Japanese lesson study approach seen previously, which can bring about a change in teaching practice and student achievement, rather than the one-off seminars and workshops which are frequently featured. It is important therefore to generate dialogues about how to ensure that teacher educators get what they want, not what experts think they need.

Finding a framework for an inductively created solution for MTE to improve its practice needs a lot of thought. Two available professional development models are those of Loucks-Horsley et al (2003) and Heck et al (2008), but they need to be modified, for the simple reason that MTE is context-specific. The former seems to be better positioned to work with teacher educators as it provides a step-by-step generation of ideas to take teacher educators through the entire process, from forming ideas of professional development in MTE to implementation (see Figure 7 for the framework of professional development).

In order to achieve this step-by-step careful planning in terms of phases, it is suggested to start with what is briefly discussed as shown in Figure 7, which is a modified framework for developing MTE (Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003). From my understanding, this is just a tool to guide and remind those involved in professional development to consider and start with teacher educators. A pool of models without a vehicle to propel them goes nowhere, and that is why I started by emphasising teacher educators' vision of what they want, which counts as professional development in mathematics. Therefore it is important to set goals, develop and carry out a plan with integrated assessment of what is being achieved, and finally do a summative assessment, which is the beginning of another cycle that reflects on and assesses the programme's objectives to see what worked, or what did not work, and why, and what needs to be improved (See Figure 7). This brings us to the crucial issue of assessment in teacher education, which is discussed next.

To conclude the discussion on teacher educators' professional development is challenging because of the many players involved. I started with the meaning of professional development, which is the process of continued deepening of mathematics teacher educators' knowledge after qualification. Next, I attempted to shed light on the broad and individual driving forces behind professional development, before suggesting a framework for the steps needed to be taken to get started. This led to the suggestion of possible professional development strategies which can take pedagogical and subject matter knowledge into the classroom: for example, community of learners, mentoring, demonstration, modelling, and lesson study. Traditional approaches to mathematics teacher educators' professional development, which are very often top-down, seem not to meet their needs, nor are they under their control. An argument often advanced is that job-embedded, practice-based and collegial forms of mathematics education professional development are widely accepted by teachers (Loucks-Horsley, 2003), and teacher educators are no exception. Therefore, professional development programmes which have been inductively developed are expected to enable teachers and teacher educators to learn in and about their practice. One general conclusion that may be applied to such programmes is that a professional development model that includes both subject matter of mathematics and pedagogy would be more beneficial to teacher educators and teachers, rather than either one on its own (Putt, Warren & Herrington, 2004).

Aligning assessment in MTE with goals of professional development

The discussion of assessment in teacher education with regard to MTE will focus on the rationale for and the purpose and modes of assessment. My topic is about teacher educators' conceptions of MTE. This phenomenon of interest starts during pre-service teacher education and goes on after qualification. All along, teachers (Williams, 1998) and in the same way, teacher educators would naturally like to know how they are performing. This calls for a certain kind of assessment. It is for this reason that assessment is integrated in the entire process of teaching and learning. Student teachers are likely to focus on what is assessed, and in the same way, teacher educators are likely to emphasise what is likely to be assessed. Concerning the rationale I will incorporate some philosophical points of view about assessment while reflecting on the Tanzanian experience. The term philosophical point of view is taken to mean theoretical ideas about assessment.

To discuss the rationale, purposes and modes of assessment in teacher education without establishing their meaning is likely to create a conceptual gap, and so I will spend time on the rationale for and purposes of assessment. The literature regarding assessment describes it as 'a formal attempt to determine students' status with respect to educational variables of our choice' (Popham, 1999). In a strict sense it is finding out how much learning is taking place. It can formally be done before, during, and after the teaching and learning process. Ongoing assessment while learning is in progress is commonly referred to as formative or continuous assessment, as it is commonly known in Tanzania. On the same basis, summative assessment is done at the end of a programme in order to determine whether the set objectives have been achieved. The notions 'assessment' and 'evaluation' are difficult to separate. Very often, evaluation is linked to the process of making a value judgment on the results of assessment - hence the statement, 'to assess is to evaluate' (Hornby, 2000). The downside and benefits of formative or summative assessment will be discussed, alongside ideas for improving assessment in MTE, following a discussion on the rationale for and purpose of assessment in MTE.

The rationale for, purposes of and a reflection on assessment

The rationale for and purposes and modes of assessment can effectively be drawn up and argued for, despite the inherent challenges (Baartman, Bastiaens & Kirschner 2007; Howson, 1993; Niss, 1993; Williams, 1998). It is not my intention to give a detailed account, but knowing that researchers have established that a strong relationship exists between instruction, learning and assessment, it can be argued that assessment reveals how student teachers learn and how teacher educators teach, with the result that both teacher educators and student teachers focus on what assessment requires, and this is what connects my research topic and the issue of assessment. Thus, assessment is ever present in many of the teacher educators' tasks of teaching about teaching and learning about teaching.

Regarding rationale, first of all there is a common understanding that the roles, functions, and effects of contemporary modes of assessment are neither clear nor well understood by teachers, teacher educators and the public. Second, the present assessment modes and practices have conflicting interests, divergent aims, and unintended side-effects. In Tanzania, for example, experience indicates that assessment creates apathy in teaching and learning and MTE because of mass failure. Third, it is challenging to devise harmonious and acceptable assessment tools because of variations in contexts. However, researchers agree on certain crucial points. The reasons for assessment seem to be cross-cutting, but are not exhaustive.

Again, in the discussion on assessment Williams (1998) goes to great lengths from a theoretical point of view, showing how assessment is gradually being considered a top priority by teachers, learners and parents. In the case of teacher education it may signal the end of a course and the beginning of a life-long career. Unfortunately, assessment has not been a traditional occupation of philosophers of education (Carr, 1998). Only recently has it started to be an area of interest, especially after the introduction of notions such as in-school curricula, and MTE is one such curriculum. For MTE, it means the competencies needed to function as a maths teacher, during and after qualification (Emanuelsson & Sahlström, 2006). In the interest of working with examples reflecting a typical teacher education classroom in Tanzania, it may include the knowledge and skills needed to prepare schemes of work, lesson plans and teaching/learning materials, provide for the active and mental engagement of students, develop an open-minded stance and question conclusions. In the traditional way of learning to teach mathematics, it has been expected that if learners are provided with mathematics knowledge, they will routinely transfer this to many other new learning situations. An assessment of this would mean seeing how the possession of numeric skills is applied to different situations as they arise. However, experience has shown that making learners (student teachers included) knowledgeable in mathematics does not necessarily help them to do this, especially when problem-solving skills are required before advancing to another stage.

To interpret Carr´s (1998) and Williams´ (1998) discussion of competency-based education, what supporters of a competency-based mathematics curriculum suggest is that internalising mathematics knowledge alone does not guarantee skills competency. The role of teacher education therefore is to help student teachers of mathematics to develop competencies which will help them in diverse classroom situations. In a fast changing world, educational systems have to cope with changing purposes and means of teacher education. Consider, for example, cases of overcrowded classrooms, and limited physical and teaching learning materials in Tanzania. By developing new ways of supporting mathematics teaching and learning, student teachers may recognise problems and find solutions for different classroom conditions. From Carr (1998) and Williams (1998), it is possible to interpret competence as the possession of knowledge and skills enabling an individual to deal with both routine and non-routine and abstract work processes. This suggests that competencies are a mix of cognitive skills, and interpersonal skills, which enable teachers to take action and make pedagogical decisions. On this basis, with the many differing views about the purpose of education, it sounds logical to assert that one central aim of educating people is to help them develop various competencies in the work they are doing or going to do, MTE being part of this important purpose.

It was not until the introduction of competency-based education systems that assessment became an issue for educational philosophers (Carr, 1996; Davis, 1995**)**. Why has assessment become an area of interest now when it was not so in the past? There are many reasons, but it is worth mentioning a few. When assessing, for example, mathematics computational skills, concepts and problem-solving abilities, there are issues of truth, accuracy (objectivity) and fairness. This view is supported by Williams (1998) and Schoenfeld (2002), who also see a connection between assessment and human learning. Experience indicates that human beings want to know (want an answer to) something as simple as the number of family members, to have the ability to count, to manage financial accounts, and to use modern technology applied to financial accounts. Mathematics teachers, on the other hand, would like to know how they have been progressing or simply learning in a given teacher education programme. On this basis, notions of assessment or degree of success have been associated with learning.

One of the most common features in determining the degree of success in learning among human beings is comparing learners with norms, and this is taken to new heights when many education systems (including Tanzania) want to compare the performance of learners, which induces intrinsic competition, as argued, for example, by Carr (1998). The issue of comparison becomes important in many ways. For example, an athlete practising alone may consider himself/herself the best if personal performance is not compared with other athletes running the same race. However, failure to meet criteria of success, as may be indicated by comparison using norms, does not diminish our value as human beings, but it simply means that we lack ability in a certain human activity, and it should not lead to condemnation. Therefore, there is nothing inherently objectionable in the notion of norm. As Williams (1998) argues further, learners' ambitions tend to exceed their abilities, and therefore assessment provides a way of establishing the relationship between their ambitions and their abilities, which in turn establishes the relationship between one individual learner and others through norms. Having shown why assessment is an area of interest at present, we now look at the purpose of assessment.

The purposes of assessment

The purposes of assessment are inspiring to examine from a practitioner's point of view because of the close relationship between teaching and learning mathematics. The integration of assessment tasks or activities in the process of teaching and learning has a bearing on what MTE is all about. Niss (1993) and many of the researchers previously cited believe that most purposes of assessment in mathematics are not unique to mathematics education, except for their focus, which is the same for MTE regarding the process of teacher development. Most of us have had the experience of being assessed at the end of our teacher education programmes to mark the end of the course and the beginning of a career as a mathematics teacher educator. Some judgment has to be made to determine the benefits and achievement. Taking Niss's (1993) reasons into account, the purposes of student assessment in both MTE and mathematics education, if we do not institute a strict dividing line between, fall into three main categories. The first is to provide information to both individual learners and teacher educators for further reflection, and perhaps take corrective measures for enhancement. The second is to establish the basis for decision-making or some other actions, for example to select individuals for opportunities, positions, jobs, and the licensing of teachers. The third is to reassure society that such people are suitable. However, to argue that the intention of assessment is only to provide information concerning these things is an oversimplification.

Unlike Niss's (1993) pragmatic view of assessment, Williams (1998) raises further theoretical questions pointing to the epistemological benefits and disadvantages of assessing abilities, taking into account his own argument and that of Schoenfeld (2002) on making mathematics work for all children, where for example, standards, testing and equity are emphasised. According to Schoenfeld, it is plausible to argue that assessment is compatible with the normal functions of teaching and learning, whilst at the same time being a tool to compare performance. On this basis, it is in line with knowledge claims because it supports learning and is therefore educative. Assessment, by its very nature, has inherent absoluteness, accuracy and fairness. Secondly, assessment may also help to check the consistency or relevance of what is taught and examined in educational institutions. In mathematics, for example, take the case of questions about the speed of a bullet train, or the escape velocity in physics classes. Certainly it raises concerns about relevance, and therefore a degree of truth, when the speed of a bullet train is used with learners who are not in a position even to imagine that context. Regarding this, there are certainly competing interpretations about what is to be selected as mathematics curriculum material in teacher education, the choice of examination questions and the pedagogy to be used. In the words of Giroux (1998), it is the same as labelling knowledge through examination. Hence issues of absoluteness, accuracy and fairness exist in the process of gaining knowledge.

Critical reflection on assessment

There is also a downside to assessment, as argued by opponents who question the soundness of the accepted ideas. What are the major arguments against assessment? A number have been advanced, and I will take one case at a time in the interest of seeing the weight of each one. First, attempts to assess learning commit teachers, test designers and examiners, to notions of complete detachment and objectivity regarding the definition of what is to be learned and the quality of learning. In more critical words, assessment is a threatening exercise and not motivating at times. Some reflective practitioners and philosophers, for example Illich (1970), Reimer (1971) and Freire (1972), caution that grades tend to degrade. Second, Broadfoot (1984) as cited in Carr (1998), as one of the opponents of assessment, argues that assessment is an instrument of social domination with assessment as its cutting edge. It seems complicated to accept this because one cannot be sure that it is done deliberately. More often than not it is a spontaneous result following decisions by those in power with a range of vested interests. The relationship of assessment and the exercise of power can be scientifically arrived at. It may be fair to assert that the status of examinations is less related to the possession of knowledge and skills than to a certain stage of accreditation. Following the arguments by Carr (1998) and Williams (1998), opportunities to pursue further education, for example, appear to be related to the possession of qualifications rather than to the character of what has been assessed as knowledge possessed. This is exemplified by associating qualifications with social status. In Tanzania, for example, at one time, certain schools were regarded as high-status schools, for instance Tabora, Mkwawa, and Pugu. It was assumed that anybody who graduated from these schools was knowledgeable because of the social status of the schools at that time.

The next question in the analysis of assessment is how do we achieve an effective system? I will use Davis's (1975) claims to present the role of future assessment in MTE. One important suggestion is how to shift from what is claimed as rich understanding of mathematics to what tasks student teachers can perform. Just like the criticism of assessment of abilities in the norm-referenced approach, there are also strong criticisms of the criterion-referenced systems of assessment, in particular with reference to MTE, where the performance of certain tasks is normally referred to as the criterion. The first criticism is that there are reservations about many systems of assessment experienced during teaching which claim objectivity. Second, assessment in education, be it in mathematics or other subjects, is inherently inexact and it should be treated as such. Why these reservations? Carr (1998) and William (1998), in support of the impossibility of objectivity, claim that human ability cannot be measured with the kind of accuracy which applies to measurement in the physical world.

On the basis of the reservations about criterion-referenced assessment, there are general concerns about assessment and performance. The first concern is the impossibility of assessing rich knowledge or abilities using any form of assessment. It is difficult to assess the quality of abilities in the minds of other people. Second, in the eyes of the public, the status of individuals is of less importance than the operations they can perform. The way I see it is that what counts as truth from a pragmatist's point of view puts emphasis on practical use. Third, there are differences between individuals' knowledge of the same subject when assessed with a range of different measures of knowledge. Fourth, there is a tendency to value the process of learning rather than the product of learning. The issue is what characterises human intelligence - it is the actions rather than their prior processes, which follow some prior understanding. How do we trace prior understanding, or what the learner already knows? It is a difficult and endless task. What is of value to the public, who will judge whether something works or not? There are also concerns specific to Tanzania.

The inflexible views held by some teacher educators in Tanzania regarding Bloom's Taxonomy (1956) need to be received with caution. In a discussion about a framework for assessing the new course for the diploma in education

(TIE, 2008; NECTA, 2008), teacher educators reached a compromise concerning the outgoing and new curriculum in a challenging situation. They kept referring to Bloom's Taxonomy (1956), probably because of the long tradition of using it. During the many years of actively teaching mathematics in secondary schools and teacher colleges, I experienced the same. The six categories of the cognitive domains as established by Bloom's Taxonomy attempt to assess knowledge, comprehension, application, analysis, synthesis, and evaluation. As teacher educators the skills part becomes a difficult area to assess and the focus is thrown onto to the six categories of the cognitive domain. Typical examples as student teachers practise using the six cognitive domains would proceed as follows: a) What is the value of the digit 5 in the number 45634?; b) Which of the following is a set of natural numbers: 0, 1, 2, 3, 4 and 1, 2, 3, 4?; c) What is the area of a room with dimensions 3.4 metres by 4 metres?; d) What is the result of multiplying 12,345,679 by 9, and how can one use the product to compute 12345679 × 54?; e) How can you proceed to establish the area of a triangle?; And finally, f) Which of the following quotients is greater than the other: 46.371 ÷ 0.3 and 927.4 ÷ 0.6?.

Students teachers are then asked to think about the questions and fit them accordingly into the six cognitive domains. Thinking more about these questions, and relating them to the six cognitive domains, it is possible to deduce that item a), for example, is assessing knowledge, while item b) is assessing comprehension, and items c), d), e) and f) are respectively assessing application, analysis, synthesis, and evaluation. I have nothing against student teachers learning about the six cognitive domains, but the interest has to be on using the results of assessment based on this style to enhance teaching and learning. The focus of assessment under this arrangement could also be the source of the fundamental variations in the meaning and purpose of assessment in MTE, because teacher educators might have different emphases in terms of the six categories of the cognitive domain.

What follows now is a round-up and a consideration of the next steps in enhancing assessment. In spite of the failure of assessment to measure student teachers' rich knowledge, examination performance seems to remain as a reasonable indication of understanding mathematics as a subject. It is therefore the task of educators to consider assessment systems which are capable of probing the quality of understanding as well as informing learners about their performance. One important fact about knowledge and skills is that both exist in intricate and open-ended networks, which are difficult to represent in a comprehensive set of assessments of the examinee. It is possible to conclude from this that it is difficult to define exhaustively the nature of the domains of knowledge and to specify comprehensively a set of tasks that make up a skill. Based on this argument, it is even more complicated to devise a set of tasks to guarantee that individuals have mastered an area of knowledge or a skill, which the syllabus was designed to enable.

Dealing with assessment policies is demanding and complex. What is now the direction of reform in assessment given a challenging situation if comprehensiveness and exhaustiveness are to be achieved? I would like to discuss the direction of assessment in mathematics teacher education by drawing on studies by Niss (1993), Schoenfeld (2002) and Williams (1998). One overarching aim is that assessment should be a means of enhancing growth to meet high expectations and supporting high levels of student learning. Regarding this point, I have two interests to declare about assessment in MTE, the first being to have working ideas on how to improve the assessment system, and the second to have some kind of guide for action in assessment. Niss (1993) seems to provide both from a practical point of view.

It can be said that any mode of assessment is context-specific. For this reason, it is important to consider a number of factors which are also context-bound in the course of assessment. First, the subject of assessment involves knowing who is to be assessed and what units – i.e. individual students, a working group of students, the class, and so on. Second, the object of assessment has to cover what is to be assessed – i.e. what type of mathematical content and methods, and what type of student ability. The focus of any assessment needs to be clear. Third, and not in order of importance, are the items of assessment, which include the kinds of output to be assessed, and the way objects are developed into assessment tasks and demands. The fourth concerns the occasions of assessment, and this is about the timeframe – for instance, during or after the course or at the end of the entire programme. Mathematics teacher educators have experience of formative and summative evaluation and of the benefits and disadvantages of each. I have seen the strength of formative evaluation when serving as a mathematics teacher educator, as it provides systematic and ongoing feedback on learning, an opportunity to reflect on teaching and learning, as well as continued learner engagement in the learning process.

Assessment is not only a matter of two people, the educator and the student teacher to be assessed, but it can also stretch to small or large assessment systems. Important consideration has to be given to procedures, circumstances of assessment, and responding to questions like what happens and who is to do what on assessment occasions. Assessment, like evaluation, is thought-provoking because it involves judging and recording assessment stages. A teacher educator, for example, needs to keep track of the progress of his/her student teacher before, during and when leaving to begin his/her career. What is emphasised is what criteria and procedures are used for judging the assessment items. This is important to avoid what other professionals in the field would label a double standard. I have in mind my experience in working with teacher educators in Tanzania when mathematics marking schemes were used as canons with very little flexibility for students who could have other options in answering a specific question.

Finally, assessment is about what to report, and to whom, and the shape of the results, which involves issues of the style of releasing assessment results, fairness, and validity. I am of the view that if this is not properly done, it could be the source of appeals. This is important to understand when dealing with both small and large-scale assessment modes. I have come to learn that assessment in all its forms, from who is to be assessed, to the focus and tools of assessment (exercises, quizzes, tests, examinations), is being continually shaped and is always challenging.

The existence of challenges should not make us shy away from the knowledge and skills we want learners to acquire and it should not be the reason for failure to specify as closely as possible the assessment forms which promote learning. In the light of all this, the following ideas may form part of the direction for reform of assessment in MTE. First, in education systems driven by examination performance, the promotion of assessment practices which have been proved to work is of crucial importance. Consider Tanzania as a case in point, where assessment final results are on a pass or fail basis and not ranked, for example, as 'excellent', 'very good', 'good', 'satisfactorily' and 'fail'. I am aware of the complications of doing this, especially when it involves the assessment of teaching skills. It may amount to labelling prospective teachers. If the assessment of teacher education involves a grading system, it is likely to attract attention and even more so in the case of mathematics, since it is an area of intense interest. Parents and student teachers believe in the value and meaningfulness of traditional skills-based tests. Any reform is therefore likely to be challenged (Schoenfeld, 2002), and so it is important to use a combination of assessment forms to strike a balance between traditional ways and the proposed reform. For example, open-ended questions, practical essays, portfolios, teaching practice, micro-teaching, class assignments, discussions and presentations where appropriate will allow for a broad spectrum of assessment methods that are appropriate for student teachers to become acquainted with. To be specific, I am referring to considering assessment as a tool to check the mastery of teaching knowledge and skills, concepts, problem-solving, reasoning and communication, using a combination of techniques.

Second, there possibly needs to be a shift away from the misguided idea that students and student teachers have to concentrate first on skills before dealing with concepts and problem-solving. I think this might create a culture of drill masters among teacher educators. On this, Schoenfeld (2002) is of the view that skills, concepts and problem-solving can be developed simultaneously rather than with a linear approach. This approach is similar to mathematics for problem-solving or even better MTE via problem-solving. Research has already shown that students in reformed curricula do just as well as those who focus on 'skills first' (Schoenfeld, 2002). This line of thinking might not be welcomed at the beginning unless parents and other stakeholders come to understand that the 'skills first' option is based on false assumptions or is only a belief. It can be more motivating for teacher educators to simultaneously test skills, concepts, applications and problem-solving than using the 'skills first' approach.

Third, teachers, parents, teacher educators, policy-makers, as well as politicians and the general public, may be brought to understand the great variation inherent in assessment. Williams (1998) is of the view that human judgment is required in evaluating students' mastery of any human endeavour, and MTE is a part of this. There will always remain an element of uncertainty because assessment is an activity conducted by imperfect humans acting in an imperfect universe (a changing environment), and imperfect test instruments may also contribute to assessment in mathematics education being viewed as highly subjective. On the same premise, if assessment by its very nature is educative, one may conclude that there is no absolute knowledge with reference to assessment. This view is coherent with philosophical positions on the claims of knowledge in general. Finally, I would like to think that assessment in MTE determines student teachers' lives in the sense that professionalism starts with initial teacher education and not after qualification. On these grounds, assessment needs to be conducted and moderated by experienced teacher educators.

Concluding thoughts

In this chapter, the changing focuses and purposes, shifts of thinking, and negotiated meanings in MTE from the 1960s to more recent developments have been discussed. This was important in order to set a chronological point of departure for the study. Discussion of the changing focuses and purposes helped to shed light on how the meaning of MTE has been negotiated along the timeline. The major shifts of thinking which have evolved from the 1960s appear to focus on computational skills, investigation and problem-solving, which were taken into the classrooms through programmes built around the commercial arithmetic of the early 1960s, modern or new mathematics in the two decades which followed, and basic mathematics in the 1980s and years after (Sichizya, 1997; Kita, 2004). This also implied differences in the focus of teacher education from principles and methods of teaching in the early 1960s (especially for those trained to teach in middle schools) to emphasis on subject matter knowledge, which lasted until the sudden pendulum swing to methods of teaching mathematics in the 2000s. The many faces of MTE manifest themselves in the form of the commercial arithmetic of the 1960s, new mathematics of the late 1960s through 1970s and basic mathematics of the 1980s and beyond.

How does the research problem show itself globally given the local situation? It was important to consider this question in order to delineate the differences and similarities between the two contexts. It has been argued that the global MTE perspectives are not the standpoint of the researchers whose research has been consulted, nor is there a one-to-one relationship between them and the perspectives developed. However, it remains a fact that there are common key processes or notions. Thus, MTE is characterised as a composite of many influences and aspects. Further, MTE is seen as 'the process of one becoming' a mathematics teacher, while others characterise it as a skilful blend between content and pedagogical knowledge with which prospective teachers are engaged (Shulman, 1986; Shulman, 1987; Lester Lambdin, 1999; Bass, 2005 & Attorps, 2006). Finally, it is seen as involving two main tasks - teaching about teaching and learning about teaching (Loughran, 2006), a perspective derived from the field of teacher education.

The global perspectives of MTE may appear the same from the outside, but different from the inside given the local conditions in Tanzania, for this may have an influence on how teacher educators think. The discussions about the composite nature of MTE, the process of one becoming a mathematics teacher, for example, means different working conditions for teacher educators in Tanzania. The quality and qualifications of 'teachers of teachers' do not necessarily mean those who have the experience and excelled in mathematics. There are differences in mathematics teacher educator qualifications, for example, level of experience, as well as quality and; not all are university graduates in mathematics education. The issue of teaching/learning materials to support both teacher educators and the over 7,500 student teachers trained each year, as well as the shock of the overcrowded mathematics classrooms they will meet after qualification, have a bearing on the meaning assigned to MTE (MoEVT, 2008). Again, the balance between subject matter and pedagogical knowledge which triggered the concerns might look a problem of the past elsewhere in the world, but the pendulum swing is not always between content to pedagogical knowledge and vice-versa; it can also swing to research in mathematics teacher education/mathematics education. When this happens, the global situation of research in mathematics education is far more demanding than has been reported by Lerman (2001) and Roschelle, Singleton, Sabelli, Pea, & Bransford, (2008). There is even a much bigger gap in research-based knowledge on MTE given the concerns raised from the voices inside Tanzania in Chapter One.

In view of the global discussion of teacher educators' professionalism regarding MTE, the notion professional development was taken to mean the process of continuing to learn after qualification. Next, followed a review of ambitions (motives) that influence teacher educators and teachers in general towards professional development and the steps in professional development from initial, induction, to in-service teacher education and beyond. For the purpose of being pragmatic, a framework on how to get started in MTE professional development has been suggested. In turn, models which could take professional development ideas into the classroom has been proposed, and they include community of learners, mentoring, and lesson study, to mention only a few. There is a general argument that professional development opportunities, which are tied to teachers' work, are under their control, and are school/college-based, which is widely accepted by teachers (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003). One general conclusion that may be applied to such programmes is that a professional development model that includes both subject matter of mathematics and pedagogy would be more beneficial to teacher educators and teachers than either one on its own (Putt, Warren & Herrington, 2004). All along the issue of aligning assessment and the goals of mathematics teacher educators' professional development has been emphasised. The place of assessment in MTE was finally analysed in more detail with respect to rationale and purposes, and some critical but forward-looking reflections were included.

The global views of MTE have similar features in many contexts, but say little on what stands in the way of maths teacher educators' professional development given local conditions in Tanzania. From a very general perspective, Galabawa, Senkoro and Lwaitama (2000), and also Stigler and Hiebert (1999), state that a number of factors (in many countries) constrain teacher educators' professional development. In the process of working with teacher educators in Tanzania for more than twenty three years, I have come to realise that teacher education, as well as MTE regard teacher education as a 'once-and-for all' activity, and this has become a strong belief (myth). Next is a general misguided view that anyone can be a teacher educator. This has a far-reaching effect in terms of resource allocation for professional development despite the fact that MTE is very often in the spotlight. Thirdly, at local level the source of teachers engaged as teacher educators is rarely defined and this makes the problem unique to Tanzania. Globally level engagement of teacher educators normally originate from among competent teachers according to subject areas with sufficient classroom experience (Lunenberg & Willimse, 2006). The sum of all these issues and specifically, insufficient expertise in teacher education, accompanied by low teacher educator qualification, their inability to integrate teaching, learning and assessment, varied teacher educator career paths, low public recognition and not to mention inadequate research-based knowledge make working conditions for mathematics teacher educator in Tanzania unique.

Finally, the changing focuses, the local to global unfolding of MTE, and teacher educators' professional development regarding mathematics tell us, first and foremost, of the uniqueness of the domain from the global point of view, and at the same time some similarities. Both the similarities and differences reveal only glimpses of the meaning assigned to MTE by teacher educators. Thoughts about MTE professional development vary and include issues around the integration of teaching, learning and assessment.

3 Methodological research solutions

This part of the study deals with the methods of inquiry. It covers the research questions, research design, subjects of the study, data collection techniques, coding process and data analysis. These methodological considerations are necessary in important ways. The research questions, for example, are central in guiding the investigation process, in a specific way they restate the research objective, and determine the research approach. On the same basis, the research plan enabled me to have a general approach on what research tasks were needed and when to carry them out, and this involved time for piloting the research questions and data collection. In a strict sense, it guides the entire process from designing and shaping a study through methods to thesis writing. The subjects of study or interviewees were involved in the collection of useful information to answer the research questions. This was captured through the use of interviews and open-ended questionnaires as data collection techniques. Finally, coding and data analysis were carried out, which involved a series of connected activities from a careful reading of teacher educators' statements in order to get a general picture, to sorting out the statements according to key words, to developing tentative categories of descriptions according to similarities and differences. Each of these tasks will be discussed under appropriate sections of this chapter.

3.1 The guiding research questions

In view of the background, motives and purpose of the study, I find it necessary to raise relevant questions about 'mathematics teacher education' to the strategic practitioners, rather than accepting straightforward solutions. This is because the community I have been working with seems to possess mixed ideas and sometimes a contradictory grasp of the situation. Posing scientific questions in order to understand a situation can be done in different ways. Some researchers may be interested in finding out what reality is like and why. Others may want to focus on what kind of conceptions and perceptions individuals have of a given object (Marton, Beaty, & Dall'Alba, 1993), or on a 'situation', an 'event', a 'programme', to mention but a few cases. This study aimed at identifying teacher educators' conceptions of mathematics teacher education and gave a description of their variations. Within this broad aim, thoughts for further development of MTE have been identified and their variations described. The theoretical background has indicated that there are concerns about the meaning assigned to MTE, as well as what ideas are held by teacher educators for further development of the field. I find this an opportunity to ask two questions to guide this study:

a) *What are teacher educators' conceptions of mathematics teacher education?*

b) *What are teacher educators' thoughts on the development of mathematics teacher education?*

The first question addresses teacher educators' conceptions, which may be revealed by teacher educators in terms of sense of meaning, perceptions, perspectives, views, understanding, ideas, images, impressions, and beliefs associated with MTE. It is important to caution that by sense of meaning I do not mean a formal definition, but rather the sense that comes to mind to make a word, a phrase which is sensible and comprehensible. The second question, which stems from the main question, seeks teacher educators' thoughts which during the interview may appear as consideration, ideas, reflections, contemplation, perceptions, perspectives or directions of thinking, understanding, and views for further developments in terms of possible knowledge and skills in MTE. Specifically, it concerns areas of focus now and for further development. The difference between the notions 'conception' and 'thought' in this study rests on the nature of the research questions. Except for the two closely related phenomena of interest (MTE and MTE development), conceptions and thoughts are equally closely related. However, I am of the view that conceptions seem to remain at the seat of conventional phenomenography, while thoughts appear to occupy the same area but are also more dynamic. Again, this does not necessarily mean a specific strategy, rather thoughts which may help to get started towards a professional development route in relation to MTE.

3.2 The qualitative research approach: Choosing phenomenography

A qualitative research approach is taken as the methodological solution to investigate teacher educators' conceptions of mathematics teacher education as a phenomenon of interest. Marshall and Rossman (1999), as well as Cresswell (1998) are among the exponents of qualitative research, where unlike the quantitative research approach, in which numbers matters most, words and pictures from participants, for example mathematics teacher educators, can be analysed to reveal meanings of a subject of interest. About a decade earlier than Marshall and Rossman (1999), Strauss and Corbin (1990) viewed a qualitative research approach as an approach in research that produces findings not arrived at by means of statistical procedures or other means of quantification. I very often associate the research approach as one capable of generating findings in terms of distinct attributes as a result of data analysis. In this case, the qualitative research approach is selected for the following reasons. To start with, it is neither based on my personal liking, nor any conviction regarding qualitative research designs, but rather on the nature of the problem under investigation. Teacher educators' mixed ideas and contrasts seem to be based on their perspectives, their lived experiences and their insights, which cannot simply be reduced to numbers. Second, the nature of the research questions suggests the possibility of dealing with a collection of qualitative data. The research questions and the motive for the study are built around differences of understanding of a problematic situation of MTE rather than the number of teacher educators with a given conceptual understanding. Thus, there is an appropriate match between the research questions and the choice of a qualitative research approach.

Furthermore, the choice of the topic also makes it difficult to identify and work with variables and hypotheses, which are common in a quantitative research approach that is often interested in studying the size of a problem in terms of numbers. A qualitative research approach is also more appropriate in studying how people experience the implementation of a programme in their workplaces, for example mathematics teacher educators. What teacher educators think about MTE now and their thoughts about its development can be better revealed through interviews and observations rather than through other means, for example surveys, which are common in quantitative approaches.

The longstanding qualitative vs. quantitative controversy is known and not yet over. Shank (2006), for example, points out that the number of qualitative journals is increasing rapidly in a number of fields; studies that previously might have been strictly quantitative are now accepting both kinds of approaches. It seems that the advantages of a qualitative approach to research are greatly appreciated. In the minds of others, things are changing and qualitative research is under attack again. I would not choose to be drawn into either side, but rather, based on personal experience, advocate that any simple classification into qualitative and quantitative is an oversimplification of practice in educational research. This issue of research approaches being largely qualitative or quantitative or completely adopting one route will be discussed a little further in Chapter 5 when dealing with a critical reflection of the methodological question. For the moment, let me focus on the methodological approach appropriate to the research questions.

In view of the interest in identifying conceptions and describing their variation, a research approach which is an appropriate match is phenomenography. The notion 'phenomenon' originates from a Greek word 'phainomenon', which is taken to mean the 'appearance' of a situation, and 'graphein' means to 'describe' in words or in pictures. Taken together, phenomenography is a description of appearances. According to Marton (1997), phenomenography is a research approach chosen in order to identify conceptions and describe their variations in specific situations, for example happenings, events, and programmes. In summary, phenomenography is a research approach that investigates how people experience a phenomenon of interest. More specifically, it is about ways of understanding and comprehending or ways of conceptualisation (Marton, & Booth, 1997). For this particular situation the phenomenon of interest is MTE. I expected various patterns of ideas or thoughts about it to be revealed. For the purpose of this study, phenomenography is also taken to mean a research approach to investigate perspectives, impressions, and views held by educators in MTE.

Much has been written about phenomenography as a research approach, and it is very often referred to as an approach defined by specific features (Eklund-Myrskog, 1996; Johansson, Marton, & Svensson, 1985; Marton, & Booth, 2000; Säljö, 1979). Despite recent criticisms of phenomenography, for example the reduction of meanings from participants through categorisation, there are substantive commonalities of features discussed among phenomenographers. Criticisms on phenomenography will be discussed in detail in Chapter 5. At present, I have a special interest in looking at what I may call the main features of the phenomenographic approach. The process and the practical details are important because they determine whether the results are defensible and epistemologically valid (Entwistle, 1997). It is worth noting that the practical details of the phenomenographic research approach may vary. The main features of the research approach chosen in this study are discussed next.

One of the fundamental features of phenomenography is to investigate and describe how individuals conceive a phenomenon (or an object chosen), using a second-order perspective. This is different from first-order experience or personal experiential knowledge, where the phenomenon is directly investigated and described as it is. Second-order perspective is the researcher's description using the eyes of a second person. Emphasis is on describing the same phenomenon as it appears to different individuals. The phenomenon chosen in this case is '*mathematics teacher education*'.

The second feature stands on how data is collected. The main data collection technique in phenomenographic studies is the interview. In this study, for example, the empirical basis for data collection was recorded interviews, and open-ended questionnaires. This was followed by transcription of the recorded interviews.

The third feature of phenomenography concerns the establishment of conceptions of a given phenomenon as the main findings. This process is commonly referred to as development of categories of descriptions. As discussed by Evans (2002), as well as Eklund-Myrskog (1996), developing categories involve identifying key concepts, breaking them down, critically examining them, and comparing one with another to see if there are commonalities between them. One aim of phenomenography is to describe similarities and differences between different conceptions. The process of finding similarities and differences between individual statements generates the categories of descriptions which form the main findings. The categories in turn have certain important features or quality criteria, including each individual category reflecting the purpose of research or the phenomenon of investigation. Important also is for each category to tell something distinct about a particular conception of the phenomenon. In other words, the category in question is mutually exclusive (Evans, 2002).

The fourth important feature of phenomenography appears in the relationship between what is conceived and how it is conceived. In phenomenography, the aspect 'what' of investigation, which implies the meaning of content, comes before the 'how' aspect - the way an individual comes to construct the meaning of a phenomenon. The interpretation of this important feature of phenomenography is that conceptions have to be identified before one does research on how they are constructed.

The final feature is seen when the different categories of descriptions are taken together to constitute an outcome space in order to get the general picture. On this basis, phenomenographers talk about 'how many qualitative conceptions (experiences)' there are rather than talking about 'how many people' have a certain conception (Marton & Booth, 1997). In a study like this one, individual conceptions pointing to a certain category of descriptions are illustrated using quotations or extracts. In a real learning situation, when conceptions constructed by learners are in conflict with socially accepted conceptions, one may call them misconceptions, or pre-conceptions (Attorps, 2006). It makes sense to add the terms beliefs and misunderstandings to the list.

The whole discussion to this point strongly associates phenomenography and conceptions. The research approach itself and the notion ‘conception’ are epistemologically associated, in that they are like matching partners in a process of studying conceptions from a second-order perspective. I stand to be corrected if this is stretching it too far. I would like to recall one of the widely used statements for defining the term ‘conception’:

> *“A way of seeing something, a qualitative relationship between individuals and some phenomenon. A conception is not visible but remains tacit, implicit or assumed, unless it is thematised by reflection. In this sense, conceptions are simply categories of interpretation in terms of which we understand the world around us”.*
> (Johansson, Marton & Svensson, 1985, p. 235).

In Marton and Booth (1997), the term ‘conception’ refers to comprehending an idea about a phenomenon. That is to say, knowledge is within the meanings individuals discern upon seeing the phenomenon. Elsewhere, Thompson (1992), in a discussion about teachers’ conceptions of mathematics, associates the term ‘conception’ with meanings, understandings, conscious or subconscious beliefs, perspectives and mental images of mathematics (note the strong relationship with mathematics education). It seems there are reasonable commonalities between the definitions. None contradicts or questions those of Johansson, Marton and Svensson (1985). Mihanjo (2004), writing mainly from a philosophical point of view, extends this discussion and differentiates between an ‘idea’ and the notion ‘conception’. According to Mihanjo (2004), the notion ‘conception’ has an inner or deeper meaning than ‘idea’. One could have an example in mind. Consider a mathematics educator thinking about a phenomenon in terms of a concept, and another one in terms of an idea. Between the two, it is possible to see that the former mental framework involves deeper thinking, seeking more precision than the latter. Conceptions therefore may involve ‘deeper experience’, not ‘surface experience’, of understanding situations, events, curricula, programmes, to mention only a few. In much of the contemporary literature consulted, the two notions appear to carry the same meaning. It is done so perhaps to simplify matters or for convenience. In short, conceptions are stronger than mere ideas about a phenomenon. I therefore consider that the definition by Johansson, Marton and Svenson (1985) and others that have been discussed are acceptable because they are compatible with the underlying root of phenomenography.

Sense-making of a phenomenon is a process which is influenced by many factors, as Treagust et al (1996) argued. There are concerns built around the interpretation process of a phenomenon to give meanings and interpretation strategies for the way forward. The person-to-person relationship is one, the researcher and the subject is another - whether conceptions in mathematics and sciences are brought to light by the subject or created by the researcher. Even more important is the fact that investigations of conceptions by themselves are learning processes. I find no quick solution to escape completely from being drawn into all these concerns, apart from adhering to strict precautionary measures in the course of investigation. Otherwise, there is a possibility of being carried away by the researcher's own conceptions rather than bringing other people's conceptions to the forefront. This is the critical path to be followed in this study, and to be sensitive to all along.

The research results of phenomenography are a systematic and reliable support for making decisions or proposing innovations if the need arises. In this particular situation, phenomenography as a research approach has an added advantage over other methods. Despite the systematic and reliable approach in studying conceptions, the same can be studied through other research methods as well. For example, Black and Atkin (1996), by using case studies, dealt with emerging conceptions in science and mathematics. Thompson (1992) examined teachers' beliefs and conceptions using ethnographic research. Duit et al (1996) investigated students´ prior understanding as a prerequisite to improving teaching and learning mathematics and science. Certainly, ethnographic studies take a long time and did not suit the proposed timeframe of this study. Despite the ability to facilitate the study of conceptions, phenomenography possesses inherent shortcomings and hence there are criticisms levelled against the research approach. The discussion about related criticisms will be extended in Chapter 5. At this point, let us consider the issue of teacher educators as study subjects.

3.3 The subjects of study

The study focused on teacher educators who teach '*mathematics teaching methods'* as subjects of this study. Specific factors which guided the choice of the subjects were as follows. First, because of the highly qualitative nature of the study, it does not lend itself to large sample sizes associated with large-scale surveys. Neither would such a large-scale sample nor the choice of subjects be realistic in terms of costs and timeframe. Second, the choice of subjects was restricted to the thirty-two teacher colleges in Tanzania responsible for teacher preparation and development, including mathematics teachers, for primary and junior secondary education. On the basis of these principles, a core group of 27 teacher educators participated in phase I of data collection. The same respondents took part in phase II, with the addition of 5 more respondents to make a total of 32 expected respondents (see Table 5 for details). The mathematics teacher educators were purposefully selected as subjects of this study, and their main tasks were to respond to the interview and the open-ended questionnaires. The word 'purposefully' selected means selecting participants on the basis of *criteria which correspond to intentions* (Cohen & Manion, 1989). It is a strategic consideration with a focus on demand for variability of conceptions, rather than on a representative selection. The geographical spread in different locations of the teacher colleges and contexts was considered satisfactory with respect to potential variability of ideas and perspectives of mathematics teacher education. It was intended to engage a reasonable number of mathematics teacher educators in both certificate and diploma teacher colleges with appropriate experiences of similar MTE systems. Diploma teacher colleges are staffed by university graduate teacher educators. Certificate teacher colleges are staffed by both university graduate teacher educators and diploma graduates. Their experiences were also considered because they were expected to be a rich source of conceptions and thoughts for further development of MTE.

The selection of the subjects in this study was based on carefully considered principles. The first consideration was to purposefully select one teacher educator from the 32 teacher colleges in Tanzania. The second consideration was to select from different geographical locations in order to optimise the variability of conceptions expressed by teacher educators who teach *mathematics teaching methods*. Third, those selected have been responsible for the teaching of the *mathematics teaching methods* syllabuses in their respective courses. Finally, lived experience in guiding student teachers in learning *mathematics teaching methods* was of interest and important as a source of ideas, and thoughts for the development of MTE.

Data was collected in two phases preceded by brief testing of the instruments, for example, interview schedules. Table 7 is a summary of the selection of subjects among teacher educators in phase, I and II of data collection.

Table 7. Subjects (participants) selected for data collection from teacher colleges

Type of subjects	**Expected participants**		**Actual participants**	
	Type of subjects Phase I	Type of subjects Phase II	Type of subjects Phase I	Type of subjects Phase II
Teacher educators from certificate colleges	17	17	15	17
Teacher educators from diploma colleges	15	15	12	15
Total number of participants	32	32	27	32

A summary of background data in terms of the respondents' gender, educational level, experience in years of teaching mathematics, as well as geographical locations according to zones is shown in Appendix IV and Appendix V respectively. Consideration of the background factors is in answer to some of the criticisms of phenomenography that it is interested in siphoning conceptions and leaves the context on the sidelines (Säljö, 1994). In this study, for example, educational level and experience in teaching mathematics is likely to influence conceptions built around the experiences of individual teacher educators. It is not the intention of this study to focus on background factors without any reason. For example, the educational background factor is presented here as part of addressing the reliability of the study results, while other factors serve the purpose of having a wide view of the respondents.

3.4 Techniques for data collection

The decision on data collection techniques was to a large extent determined by the choice of phenomenography as the research approach. In this case, recorded individual interviews and responding to an open-ended questionnaire were the data collection techniques (see Appendix III). These were administered to individual mathematics teacher educators to obtain statements (expressions) about MTE and thoughts for further development. The focus of the interview was on the individual teacher educator rather than groups in order to avoid influence among the subjects. An interview in this sense is taken to mean a two-person conversation based on a systematic set of questions which were initiated for the purpose of obtaining relevant statements in answer to the interview questions. There are some known limitations of individual interviews, for example the interviewee dominating the schedule, and therefore making it time-consuming. However, individual interviews have a comparative advantage over other techniques of data collection. Consider, for example, the limited power of survey questionnaires and observation schedules in facilitating in-depth responses, personalisation, the opportunity to probe more, the language used and the possibility of getting educators' meanings and actions in their natural places. Since an interview involves a person-to-person conversation, the interviewee has the opportunity to know the purpose and reasons behind the interview, which provides mutual benefits.

If the interview is adequately conducted, it will help reveal conceptions, as well as thoughts for understanding areas of focus in mathematics teacher education One obvious limitation of an interview is that you cannot reach many people at the same time as with a structured questionnaire. Since the study focuses on qualitative data, a one-to-one conversation between the interviewer and the interviewee is a better way to achieve the aims of the study. The possibility of bias has to be addressed through awareness well in advance.

To achieve what is said in the preceding paragraph it was important to allocate time for establishing rapport, explaining the purpose of the research in general and of the interview, and learning from the experience of each other (the interviewer interviewee). Figure 8 represents the procedures for data collection.

The collection of data in Phase I was done using an interview questionnaire (interview guide) as indicated in Appendix II, and the interview was recorded. Phase II of data collection requested teacher educators to respond to a written an open-ended questionnaire as indicated in Appendix III. Five of those who responded to the written open-ended questionnaire were further interviewed after this exercise, for two main reasons. First, it provided an opportunity to clarify some of the statements given, and second, it was a way of data triangulation from the mathematics teacher educators. The details of the research tasks, from piloting of the research instruments to data collection were carried out as explained below.

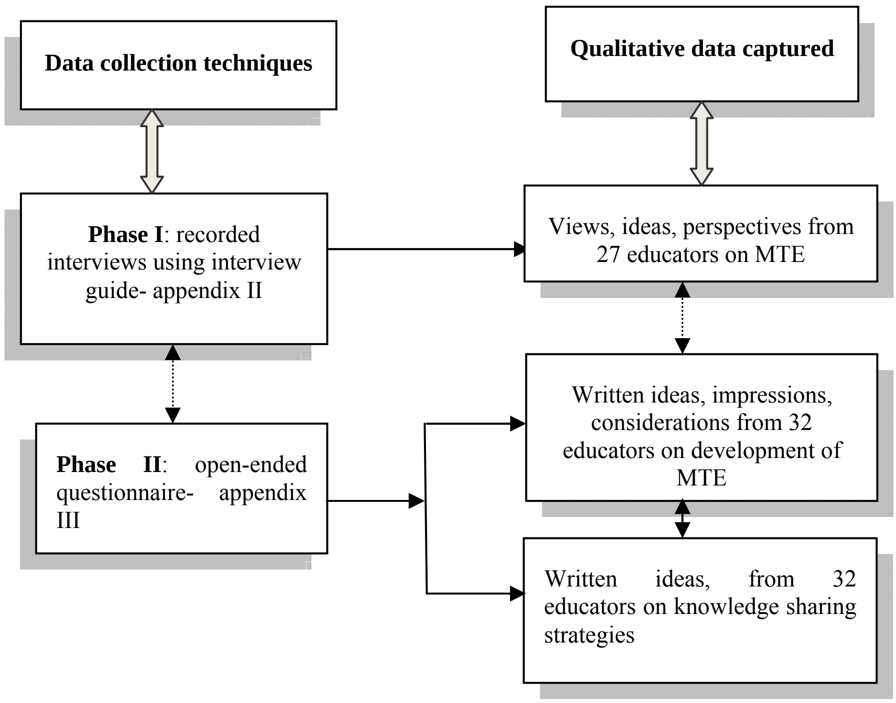

Figure 8. Procedures for data collection from mathematics teacher educators

Figure 9 represents the summary of research plan from Phases I and II to coding and analysis. This may not tell it all about the research design. It is challenging to define it given the background and the methodological solutions under discussion. However, setting the scene is important in order to make sense of it in the context of this study. I would associate a research design with a set of connected research tasks from the study inception to submission of a comprehensive report. Specifically and, in the context of this study, it covers setting the aims, research questions, appropriate research methods, principles of selecting subjects, execution of data collection (techniques), data analysis (unit) and, finally report submission, all guided by some criteria of acceptable work.

As part of the study preparation, I visited three teacher colleges (colleges A, B, and C) during August 2006 in order to test the interview guide. The purpose was to see if it would capture the intended information. I regarded this as part of Phase I of the study and the main focus was to collect data in line with research question one. Actual data collection for Phase I was done between December 2006 and March 2007. Simultaneously with this task, I used the opportunity to test the open-ended questionnaire for Phase II of data collection (data related to research question two).

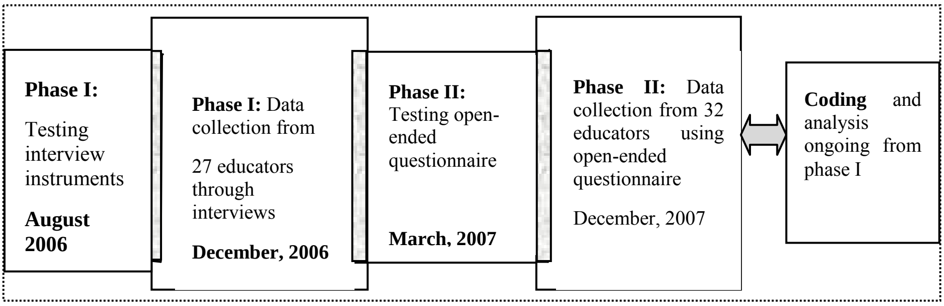

Figure 9. Study plan and a summary of research tasks

The result of the testing of the questionnaire helped to shape research question two into two sub-questions to which mathematics teacher educators responded. The two refined sub-questions related to research question two could then read as: Think of some mathematics teacher education development ideas you have really implemented well as an individual teacher educator or as a team. Describe them, and say how you prepared yourself and actually put them into practice.

In what ways do you share your development ideas about mathematics teacher education with colleagues? (See Appendix III). The first sub-question addresses development thoughts and ideas in MTE as reflected by what mathematics teacher educators actually do. The second sub-question addresses thoughts on knowledge-sharing strategies, plans, mechanisms, and professional development models by teacher educators. The actual data collection for Phase II was done in December 2007 and the coding process followed thereafter. The coding and analysis of data for Phase I ran concurrently. The details of the coding process and data analysis are described in the following section.

Interview is the technique taken as a methodological basis of phenomenography in data collection. In this study, it was reasonable to widen this data collection base to include an open-ended questionnaire. The use of an open-ended questionnaire would make the analysis deeper and in a way enhance phenomenography as a method. The advantages as well as disadvantages of using open-ended questionnaires have been widely discussed (Cruz & Garret, 2006; Hannan, 2007; Reja, Manfreda, Hlebec & Vehovar, 2003). Questionnaires, whether open-ended or structured, are employed as devices to gather data about people’s views, ideas, and conceptions. Before proceeding to describe the coding process and data analysis, it should be mentioned that a discussion of the strengths and the challenges of the data collection instruments can be found in Chapter 5 as part of a critical reflection of the research methodology.

3.5 The coding process and data analysis

In the view of Strauss and Corbin (1990), science (mathematics teacher education in this case) could not have existed without a consideration of concepts (structural meanings). Concepts are important because of the need to draw continuing attention to knowledge and skills in a specific area. As said before, the interest was not on the structural meanings of MTE, rather on sensible and comprehensible words, phrases or statements that came out of the minds of the teacher educators. On this basis, the conceptual labels on MTE that came out from teacher educators' minds formed the initial activities of the coding process.

In a discussion about coding, Evans (2002) considers it as a process of analysing data from question responses and other information into categories according to commonalities that they share. In this case, the questionnaire responses were carefully read to determine specific key conceptual labels of MTE, thoughts about development, and knowledge-sharing strategies of MTE. These statements were broken down, compared one with another, examined and classified, eventually leading to the establishment of categories of thinking about MTE, thoughts about further development, and knowledge-sharing strategies.

In a similar discussion, Strauss and Corbin (1998) refer to the process of breaking down, critically examining, comparing, conceptualising and categorising data as open coding. Marton and Booth (1997) refer to categories as patterns of certain properties which are discovered through the process of classification of different conceptions of a phenomenon of interest through grouping, comparison, and naming. It is the categories that form the basis of analysis in this study. In short, categorisation is expected to emanate from the classification of compared key conceptions.

In this study, the process of coding and analysis to generate categories of descriptions and thoughts about further development of MTE involved several detailed tasks. One of the cross-cutting tasks was the word-processing of recorded interviews into text statements for research question one, and partly research question two. First, all the 27 out of 32 expected interview responses for research question one had their contents critically analysed. In the same way, the content of all 32 educators' written statements from the open-ended questionnaire for research question two was systematically analysed. Second, in each case, statements, sentences, and parts which reflected or indicated teacher educators' conceptions/ thoughts were marked and all key words characterising teacher educators' conceptions/thoughts were written down (conceptions and thoughts relate to research question one and two respectively). Third, I made a critical comparison of similarities and differences of concepts, together with statements already labelled with category names with the support of three research assistants, who had full knowledge of the study from inception to this stage. The category names, deriving mainly from the subjects' statements, enabled me to build patterns of meanings, or categories. Fourth, every statement was considered to belong to a category. On the whole, the guiding principle was a critical examination of what features make one statement similar to or different from another.

During the entire process, I kept on reflecting on the meaning of each category of description without interfering with the subjects' revealed meaning that is, allowing the informants' statements to speak for themselves with minimal reduction. The detailed categorisation step done in collaboration with the research assistants (colleagues) is described next. The involvement of colleagues was necessary in important ways, one of which was to address the issue of validity and reliability right at this very early stage. It was a way of encouraging open-mindedness and minimising bias as opposed to the usual phenomenographic approach, where a co-judge is engaged at the end of the process. This idea follows my reflection of a study done at Sydney, Latrobe University and Canberra in Australia on conceptions of mathematics and how it is learned by students (Crawford, Gordon, Nicholas & Prosser, 1994; Åkerlind, 2005).

The details of the process of what was done together with the research assistants are explained. It is important to note that this process was the same for the statements of both research questions. For the purpose of describing the steps, let us consider research question two (the same procedures for each sub-question) as an example. Research question one followed the same route. The process of reaching a compromise on shared meanings involved all four members (myself and three assistants) of the research team, as follows:

Step one:

Sixteen representative questionnaire responses for research question two were chosen. One half of the 16 questionnaire responses were taken from teacher colleges offering certificate teacher education programmes. The remaining was taken from teacher colleges offering diploma in teacher education programmes. In each case the selection of questionnaire responses in one type of teacher college (certificate or diploma) involved picking the first eight on the list. The intention was to consider the issue of variations of the first draft of agreed categories of descriptions.

Step two:

The four researchers each individually worked on the sixteen responses to identify an initial set of categories.

Step three:

Each researcher came with his/her categories on the given sixteen responses for discussion.

Step four:

The researchers met and each researcher presented his/her categories. The researchers compared the categories to see the similarities and differences. Overlaps were removed. After removing the overlaps, new categories were formed. There was a stimulating discussion regarding the agreed categories at this stage.

Step five:

Each researcher was given the sixteen responses to re-categorise according to the agreed categories. The researchers categorised the questionnaire responses accordingly, and aspects were coded with respect to the identified categories.

Step six:

The researchers met again to compare the categorised statements and aspect agreements. Aspect statements that did not appear to be marked by all four researchers in the same categories were left pending, since the lead researcher (myself) had to continue with the remaining sixteen questionnaire responses. A similar process had taken place for research question one.

In the interest of a more detailed description of the coding and data analysis, let the conceptual labels (Col) reflecting teacher educator statements for MTE be labelled as conception A (Col-A), conception B (Col-B), conception C (Col-C) up to statement twenty-seven (Col-G) of research question one. Figure10 is a conceived framework to represent the coding and data analysis of mathematics teacher educator statements. Again, following the same logical sequence of the coding process, the conceptual labels reflecting thoughts about further development of MTE are named development thought 1 (Dev/Th 1, Col-A), development thought 2 (dev/Th 2, Col-B) in that order up to dev- thought 32 denoted by conceptual label Col-E for research question two.

The actual process of coding and analysis of data can be much more complicated than is shown in Figure 10. In actually doing it, there is a constant reflection of the statements. There is categorisation and re-categorisation, classification and reclassification for the purpose of being exhaustive and exclusive. I kept on doing this until I arrived at the refined categories of descriptions and related aspects (in some cases referred to as sub-categories). I will tend to use ‘aspects’ because it is more in line with phenomenographic studies. The first two columns of Figure 10 have been discussed before. Aspects for each category are shown in the far right column of Figure 10. Aspects represent the key characteristics of each category of description. The aspects carries the ‘what’ and ‘how’ of a given category (Marton & Booth, 1997). We have seen this during the discussion about features of the phenomenographic research approach. That is, in the study of conceptions or people’s experience, what meaning is assigned to a phenomenon of interest comes first, before ways of learning (how) about the phenomenon of investigation. In other words, ‘what is conceived’ comes first before ‘how it is conceived’. In this way, aspects constitute the structures of categories and might carry deeper or richer meaning of the parts that make up the experience. Another important feature of aspects is that they can be found in more than one category of description. The differences in understanding happen because people may focus on the same object differently.

It is asserted by Hales & Watkins (2004) that aspects define conceptual variation because people focus on them differently. This is captured in the discussion of components of conceptualisation or experiences of a phenomenon. Although they keep coming back to the same idea of Marton and Booth (1997), conceptions exist in referential and structural components. The referential component describes the ‘whatness’ of a phenomenon of interest, i.e. what this phenomenon means in everyday language, whereas the structural component refers to a deeper level of meaning. In turn, the structural component is made up of three more aspects. These aspects are what parts make up an experience, how these parts appear to be organised or arranged in awareness, and how the phenomenon is delimited from other phenomena. Differences in meanings within the same individual or across other individuals reflect differences in these three aspects.

In summary, the discussion of the methodological question has made it important to consider the appropriate research questions, the basis of selecting phenomenography as a research approach and, of course, the subjects of study. To reach this point it was also important to think about the techniques of data collection and finally how the data could be coded and analysed. In the interest of making thorough methodological research considerations, the issue of validity and reliability of research findings cannot be left untouched. However, this will be appropriately discussed in Chapter 5, together with a critical reflection of the research methodology.

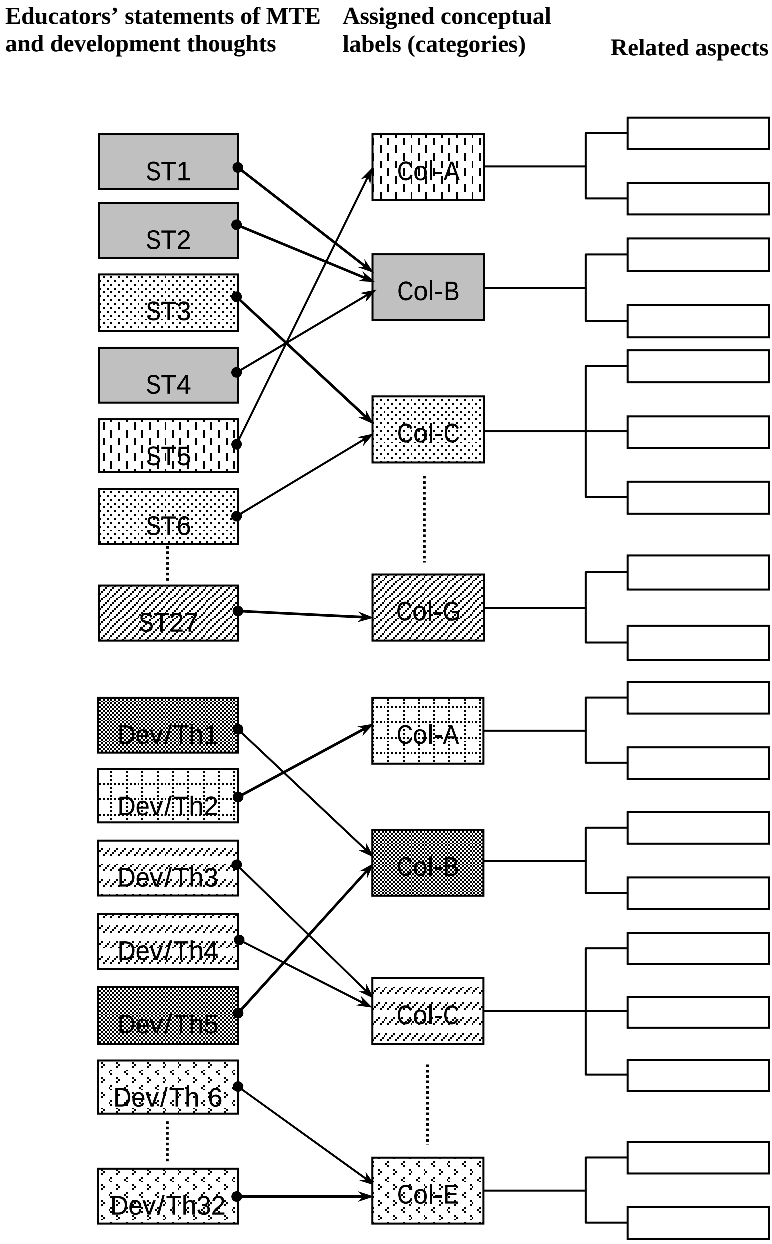

Figure 10. Framework for the coding process and analysis of data

4 Presentation of research results and data analysis

This study aimed at identifying teacher educators' conceptions of mathematics teacher education and provides a description of their variations. Within this aim, thoughts for further development in MTE are identified and described. In this chapter, I begin with a presentation of results on the basis of the research questions raised in Chapter three. The findings in terms of categories of descriptions are briefly presented first and then in detail. Next, a comparison of the number of (variation) teacher educators according to conceptions is also presented, reflecting the two research questions. After that, a detailed analysis of the results according to each category of description is presented. In addition and in order to obtain a clear picture of the results, an analysis of the results is presented, supported by a brief review of literature for the purpose of clarification. It is also important to note that the categories of descriptions are not an imposition of the researcher. The categories of descriptions have been generated from the responses of each of the mathematics teacher educators. On this basis, they form part of the research results in answering research questions one and two. It is also important to note that, in some cases, teacher educators' statements carried conceptual aspects which were classified in more than one category of description. In others words, identification of teacher educators' ideas and the category of description did not bear a one-to-one relationship.

4.1 Teacher educators' conceptions of MTE

Before I proceed to present the results of this study for both research questions, it is necessary to set a few ideas in the right perspective. With the two research questions in mind, coding and data analysis were carried out, and in the process three category systems were developed: one category system in the first research question and two for the second research question. The term '*category system*' is taken to mean the qualitatively different ways of conceiving a phenomenon (Eklund-Myrskog, 1996); in this case MTE is the phenomenon. In my view, a category system may also be seen as a set of categories reflecting the same phenomenon under investigation. For the first research question, the category system is teacher educators' conceptions of MTE. The second research question generated two category systems, which are teacher educators' thoughts on development of MTE, and thoughts on knowledge and skills-sharing strategies in MTE. Each of the three category systems revealed in turn categories of descriptions and aspects. Aspects which carry the key features of a category tend to point in the direction of the conception or the process of acquiring the conception. The conceptions, thoughts, and aspects were further described using extracts or quotes drawn from the teacher educators' statements. As one reads different literature on phenomenography, it is not difficult to find the notion category of description used in the presentation of results of the empirical studies. More often than not, the notion category of description invites questions on the criteria for the quality of the categories of descriptions. Phenomenographers seem to suggest quality criteria in different ways. Interestingly, they seem to converge with Marton and Booth's (1997) description of quality criteria. The following quote summarises the fundamental quality criteria for categories of descriptions in phenomenographic studies:

> *"… The individual categories should each stand in a clear relation to the phenomenon of the investigation so that each category tells us something distinct about a particular way of experiencing the phenomenon".* (Marton & Booth, 1997)

This statement highlights at least two relevant quality criteria used in phenomenography to judge the degree of excellence of categories of descriptions. The two possible quality criteria are for the individual category to stand in a clear relationship with the phenomenon being studied. In this case, each of the categories of description which have been developed has to bear certain aspects of MTE. Otherwise it is a discussion of relating the unrelated. The relationship between each of what is claimed to be a category of description tells other researchers something different in kind (distinct) about a specific way of experiencing the phenomenon being studied. I am of the view that a third quality criterion can be developed, using some kind of skilful interpretation of the first two criteria. Logic may convince us that if all the categories of description stand in a clear relationship with the phenomenon of interest, then in a way each category should stand in a clear relationship with one another in whatever arrangement – whether hierarchical or not. Thus, these are the benchmarks, and I have tried in a modest way to put them into practice.

In order to gain an insight into teacher educators' conceptions of MTE, the relevant question related to the basic research was asked. The main question which led the interview was: if I ask you what mathematics teacher education is, what would you say? This question was asked after a few preliminary questions focussing on the experience of the teacher educator, and the context. From the very beginning, I assumed that mathematics teacher educators as practitioners and from lived experience have their own way of understanding, reasoning, and reflecting on MTE. That is to say, they make certain meanings upon reflecting on MTE as a phenomenon. The next step was for me to develop a set of categories of descriptions on the basis of teacher educators' statements (responses). On this basis, there were seven qualitatively distinct categories of descriptions on how educators conceived MTE, each with certain features. The categories of descriptions are labelled A, B, C up to G, in that order for easier reference.

Conceptions of MTE in short

The results indicate that teacher educators conceive MTE first as a process of teaching and learning mathematics via investigation, and second as a process of inspiration in the course of teaching and learning mathematics. Thirdly, some regard MTE as an approach to teaching with a focus on problem-solving. MTE is conceived primarily as a didactical process in teaching and learning mathematics, with a focus on pedagogical knowledge and skills. Besides this, MTE is conceived strongly as a process of learning with emphasis on subject matter knowledge. Finally, MTE is seen as an approach in teaching and learning that integrates subject matter, pedagogical knowledge and skills. Interestingly, some teacher educators strongly identified themselves with particular ways of conceiving MTE. It is also important to note that the set of categories identified are characterised by certain features, which I would like to point out before a description of each one is given. Primarily, the categories reflect the purpose or topic of this study, and hence meet the requirement of relatedness as in phenomenographic studies. Second, each category stands on its own, though derived from one major classification principle. In a strict sense, all are either 'processes', 'approaches' or 'development of' and only distinguished by qualifying labels like investigation, inspiration, problem solving, and pedagogical knowledge. Thus, the categories generated are briefly described in Table 8, followed by a detailed account.

Table 8. Categories of descriptions of MTE and related representative quotes

Category of description	Aspects	Representative quotes
A: MTE as a *process of learning via investigation*	creating, discovery, inquiry, activity-based	*"..maths teacher education is about academic knowledge we gain, through experiments, research or inquiry,… activity-based… involves learning by investigation, and the process to make others learn"*. (Arthur)
B: MTE as *a process of inspiration in learning*	Stimulating, amusing, enterprising	*"..mastery of content, ability to communicate maths concepts, use a variety of teaching/learning aids to manage the students' learning process motivate, inspire learners, and attract them".* (Kahe)
C: MTE as a *process of learning focusing on problem solving*	Application, understanding, procedures, process, methods, reflection, using problems to teach mathematics	*"..how to solve problems in the teaching/learning process. It is about subject content, supporting learners to develop the necessary skills in problem solving, lead process, procedures, methods and how to arrive at solution, reflection on final solution".* (Mbilia)
D: MTE as *a process of teaching and learning with a focus on development of pedagogical knowledge and skills*	Strategies of teaching and learning mathematics	*"..It is about pedagogical knowledge and skills in maths teaching, a way of bringing up and developing students' knowledge and skills in teaching maths".* (Asha)

Category of description	Aspects	Representative quotes
E: MTE as a *didactical process of teaching and learning maths*	Teaching, studying about teaching and learning, what to teach? Why? And how?	*".. the what, how and why of content and teaching strategies. it is instructing, a didactical process of teaching and learning maths"*. (Miraji)
F: MTE as a process of *teaching and learning with emphasis on subject matter*	Subject matter, knowledge base, solid subject, principles, rules, structures	*"I think it is more or all about content mastery, subject matter, it is a subject guided by rules and formulae"*. (Iddi)
G: MTE as an approach to teaching and learning which integrates subject matter, pedagogical knowledge and skills	Subject matter, methods	*"For a maths teacher educator there is added value in combining subject matter and pedagogy. Knowingly, they attempt to combine maths subject content and pedagogy to get… a hybrid subject specially for teacher education"*. (George)

When the statements from the categories were critically examined and compared, it was found that they inclined towards a specific category of description. The number of teacher educators in a given category of description formed the basis of the category variation. This is indicated in Table 9.

In respect of teacher educators' conceptions of MTE described previously, Table 9 represents teacher educators' variations according to the identified qualitative conceptions of MTE. The word 'variations' is taken to mean differences among teacher educators in understanding the same phenomenon of interest (be it MTE or desirable development ideas in MTE. These quantitative data, in simple numbers, are only intended to help give a detailed description of the analysis of the results, and not by any means for the purpose of generalisation beyond this study.

In the interest of logical order and the major findings, Table 9 shows the distribution of teacher educators according to qualitatively different categories of descriptions. The major qualitative categories of descriptions (in terms of counts, not importance) fall under category G (9 out of 27) and category D (6 out of 27). Teachers in category G conceive that mathematics content and pedagogy integration is the meaning and purpose of MTE, while those in category D see it as a pedagogical process to facilitate teaching and learning. In addition, only 4 teacher educators were classified in category A, and understood MTE as a process of mathematical investigations. Furthermore, 3 of 27 teacher educators, classified in C, understood MTE to be a process of learning mathematics with a focus on problem-solving, that is, application of knowledge and skills in solving teaching and learning problems as well as lifelong problems. Category F indicates that 2 teacher educators conceived MTE as emphasising subject content in the process of teacher preparation. They preferred teaching mathematics in isolation of the methods or the academic part of it, which they viewed as the same as school mathematics. The least of the categories of descriptions, but not a minor one, was category E, which revealed only 1 teacher educator conceiving MTE as a process of didactics as well as studying about MTE.

Table 9. Teacher educators' variations according to categories of description on conceptions of MTE

Categories of description	Count	Category variations
A: MTE as a process of learning via investigation	4	4/27
B: MTE as a process of inspiration in learning	2	2/27
C: MTE as a process of learning focussing on problem-solving	3	3/27
D: MTE as *a process of teaching and learning with a focus on development of pedagogical knowledge and skills*	6	6/27
E: MTE as a *didactical process of teaching and learning mathematics*	1	1/27
F: MTE as a process of *teaching and learning with emphasis on subject matter*	2	2/27
G: MTE as *an approach to teaching and learning that integrates subject matter, pedagogical knowledge and skills*	9	9/27
Total	**27**	**27/27**

A detailed account of the qualitatively different conceptions of MTE

Category A: MTE as a process of learning via investigation

Like the rest of the other categories, this one has been derived from the same categorisation principle. First of all, learning via investigation is a process like MTE is. Next, it stands on its own given the qualifying aspects used to classify teacher educators to belong to this category of description. To emphasise this conception, key aspects used to define the category of description as revealed by teacher educators' responses were: creativity, discovery, activity-based, and inquiry into teaching and learning mathematics. Aden (a given name) was one of the several teacher educators who understood MTE as a process of learning through investigation. In response to the interview question, Aden thought:

> *"First of all, I am teaching mathematics education, I am not teaching mathematics as taught in secondary or primary schools, especially after attending some project-financed in-service training. I can say that mathematics education deals with professional aspects of teaching and learning. Mhhh…It is learning through inquiry, investigation…is a teaching and learning process of mathematics".* (Aden, December, 2006)

When pressed to expand further on this point, the interviewee, here referred to as Aden, further explained and said:

"….I think we are dealing with two major things, learning the content and context of teaching and learning mathematics... by enquiry, evaluation, and reflection on the various topics for relevancy". (Aden, December, 2006)

Aden's two statements seem to strongly focus on the term 'investigation', and appear to mean a pedagogical approach to teaching and learning mathematics. However, acting with an open-mind, one may sometimes connect investigation to studying about the context of teaching and learning. This in turn links MTE to studying the contexts of teaching and learning mathematics, correctly exemplified by notions like 'inquiry', 'evaluation', 'reflection' in teaching and learning. I am aware that investigation could also mean activity-based learning, which also implies a pedagogical approach. Above all, investigation is strongly associated with pedagogical approaches in learning mathematics. As an example following a probing question, one of the teacher educators described an activity to establish the Pythagorean theorem using three squares with one side marked *a, b* or *c*. With the help of a square board, flipchart and marker, student teachers are guided to write their ideas about an investigation into the relationship between squares A, B and C. This can be done through a number of activities to finally arrive at $a^2+b^2=c^2$ as indicated in Figure 11.

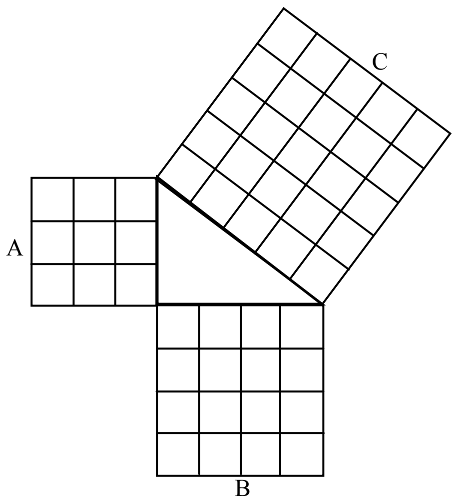

Figure 11. A typical activity-based investigation for proof of the Pythagoras theorem

If investigation is treated in this way, it appears more of a pedagogical approach than research for the purpose of knowledge production. MTE as mathematical investigation is not confined to activity-based or learning through discovery only. To the teacher educators it appears to be a broad concept concealing other things. Anna was another teacher educator who had been reflecting on mathematics teacher education when she said:

"I think science education or mathematics teacher education is about the academic knowledge we gain through science in action, for example, through experiments, research or inquiry, or activities for mathematics education involve investigation into teaching and learning

and in the process problems are identified. The difference is, in college I deal with subject matter and the process of how to make others learn mathematics. In schools some deal with what content to learn. Mathematics teacher education is a science which connects science subjects and many others". (Anna, December, 2006)

The two statements, one by Aden and the other from Anna, have certain features in common. Anna's description is broader in the sense that MTE is not an issue of content selection only but also investigation or inquiry as a means of adding value and enhancing practice in MTE. However, the middle part of the statement indicates a strong linking of MTE to investigation as its pedagogical carrier. To elaborate further, Salum was another teacher educator who has been reflecting a great deal on MTE, and his conceptual image is illustrated in the following extract:

"… I would say it is a field which deals with subject matter, pedagogy and the constant improvement of the field, sometimes through action research. I think the major elements of mathematics teacher education are to develop learning through investigating more into mathematics education and constantly reflecting on it. The target group is what differentiates mathematics teacher education from school mathematics. Mathematics teacher education has the additional task of supporting the group on how to support others learn through constant improvement of the subject". (Salum, December, 2006)

Again, the first part of this statement links mathematics teacher education to research, and the second part of the same statement overshadows the former. From the quotation above, the following features seem to be characteristic of the conceptions of MTE as 'constant improvement of the field through action research'. In this case I am referring to research done in order to bring about change in classroom practice and to provide a better reality. Between the three teacher educators' statements, the one by Salum appeared to be more comprehensive than the first two, but not necessarily related to the category under discussion.

The conception of MTE as investigation with a focus on learning through activities appears to be more distinct and identifiable with the category. Teacher educators' statements in this study indicated additional features of the term. A number of studies discuss MTE in terms of research in order to add knowledge and enhance practice as the primary role of MTE. I would like to bring forward a few as evidence of this view.

Teacher educators who described MTE as investigation, or as a way of organising teaching and learning mathematics, add expressions of including reflection on what goes on in the classroom, being informed and taking action through investigation. This way of thinking about MTE as investigation is illuminated by the work of Sahlberg and Berry (2003). Mathematics investigations refer to:

".. tasks that often require students to explore their own conjecture in order to meet some criteria. Mathematics investigations are pure mathematics puzzle type problems which involve the exploration of

> *mathematical problems which do not necessarily involve real application. They need not be long in pure mathematics. Investigation provides opportunities for pupils to express and explore for themselves and encourages children to follow their own lines of inquiry”.* (Sahlberg & Berry, 2003, p.68)

Although this example was taken from teachers teaching school mathematics, the notion investigation as used in the context of school mathematics may simply mean a strategy of teaching and learning mathematics: that is, learning mathematics through activities. I am of the view that there is added value related to the pursuit of studying teaching and learning mathematics in teacher education. That is to say, it is beyond the tasks arranged to explore specific mathematics concepts. Investigation in this case is preferred as a shift away from the traditional presentation-recitation model of teaching and learning mathematics to investigation or ways to organise the teaching and learning of mathematics.

Enquiry, if related to ‘investigation’ and argued along the lines of Charles (1989), as well as Thompson (1989), concerns the reasons for research activity. In this case, enquiry is represented as the systematic quest for knowledge and understanding, giving dynamism to the investigation. Investigation, therefore, is expected to be intentional. This match the views of the teacher educators in the sense that investigation on the one hand is about ways of organising in order to help student teachers learn mathematics. On the other hand, it implies studying the contexts of teaching and learning for the purpose of addressing problems in the process of learning. The former meaning appeared to be coming more to the fore. The link between mathematics teacher education and research will be discussed further in its appropriate category.

Concluding remarks on category A: MTE as a process of learning through investigation

In this category, teacher educators’ conceptions of MTE focus on investigation, supported by aspects like activity-based learning, discovery, and inquiry in teaching and learning mathematics. In this category, teacher educators’ conceptions of MTE were inconsistent. The variation that exists within the same category is that some revealed concept images focussed on activities and processes leading to the understanding of key concepts in mathematics, citing the proof of the Pythagoras theorem as an example. Yet others had a broad view and went further to include conducting studies in order to be better informed about their classroom actions. This was signalled by aspects like action-research and inquiry into teaching and learning. The outstanding issue to reflect on and discuss at a later stage is the conceptual differences that exist among teacher educators. First, there are teacher educators who view investigation as a pedagogical approach to teaching and learning mathematical concepts and procedures. This is to insist that the role of mathematics teacher educators is not to act as drill masters but act as sources of how to learn through investigation. Second, others view investigation as a research activity to enhance practices in MTE. The variation in the process of learning reflects the purpose and meaning of MTE. Despite the variations, investigation as a way of organising teaching and learning in MTE came more to the surface. The notion appears to have been sourced from earlier projects on mathematical investigations.

Category B: MTE as a process of inspiration in learning

In this category the focus is on inspiration as a process of learning and closely reflects the purpose of this study, despite being derived from the same first level categorisation principle as the previous category – a process. The category of description is distinguishable from others on the basis of the aspects which define it. Inspiration as a category stands on its own because its ultimate aim is to cultivate interest and motivate student teachers in MTE through stimulation, amusement, enterprising activities, as well as use of puzzles. According to the teacher educators, it is a way to address phobias, negative attitudes, low self-esteem, and mathematics-avoidance syndrome. It is also meant to address dilemmas among teachers that some students can learn mathematics, some cannot, and so on. Kahe (a given name) was one of the teacher educators who conceived MTE as a process to inspire teaching and learning. In relation to the interview question, Kahe's response was:

> *"…. Mathematics teacher education is knowledge given to student teachers that helps them know how to present mathematics lessons in the class. If I can say it in a few words, I think what is central is methods and attracting student teachers to learn".* (Kahe, December, 2006)

Kahe was probed further to respond to the question: *what comes to your mind when you think of an ideal mathematics teacher educator?* The aim of this additional question was to capture the meaning of 'attraction' and its features in relation to MTE. Interestingly, the features revealed by Kahe were:

> *"… characteristics of an ideal mathematics teacher educator are love for the subject and students, knowledge of the subject, how to present the subject, creativity with regard to the subject".* (Kahe, December, 2006)

In the same vein, Yambo was another respondent who revealed a similar conception indicating inspiration as a focus when she narrated:

> *"…I think an ideal mathematics teacher educator is one who has mastery of subject content, ability to communicate his or her mathematical concepts, ability to use a variety of teaching and learning aids and how to manage the students (I mean including all classroom corrections and assessment). This includes self- discipline, a good relationship with students and not to use harsh language. I think these motivate or inspire learners and attract them".* (Yambo, December, 2006)

The two teacher educators, who belong to the same category of description, differ in the sense that inspiration is built on different foundations. Kahe's inspiration is based on fascinating mathematics teaching and learning, while Yambo's inspiration is built on a much stronger base, originating from mastery of mathematics content, appropriate pedagogy, ability to communicate mathematical concepts and assessment.

It is worth mentioning the issue of motivation because it is one of the sources of inspiration. More than a quarter of a century ago Kline (1973) recommended enhancing the relationship as a remedy to the defects of the traditional mathematics curriculum. I find that these views expressed 25 years ago are still valid today. The relationship between mathematics and other human interests is of prime importance. Why is this so? This can be looked at from different angles. Certainly good relationships serve as motivation and, even more importantly, an emphasis on life application could make mathematics stimulating and relevant. Both motivation and application are aspects related to inspiration and relate to the teacher educator's thinking.

Teacher educators' emphasis on inspiration is not far from recent research ideas. In an attempt to humanise calculus, Cirilo (2007), for example, raised the issue of helping students to see mathematics as a body of knowledge developed by human beings, and cites an interesting situation of a student studying calculus:

> *"I know how to use the power rule to find derivatives, but I do not know where it came from, or who made it up, or why we have to use it except that this is what the books and teachers want us to do when we do calculus".* (Calculus student, spring 2005)

This view suggests problems of low-self esteem among learners in teaching and learning mathematics. The way I see it, low self-esteem is an indicator of diminishing inspiration. In the same vein, Kline (1973) questioned the tendency by teachers to neglect motivation and application, very often on the pretext that motivation and application is a departure from mathematics content, and could possibly be a non-starter. Teachers and MTE have caused students to have little interest in the subject. What has been done is a presentation of the stem but not the flower and has caused students to fight battles without telling them why they are engaged in a difficult situation in learning mathematics. It is reported that one of the poorest forms of teaching mathematics is that of treating mathematics as though it has no connection with anything beyond its technical confines. Kline's (1973) argument, though old, is still valid today. The connection between mathematics and other fields may include examples like the study of parallel lines in elementary geometry, the calculation of the circumference of the earth, the parabola as a curve taught as a locus of a point, radio waves, automobile headlights, and radio antennae. Linear and quadratic equations and the calculation of how high a ball or projectile projected straight up will go and whether the projectile will reach a certain height are further examples. It is further argued that knowing what mathematics does is part of knowing mathematics and I would argue the same for mathematics in teacher education because of the complementarities. Without motivation or inspiration, student teachers are not introduced to the proper mathematics. So goes the wisdom of practice, as argued and emphasised in Kline (1973), that the mind is not a vessel to be filled but a fire to be rekindled, –and motivation rekindles the fire. In this way, inspiration is seen as an important means to resolve issues of low self-esteem, stigmatisation or what is very often stated as shrinking interest.

Concluding remarks on category B: MTE as a process of inspiration in learning

In category B, teacher educators conceived MTE as a process of raising the interest of student teachers. In a strict sense, it is a focus on strategies of teaching which invite readiness to learn mathematics, and which minimises low-self esteem. As argued by Lee (2008), teachers are expected to adapt a new role from being mere tellers or transmitters of knowledge to sources of inspiration to student teachers in the construction of knowledge. This involves a selection of methods which do not remove MTE from the context in which learning is situated. The issue worth discussing in communities of mathematics teacher educators is how can teacher educators make inspiration happen? Inspiration, I think, is a consequence of motivation, and in the middle of this there is a misconception by teacher educators that motivation or inspiration is a departure from the preferred solid mathematics. For some, a discussion on inspiration is like moving away from mathematics. As suggested at the beginning, inspiration, motivation, and building relationships in mathematics may be taught as part of a MTE topic. This issue of how to make inspiration happen will be discussed in detail in Chapter 6. In the absence of this, one will need to work hard to find solutions to phobias, stigmatisation, low self-esteem, mathematics-avoidance syndrome, and so on. Unfortunately, what we call educational studies, and teaching subjects where solutions could be sourced, are taught separately. I will try more to raise questions about this arrangement, than to provide answers. Who knows where these important fields meet? To answer this question may involve analysing of some of the current MTE practices. Perhaps it may warrant a study of its own.

Category C: MTE as a process of learning with a focus on problem-solving

Again, this category points towards my topic of study on the grounds that problem-solving is a process of learning, just as MTE is a process of developing or learning to make a mathematics teacher. Though this category seems to encompass categories A and B when one views it as a process, it is distinguishable from others when aspects which define it are considered. Aspects which have been used by teacher educators to qualify their conception of problem-solving is the application of mathematics knowledge to practical situations, the argument to support the teaching and learning of mathematics for example in solving real life problems and taking into account understanding, deciding on procedures towards finding a solution, and finally reflecting on the solution. Mbilia was one of the teacher educators who described MTE as a way of learning through problem-solving. This is what she described in relation to the interview question:

> *"I think in this case mathematics teacher education deals with helping student teachers to acquire skills on how to teach and solve problems in the teaching and learning process. It is about subject content, supporting learners so that they develop the necessary skills to solve practical problems, leading the process of these, procedures, understanding, the methods and how to arrive at solutions and finally reflecting on how one has arrived at the final solution. To achieve this, communicating well and logically in mathematics, accepting other*

learners' ideas and respecting other options". (Mbilia, December, 2006)

When Mbilia was further probed to expand on this point, she was not short of ideas to support her position. Her elaboration went like this:

"This is different to from school mathematics, where the interest is to get correct answers in examinations, while teacher education is expected to develop student teachers to teach concepts, skills and problem solving". (Mbilia, December, 2006)

A cross-examination of Mbilia's statements reveals a focus on standard procedures or algorithms and application respectively. In short, student teachers may learn about mathematics (focus on standard procedures), and may also learn mathematics for problem solving (focus on application). It also indicates that MTE may be learnt through exposure to specific problems rather than the standard procedures of lesson presentation. To clarify this idea, it seems reasonable to compare a few patterns of lesson plans (USA, Germany and Japan) as articulated by Stigler and Hiebert (1999) in a study about 'The teaching gap'. The standard German lessons, for example, usually unfold through a sequence of four activities like reviewing the previous lesson, presenting the topic and the problems of the day, developing the procedures to solve the problems and finally practising. On the other hand, a typical Japanese lesson often follows a sequence of five activities, which are reviewing the previous lesson, presenting the problem of the day, students working individually or in groups, and discussing solution methods with the teacher, finally highlighting, and summarising the major points at the end of the lesson. Furthermore, the US pattern of eighth-grade mathematics instruction is characterised by four activities, which are reviewing previous material, demonstrating how to solve problems for the day, practising, and finally correcting class work and assigning home-work.

It suffices to say that the three lessons share some basic common features. These are review of previous material, presentation of the day's problem and students solving problems at their desks. It seems there is international agreement about the importance of these activities. Further reflection reveals that the German presentation of the problem sets the stage for a rather long development of the solution procedure, and whole-class activity guided by the teacher. In addition, Stigler and Hiebert (1999) argue that the Japanese case of problem presentation sets the stage for students to work individually or in groups in developing solution procedures. The US case of problem presentation involves demonstration of a procedure and sets the stage for students to practise the procedure. The Japanese pattern may be closer to the meaning of problem-solving as revealed by the teacher educators. With this background, Shyrose's conception of MTE as problem-solving is illustrated by the following extracts:

"..Mathematics teacher education…deals with mathematical skills, concepts, and problem-solving. It is about innovations and the use of teaching and learning aid as we have been oriented in many innovations in the past and some ongoing from within and outside the country. Though the projects keep coming and going and some overlap it is learning mathematics through posing problems, then you think

> *which principles to use and some may come outside mathematics itself. To student teachers you may pose a topic in the form of a problem, and find how to teach it. I consider that the role of teacher educator is to help student teachers learn mathematics using an actual problem".* (Shyrose, December, 2006)

It may be difficult to differentiate between Shyrose's and Mbilia's thinking, though they fall under the same category of description. However, it is still possible to see their different focuses converging towards problem-solving. While Mbilia focuses on standard steps in mathematics problem-solving, Shyrose's emphases the application of mathematics knowledge to solve life-related problems. Neither Mbilia's nor Shyrose's conception of problem-solving is unique to mathematics teacher educators in Tanzania. Elsewhere, Lester and Lambdin (1999) strongly called for an emphasis on problem-solving among teacher educators. To use their own words, they reveal their insights about problem solving:

> *"….students should be actively engaged in solving non-routine problems; in exploring, testing, and making conjectures about mathematical ideas; and in being responsible for their own learning".* (Lester and Lambdin, 1999 p. 41)

This is the voice of experienced researchers-cum teacher educators about the way they see the notion of problem-solving. Problem-solving appears to them to be the meaning and purpose of mathematics education. I am using the term mathematics education on the basis of the close relationship to mathematics teacher education.

Concluding remarks on category C: MTE as a process of learning with a focus on problem-solving

It is important to note that problem-solving is only an approach to teaching and learning mathematics. It is not a subject by itself but rather an approach to how to guide learners. There are variations in understanding and emphasis among teacher educators. Some teacher educators' conceptions point to, and are more interested in, procedures, rules, and techniques (first-level conception). Yet others understand and emphasise the application of knowledge gained or transferability (second-level conception). Further, some teacher educators' conceptions point to immersing students in project-style problem-solving in MTE. Teacher educators' ways of conceiving the notion 'problem-solving' seems to be compartmentalised if compared with the views of Lester and Lambdin (1999). The argument is to optimise complementarities rather than each standing on its own.

Up to this point, three different qualitative categories of descriptions have been dealt with. These are mathematical investigations, inspiration, and problem-solving. It is possible that they are all generic products of a shift away from mathematics teaching to more open-minded approaches to teaching and learning mathematics. This is what ties them together.

Category D: MTE as a focus on development of pedagogical knowledge and skills

This conception is rooted in methods of teaching and learning among teacher educators. For them, MTE is grounded on methods of teaching and learning mathematics. Since development of pedagogical knowledge and skills essentially reflects possible tasks done in MTE, the category bears a relationship with the phenomenon of study - MTE. Aspects given to qualify and therefore distinguish it from others include emphasis on strategies of teaching and learning mathematics. To the interview question, one teacher educator named Omari had this to say:

> *"I think what we are doing is to prepare or develop students to be mathematics teachers – it is about the how, what and why of teaching mathematics. It is all about methods or pedagogy. Therefore, I think an ideal mathematics teacher educator has certain characteristics, for example, confidence in what she/he is doing, mastery of the subject matter, and tendency for students to identify with him/her because of the style of teaching and learning. In schools, for example, we teach concepts, while in teacher colleges we deal with how to teach mathematics to someone who is going to facilitate learning mathematics".* (Omari, December, 2006)

Although the first part of the statement gives an impression of subject didactics (reflecting on the questions what? why? and how?), the opening remarks and the remainder of the statement is more focussed on pedagogy. In a similar situation, Safari is another teacher educator whose conception of MTE is similar to Omari's, but from a different angle. As a response to the interview question Safari further expressed MTE as:

> *"….knowledge and skills on how to teach and learn mathematics, and seeking solutions in the teaching and learning procedure. It is about enabling student teachers to develop strategies for facilitating teaching and learning mathematics. It is a process of mathematics teacher preparation…and the important components are content or subject matter, but more on strategies or methods of,… assessment".* (Safari, December, 2006)

Safari's statement suggests that another dimension of MTE is a learning course in teacher education programmes, specifically, when he states it as a process of mathematics teacher preparation. This is beyond the methods view indicated by Omari. Asha is another teacher educator who attempted to present the broad view of MTE as pedagogical knowledge and went on to describe the importance of hands-on skills. This is exemplified by preparation of lesson plans, presentations, assessment and utilisation of the environment. She voiced with confidence that:

> *"It is about pedagogical knowledge and skills in mathematics content coverage, and a psychological approach to teaching. To my way of thinking, it is a means of developing student teachers to acquire skills and knowledge on how to teach mathematics Some of the things which constitute mathematics teacher education are: skills and knowledge on*

how to guide learners, preparation of lessons and presentations, evaluation or assessment of students' learning, as well as utilisation of the environment to support learning motivation, and building relationships with other subjects, for example, physics, chemistry, biology and geography". (Asha, December, 2006)

The three expressions by Safari, Omari and Asha on MTE as pedagogical knowledge are interesting. At the same time, the transition from operational to formal conceptions of MTE makes heavy demands on teacher educators' pedagogical knowledge. I find it important to investigate more fully pedagogy as a contested concept in relation to inquiry-based pedagogy. But before this is dealt with it, is important to establish an entry point and I choose the meaning of the term pedagogy, first through the eyes of the teacher educators and then from a global point of view.

Pedagogy is a global concept and one of the most common words used by teacher educators in Tanzania. The starting point perhaps is to ask what meaning is assigned to the notion of pedagogy. The answers to this question are varied. Mortimore and Mortimore (1998), for example, refer to it as all aspects of teaching, not simply instruction. Freire (1977) and Smyth (1985) strongly question that, and define it as a political tool for the enculturation of students. The third conception is one argued for by Hamilton and McWilliam (2001), who state that pedagogy is related to student-centred teaching and learning which specifically excludes didactics. Didactics addresses the questions of what to teach and learn in mathematics - why as well as how. These questions are the corner-stone of didactics. In the middle of these conceptions, it is important to consider the meaning of didactics, which comes from the Greek word didactiko's, which means to teach. Amidst all these definitions, it can be said that didactics is a contested notion and covers a wider range of that aspect of teaching than instruction. In a similar vein, MacNeill, Cavannagh and Silcox (2003) advocate that its role should encompass cultural and societal aspects of what is learnt, and why. It takes into account aspects of learning that were previously described as the 'hidden curriculum'

Concluding remarks on category D: MTE as a focus on development of pedagogical knowledge and skills

Teacher educators view MTE as pedagogical knowledge which in a strict sense refers to the science of mathematics teaching and learning. Notions like guided discovery, problem posing, as well as investigations are pedagogical or methodological approaches to teaching and learning mathematics. The unanswered questions are what constitute pedagogical knowledge? What is subject matter? Where do they meet? These questions are to form the issues for discussion in Chapter 6.

Category E: MTE as a didactical process of teaching and learning mathematics

In category E, MTE is seen as a didactical process of teaching and learning mathematics. This view reflects the purpose of this study since what is done in didactics is to take prospective teachers through a certain process as in MTE. Further, like other categories revealed before, it has been derived from the same stem – a process. But it stands on its own since it is emerging from a different tradition, though from the outside it may appear the same as pedagogical knowledge and skills. Some aspects which have been used to define didactics and therefore qualify it to stand on its own are what to teach or instruction, study about teaching and learning and the actions which teacher educators take (how) in the course of teaching. Freddy was the only teacher educator who described this concept with great interest, as illustrated in the following quotation:

> *"I think it is about imparting mathematics knowledge, skills (thinking logically, being able to give reliable interpretation skills) on how to teach, what to teach, and the reasons for selecting certain strategies for teaching certain topics (didactics). In short, I think it is about the didactics of mathematics".* (Freddy, December, 2006)

I probed more into the subject of didactics. I tried to raise a very natural question - what is subject didactics? The immediate response was:

> *"A term of many meanings. It is teaching or study about teaching and learning, teachers' actions in conducting their lessons. I had an opportunity to learn it from outside the country during our mathematics upgrading programme".* (Freddy, December, 2006)

It was even more important to understand the features of an ideal mathematics educator from the didactical perspective in order to link the meaning and possible actions. On this, Freddy again was able to explain that:

> *"I think it is one who will make mathematics become real, who is conversant with subject content, exploits different methodologies, and techniques, and has the ability to build on prior knowledge. One who assumes that learners know something, and who can decide and have a variety of teaching and learning materials".* (Freddy, December, 2006)

Freddy's way of understanding didactics is part of a long-standing debate about two important approaches in teacher preparation, namely the American curriculum tradition and German didactics. To some extent this came to light in the discussion of perspectives in MTE in Chapter Two. Neither needs repeating here. However, in the interest of being more focussed, it is worth mentioning some of the possible advantages of the didactical approach over the American tradition, in which subject content and instructional strategies are treated differently. Van Dijk and Kattmann (2007) seem to question how the various parts of didactics are addressed in the American tradition. In practical terms, the American tradition, for example, presents a top-down relationship between scholars and teachers as practitioners. For this reason, knowledge has been produced at the top and used in schools with minimum teacher questioning. On the other side, German didactics guarantees teachers professional autonomy and the freedom to teach, and teachers experience little control by the curriculum, as reported in Westbury (2000). It is further reported that despite state control of the curriculum content for teaching, it is not binding on teachers as they are able to translate this into teachable content.

Concluding remarks on category E: MTE as a didactical process in teaching and learning

In this category of description, the main idea is teacher educators' conception of MTE as a didactical process. Subject didactics, as it has also been referred to, is a term of many dimensions. Foremost, didactics is research into mathematics teaching and learning and what teacher educators regarding what content to teach and how. The other important dimension in the didactical process is teacher development. This is taken to mean what teachers do to perfect their teaching styles and standards, and even more specifically didactics as a course of study. Perhaps the second and third dimensions raise concern, for it may be the source of the gap between theory and practice. If subject didactics and educational studies are taught in isolation, how will student teachers of mathematics see the linkage? For me, this is a complicated situation worth discussing at a later stage.

Category F: MTE as a process of teaching and learning with emphasis on subject matter

In category F, the revealed conceptions relate to the topic being studied in the sense that subject matter is one of the main considerations in MTE. What made this category of description stand on its own is for the teacher educators who shared this view to incline towards developing sound academic knowledge as the basis of sound pedagogical knowledge. Combining it with other elements of pedagogy was not a preferred idea. Aspects exhibiting key features of this category system as revealed by teacher educators are subject matter knowledge base, solid mathematics, principles, rules and structures. Further, it is about knowledge of relationships between topics, concepts/definitions, skills, principles and rules. All in all, an emphasis on a high standard of subject matter knowledge in the process of teacher education is the norm. Hamad and Iddi are among teacher educators who indicated a strong conviction that MTE should focus on subject content. They both take the position that real mathematics is sure, solid and true. They conceive mathematics as a universal truth. Hamad expresses it this way:

> *"I think some of the important features of mathematics teacher education include the art of teaching, academic mastery, mostly but not always a determinant. It concerns academic methods or art of teaching, experience, and for the teacher educator to be a role model or an example so that students can identify themselves with their teacher. Mathematics teacher education implies demonstration of a high standard of knowledge of subject matter, principles, rules, structures and relationships between topics as key".* (Hamad, December, 2006)

Iddi, the other teacher educator, took a rather classical perspective of mathematics teacher education, and further remarked:

> *"I think it is all about content or mastery of subject matter, it is a subject guided by rules and formulae. Why deviate? In mathematics we are certain, and follow a logical order, it is solid knowledge and*

difficult to challenge, guided by universal acceptance. Very often you are diluting the subject". (Iddi, December, 2006*)*

There are three common features in both statements (from Hamad and Iddi). First, both seem to take positions on what MTE is, and second, MTE appears to have no relationship with teacher education in general. In addition, both statements seem to include a range of qualities for an ideal mathematics teacher educator, whose main qualities are having mastery of subject matter, using appropriate methods and supporting students in their desire to become mathematics teachers. In the middle of the statement, Iddi indicates with confidence that *'it is all about content or mastery of subject matter'*. In the same way, Hamad uses the expression *'academic mastery but not always a determinant.'* Both statements indicate difficulties in having the generally accepted conceptions of MTE. This is a challenging situation in relation to making a choice on what to emphasise between subject matter and mathematics teaching knowledge. It is plausible to infer that they are in a real dilemma, and the two are taking firm pedagogical positions. The situation is one of conflicting ideas, a no-win situation, and many more. To augment this view I consider it important to re-examine views of advocates of subject matter on this issue without ruling out the dilemma factor.

Hamad's and Idd's conception of MTE as a process and their emphasis on subject matter is not unique to teacher education in Tanzania. In the study about 'the many faces of mathematics', Sfard (1998) raises an interesting question about this issue. Allow me to use mathematics education to substantiate this point because of the close relationship between the two. Let us consider a situation where both a mathematician and a mathematics educator are studying mathematics. The question is, do they have the same focus? Without losing touch with this question, Sfard (1998) continued to argue that thirty years ago the two could claim that there is only one focus for mathematics. Mathematics has been viewed as a well-defined body of knowledge, faithfully mirroring a certain independent reality of abstract ideas. It was only in the last decade or more, when mathematics education emerged as a discipline in its own right that a serious conceptual gap started to exist between the two communities. For mathematics education, I would say it was like systematically pulling out from a classical philosophical position. In an attempt to question the identity of mathematics education, Sfard (1998) traces its origin to a breakaway of philosophers of science establishing themselves as psychologists, sociologists and many more. It was found epistemologically (philosophically) better this way than being scientists and mathematicians in order to be in line with the general conception of human knowledge advocated by works of Lakatos (1976), Davis and Hersh (1980), with interest in human inventiveness. The same author claims that mathematics educators are now torn between two incompatible paradigms to which they have to respond. It is not the purpose of this section to extend the debate of viewing MTE as subject matter only, thereby triggering further debate. The purpose is only to show that mathematicians have their own individual views.

The view that mathematics education refers to sure subject knowledge as revealed by teacher educators is strongly contradicted by Ernest (1991). Teacher educators held the view that MTE is a body of knowledge which is certain, solid and universal. This conception of MTE is in line with what Ernest (1991) calls an absolutist view of mathematical knowledge. To clarify his point, the author challenges the absolutist view of mathematics, and for this reason MTE in the view of teacher educators is not safe. It is asserted that mathematical knowledge is taken as unquestionable truth. Despite the challenge of the absolutist view of mathematics, Ernest (1991) still argues that both traditional and up to-date thinkers in regard to mathematical knowledge to some extent hold the absolutist perspective of mathematical knowledge. I am of the view that Ernest (1991) is likely to be led by the challenge of locating a demarcation line between the constructive views of mathematics as opposed to the absolutist view of it.

Concluding remarks on category F: MTE as a process of teaching and learning with a focus on subject matter knowledge

In category F, mathematics teacher education is conceived as an approach to teaching and learning mathematics with an emphasis on subject matter knowledge. Teacher educators in this category prefer to concentrate on content, and this is in line with the communities of mathematicians. In many ways, this view seems to be a clear pedagogical contrast to category D (MTE as pedagogical knowledge and skills). The perspective that MTE is sure, certain and a subject whose rules and principles are universally accepted might be questionable. For this reason, this is an issue of discussion at a later stage. Perhaps the following category of description is a better extension of this interesting discussion.

Category G: MTE as a process to learn with emphasis on integration of subject matter, and pedagogical knowledge and skills

In this category of description, MTE is understood as an organised combination or integration between MKT and subject matter knowledge to reflect an integrated discipline in the name of ‘mathematics teacher education’. Aspects used to define and distinguish this hybrid category (a cross between two or more disciplines) are subject matter and pedagogy, or simply methods of teaching and learning mathematics. This way of conceptualising MTE was confirmed by a considerable number of teacher educators. For a better understanding, let us consider a few conceptions from teacher educators, one at a time. George said:

> *“…it is the preparation of mathematics teachers of any level. When I am talking about mathematics teacher education, I am looking at it from a practical point of view…. I think it is about mastery of content, and variety of teaching methods to use when dealing with contents. It is about competency in delivery, confidence and friendly way of presentation and being as a model teacher educator. For a mathematics teacher educator, as compared to a mathematics teacher, there is added value in skilfully combining subject matter and pedagogy. Knowingly, one attempts to combine mathematics subject content and pedagogy to get some kind of a hybrid subject, especially for mathematics teacher education”*. (George, December, 2006)

One important message from George's response is the issue of an organised integration of subject matter and pedagogical content knowledge to form what I may call the crossbreed discipline under discussion. Faith was another teacher educator whose views support MTE as an organised integration between subject content and pedagogical content knowledge. As a way of confirming her ideas she stated:

> *"Mathematics teacher education includes strategies, techniques and knowledge of the subject content, as I explained previously. Mathematics teacher education deals with development of subject content integrated with teaching strategies. I therefore expect a competent or perfect teacher educator, if that is possible, to have certain characteristics, including subject matter knowledge, appropriate strategies to teach different content, knowledge of his/her students' needs, and their pre-knowledge. A mathematics teacher educator would therefore put emphasis on the process of how to teach, and seek students' ideas".* (Faith, December 2006)

In this statement, Faith seems to answer at least the what and how part of pedagogy of mathematics teacher education. This is typically exemplified by 'techniques and knowledge of subject matter' and 'emphasis on how' in Faith's statement. However, there is an omission of the why part. Unlike Faith, Nesse was another teacher educator who described a similar concept of MTE, yet of a different dimension. To register her views on MTE, she affirmed:

> *"I would say it is a body of knowledge which enables student teachers to be teachers. I mean knowledge of the subject matter and pedagogy as one holistic package. It is a process of becoming a teacher, which may mean studying about teaching and learning as research and in another way as a programme student teachers undergo at a specified time".* (Nesse, December, 2006)

The different dimensions of MTE revealed by this statement are 'subject matter and pedagogy as one holistic package', 'a process of becoming a teacher' and 'mathematics teacher education as programme student teachers undergo'. With that in mind, I find it necessary to review a few studies and writings about MTE as integration between subject matter and pedagogy.

What is the central issue in this category of description? The issue at stake is viewing MTE as integration of mathematics content and pedagogical knowledge. There is a lot of research literature along this line of thinking. Lester and Lambdin (1999), for example, worked on a mathematics project on how to prepare progressive mathematics teachers to use the problem-solving approach. It has been reported that they designed a programme which focussed on the development of teaching and learning material and a philosophy that involved the integration of mathematics content, pedagogy and school-based practice in the university preparation of elementary mathematics teachers. My impression is that this could be a solution to the longstanding problem of the theoretical knowledge and practice divide in teacher education in general.

Again, I would like to learn from the founding disciplines of mathematics education. Ernest (1991) brings to light and distinguishes four sets of problems which mathematics education attempts to address. The foundation disciplines which form the basis of integration of mathematics education are the philosophy of mathematics, the nature of learning, the aims of education and the nature of teaching. What this means again is that it sounds broad, but it is reasonable to assert that in mathematics as a discipline, teachers operate with knowledge and beliefs, and there are various approaches to teaching and learning (this addresses questions about the nature of teaching and learning). On the aims of education, mathematics education has to define the purpose of education: mathematics education for the individual student or for social transformation? It is possible to see that mathematics education is made up of foundation disciplines and each discipline addresses a specific issue. This is the basis of mathematics education which is integrated in nature. Regarding the philosophy of MTE, we could be talking about teacher educators holding certain beliefs and various approaches to teaching and learning mathematics.

In addition, Hudson et al (1999) state the case that, in many European countries, teacher education in the 1960s consisted of pedagogy, disciplines and praxis, with 'methods' being added as one of the topics in the disciplines. This arrangement was highly questioned by teachers of pedagogy, because, contrary to expectations, the competencies were not automatically transferred and used in the classroom. They demanded a more holistic approach. Didactics (from a Nordic perspective) as opposed to method (the British tradition perspective of pedagogy) was suggested to provide a solution for a more reflective and holistic understanding of mathematics education. In the course of this struggle a new discipline in teacher education was developed in the name of subject didactics, which is a combination of the discipline and didactics. This view, if compared with teacher educators' sense of MTE as integration of subject matter and pedagogy, seems to be of a similar understanding. The major added value in didactics seems to be more focussed on the what, how and why questions in the course of teaching and learning MTE. All in all, it indicates that MTE is an integration or combination of a number of disciplines, for example, mathematics content, pedagogy, psychology (motivation, inspiration) and teacher education, to mention but a few.

In a related discussion on MTE as an integration of disciplines, Gjone (1998) recommends a better positioning of mathematics education and uses mathematics didactics as the point of departure. Mathematics didactics includes central elements, such as theory of the discipline, pedagogy/didactics, psychology (learning), discipline (mathematics), methods (practical), language (communication) and critique (social). Figure 12 from Gjone (1998) has been modified to show the positioning of MTE in relation to other fields. The purpose of Gjone's ideas is to enrich the discussion on MTE as integration of related fields.

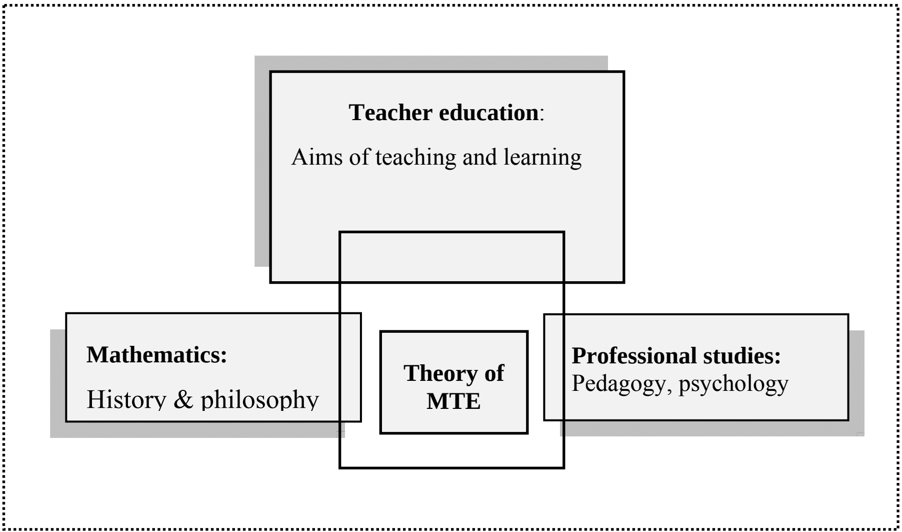

Figure 12. MTE as organised integration of mathematics, education professional studies, and aims of teacher education (Adapted from Gjone, 1998)

Concluding remarks on category G: MTE as a process to learn with emphasis on integration of subject matter, and pedagogical knowledge and skills

From the teacher educators' point of view, MTE is an organised integration of subject matter and pedagogy. Other researchers cited before take it even further and consider it as a philosophy that involves an organised integration of content and pedagogy. Yet some, as mentioned earlier, regard focusing on teaching and learning, as well as studying MTE, as being closely related to didactics. Of course, it is possible to see that teacher educators' conceptions seem to lack fundamental didactic-related characteristics: for example choosing what subject matter to select, how to present it and why that content was selected and presented in that manner. The issue of skilfully blending subject matter and pedagogy to form an 'integrative transformative science' will be brought up again for discussion in Chapter 6.

Variations of understanding exist among teacher educators on the basis of statement descriptions. This is also supported by the count distribution according to categories of descriptions of MTE. Teacher educators' conceptions of MTE tend to polarise between subject matter and pedagogy integration and those who conceptualise MTE as pedagogy-oriented. Between the two, the latter is taken to mean 'methods of teaching and learning', which, when analysed refer to knowledge about classroom processes and the different strategies of facilitating teaching and learning. Analysis of statements also provides evidence that innovative ideas arising out of projects and innovations have an influence on teaching and learning mathematics. Among the examples are investigation, inspiration and problem-solving, which have their roots in past and ongoing innovations in MTE in Tanzania. All in all, they are viewed by teacher educators as the meaning and purpose of MTE, rather than inquiry-oriented pedagogy or approach to learning mathematics. Prominent issues raised in categories A to G are to form the basis for discussion in Chapter 6. One of the main reasons for studying conceptions is that they very often form the basis of pedagogical decisions and taking positions, as we have seen in some cases. This in turn acts as a source of dilemmas.

The following section addresses the issue of thoughts for further development of mathematics teacher education as a response to research question two. In a strict sense, it is about possible options for development of MTE. All in all, it is like a process from the 'meaning' of MTE to 'areas and direction of growth,' and 'how to grow' in MTE. This is like a conceptual critical path of mathematics teacher education.

4.2 Teacher educators' thoughts on development of MTE

In order to gain an insight into teacher educators' thoughts on the development of MTE, it was important to raise a question in that area in order to seek clear solutions. The appropriate question was: What are teacher educators' thoughts on the development of mathematics teacher education?

Thoughts (ideas) on the further development of MTE were based on the assumption that teacher educators are informed and concerned practitioners, who reflect on MTE. With this assumption in mind, it was logical to think that they have ideas, first, on what knowledge and skills to emphasise for the development of MTE, and second, ideas on how to put the developmental ideas into practice in line with the process of MTE. This way of thinking resulted in the generation of a two-category system, the first with a set of five categories of description, while the second generated two categories of description on mechanisms for sharing knowledge and skills in MTE. This followed as a result of asking the teacher educators first to think of some mathematics development ideas in teacher education which they have really implemented well as individuals or as a team of mathematics teacher educators, and second, to provide a description of how they could put these thoughts into practice and share them with colleagues. These ideas are briefly presented first in Table 10 and Table 11 respectively and followed by a much more detailed account of the categories of description and a corresponding summary at the end of each.

Development ideas for MTE in short

The results indicate that teacher educators conceive development of MTE as first, the development (enhancement) of pedagogical knowledge and skills, and second, a focus on subject matter knowledge. Besides these, teacher educators' thoughts on the development of MTE also include enhancement of assessment because it supports learning, and finally, development of MTE is about building relationships between learners, teacher educators and contexts of learning. Given the categories of description, what are the common features in relation to MTE? It is possible to see that the categories of description reflect potential views in areas of MTE development. Hence the relationship between the category developed and the purpose of the study. In addition, each category represents distinct ideas worth pursuing in terms of MTE development. Table 10 is a summary, followed by a detailed account.

On the basis of Table 10, teacher educators in category A hold the view that the development of MTE implies an emphasis on pedagogical knowledge and skills. For teacher educators this involves encouraging classroom interactions; preparation and use of teaching and learning materials; team teaching; pedagogical reflections on activity-based teaching and learning; and application of ICT in learning mathematics. In addition, teacher educators need to understand student teachers´ learning gaps, the syllabus and related teaching and learning strategies. I find it logical to assert that pedagogical knowledge and skills are a natural link to subject matter as they are practically inseparable. In the words of Ball (2000), teachers who teach without understanding of content need opportunities to learn to equip themselves with the mathematics knowledge and skills that will enable them teach mathematics effectively. In the same way, Wu (2005) argues that sound pedagogical knowledge has to come from a strong knowledge of subject matter.

Category B reflects a focus on subject matter knowledge and skills. In specific terms, it pulls together the different aspects, which involve mastery of mathematics content, problem-solving skills, and connection of mathematics ideas to topics and disciplines. Furthermore, it comprises knowledge of mathematics (richness in relationships, understanding concepts) and procedural knowledge (computational skills, procedures, algorithms, principles, rules).

Category C refers to the development of MTE and assessment as a way of monitoring teaching and learning progress, as well as the strong relationship between teaching, learning, and assessment. Outstanding development ideas generated along this line include the use of portfolios, and continuous assessment (formative evaluation). According to category C teacher educators, assessment is educative and supports teaching and learning, and hence is one of the options for further development.

Teacher educators' thoughts in category D appeared to focus on the need for building relationships between student teachers, mathematics curriculum materials (texts), and educators. This includes a range of things like what, how and why MTE topics are taught. For them, it is not a question of mastering mathematics formulae and rules, but possibly humanising the subject through the contextualisation of mathematics and MTE.

In category E, teacher educators' thoughts on the development of mathematics education focused on conducting studies to inform and enhance practice in MTE. Certainly, this is a suggestion about the role of research in MTE. To be precise, they referred to identifying area of concerns or problems and making surveys about teaching and learning within MTE. This strongly implies that a platform is needed to conduct research in MTE, with the purpose of feeding into or enhancing practice.

Table 10. Categories of descriptions of development of MTE and related representative quotes

Category of descriptions	**Aspects**	***Representative quotes***
A: Development of MTE *as an emphasis on pedagogical knowledge and skills*	Classroom interactions, preparation and use of teaching and learning materials, team teaching, pedagogical reflection, activity-based teaching and learning, application of ICT in learning maths	*"I was involved in facilitating student teachers to prepare relevant examples of teaching aids for various topics found in teacher grade A maths teaching methods. The topics are geometry, statistics, algebra, co-ordinate geometry, counting numbers, real and roman numbers."*. (Fadhili)
B: Development of MTE as a process *focusing on subject matter knowledge and skills*	Mastery of mathematics content, concepts, procedures, problem-solving, connection of mathematics ideas to topics and disciplines	"*..Use of extra time to solve challenging problems in maths with learners by sharing ideas with them. For example, during the evening we could discuss with students, share different ideas and work out solutions to the challenging problems*". (Yusufu)
C: Development of MTE as a process *integrating assessment*	Use of portfolio, use of continuous assessment or formal evaluation	*"I learned to use a system of student self-evaluation of learning commonly known as portfolio assessment… use of the portfolio with student teachers was a very tough task, as students did not like changing from using a normal exercise book to using a file,.. Importance of portfolio is to make students keep track of their regular learning through their own efforts. Also to integrate results to provide systematic feedback to both the teacher educator and the student."* (Kezzy)
D: Development of MTE as a process of *building relationships*	Classroom motivation, school motivation techniques, inspiration, relationships between educator-students	*"I have been helping student teachers to use the internet as a source of information or resources relevant to mathematics teacher education. From the digital library we have developed/ made soft copy in maths teacher education and this attracted students who became interested in the subject"*. (Kia)
E: Development of MTE *as a process of studying about teaching and learning*	Identifying areas of concern or problem identification, making surveys about learning problems	*"As a team (department) we decided to discuss the challenges which are in our department and the solution to those challenges...Also to conduct some small-scale studies on teaching and learning maths to find out problem areas"*. (Violet)

Before I give a detailed account of the categories of descriptions, this may be an opportune time to see how the categories compare in real numbers. I am cautious enough not to make any generalisation from the numbers, but only to support the analysis for an in depth understanding of the phenomenon of interest. Table 11 indicates teacher educators' thoughts or views on the areas of focus for the development of mathematics teacher education.

Table 11. Variations of categories of descriptions according to number of teacher educators' thoughts on development of MTE

Category of description (thoughts)	**Count**	**Educators' variations**
A: Development of MTE *as emphasis on pedagogical knowledge and skills*	15	15/32
B: Development of MTE *as focus on subject matter knowledge and skills*	5	5/32
C: Development of MTE *as a process integrating the results of teaching, learning and assessment*	3	3/32
D: Development of MTE *as building relationships between educator, learner and contexts*	7	7/32
E: Development of MTE *as studying (research) about teaching and learning*	2	2/32
Total	32	32/32

Category A indicates that, given the opportunity, teacher educators thought this could be the better option for the development of MTE (15 out of 32 thought that). Category D shows that building relationship (motivation, inspiration) in teaching and learning mathematics is a concern, as 7 out of 32 teacher educators thought that Category B indicates that teacher educators are confident about a focus on subject matter knowledge and skills as a direction for the development of MTE (5 out of 32 had thoughts on subject matter). Category C implies thoughts about giving assessment its due weight in MTE (3 out of 32 revealed thoughts with an emphasis on assessment). Finally, 2 teacher educators in Category E revealed that studying (research) the contexts of teaching and learning mathematics could be an opportunity to develop MTE. Without being affected by strong preference, these are areas that teacher educators think of as directions for the development of MTE. The following is a detailed account of the categories of description, followed by ideas on how to share some of the development thoughts (knowledge).

MTE development ideas in detail

Category A: Development of MTE as emphasis on pedagogical knowledge and skills

Teacher educators in this category of description revealed thoughts reflecting a greater emphasis on pedagogical knowledge and skills. Ideas along this line relate well with the purpose of the research task. Apart from the relationship to the topic of study, this category stands on its own in terms of the aspects explaining it. Danny was one of the teacher educators who described the role of pedagogical knowledge and skills in MTE with confidence when he said:

> *"I implemented mathematical development ideas in teacher education as a team member in Korogwe Teacher College through a project known as the primary mathematics upgrading project. The idea was to try to make mathematics simpler by trying to utilize local resource materials to make teaching and learning more child-centred. To develop such pedagogical knowledge and skills we produced teaching and learning materials which were tested in ten pilot schools and as a result the team managed to design and write mathematics books for STD I – VII, both text and teacher guide books. Results from teachers and pupils from the schools where the materials were tried showed some success, the interest in learning and teaching mathematics grew and the performance rate increased in both internal and national examinations".* (Danny, December, 2007)

Danny's concept image of development in MTE, as reflected in his statement, seems to emphasise pedagogical knowledge and skills. Danny's criterion of 'deep understanding' of mathematics is based on examination results. It is, however, not known whether there are serious questions concerning the possibility of shortfalls in assessment. Anna is another teacher educator with the same emphasis on pedagogical knowledge and skills, but with a different focus. I would say her statement is mathematically pedagogical but takes a different dimension in the form of active learning, when she narrated that:

> *"…The following ideas have been implemented by me as a tutor to make sure that participatory methods are used during the process of teaching and learning. This was done by preparing several activity-based mathematics lessons and giving them to learners for presentation. The learners with the guidance of the tutor prepared and carried out the activities by presenting them in the classroom and other students were requested to give their views at the end of the lesson. Practically, I use… teaching aids prepared by myself depending on the topic I'm teaching at a particular time… student teachers became active participants and not passive participants by involving them throughout the lesson and motivating them to learn mathematics as other subjects".* (Anna, December, 2007)

I am of the view that Anna's use of the terms 'active participants', 'passive participants' and 'participatory methods' is a reference to the level of mental involvement and cognition, and this is subject to pedagogical choice. On the basis of this understanding, it is plausible to say that Anna added another dimension by emphasising more the mental engagement of student teachers in MTE in the interests of actively engaging them and not letting them be 'passive recipients'. When the concept was probed further, passive recipients was taken to mean the inability to construct own meaning or only surface understanding of mathematical concepts. Anna's whole issue is interaction and this has something to do with the pedagogical approach.

David is another teacher educator within the sphere of pedagogical knowledge thinking but has a different area of focus. David brings to the surface the idea of pedagogical reflection, as illustrated by the following extract:

> *"…I tried to put into practice the following: first, I used pedagogical reflection as a way to improve mathematics. This is because for student teachers to be able to do their work efficiently they need to be equipped with professional skills, competencies and self-assessment in teaching and learning mathematics. Secondly, I observed student teachers practising classroom interaction on their own to improve mathematics teaching. I started the lesson by explaining the importance of mathematics in real life situations. Thirdly, I introduced the idea of relating the ideas of one topic to the other in order to help them solve the problems of other topics".* (David, December, 2007).

David's concern in this case is not about performance in the examinations, nor interactions, as was the case with Danny and Anna. David takes pedagogy as an approach to teaching and learning to another dimension. This is about using self-criticism or 'self-assessment' of the lesson, to use his words. Pedagogical knowledge and skills is a central issue not only in MTE but also in teacher education as a whole. There are interesting studies along this line which can throw some light as we discuss this important issue. Dalgarno and Colgan (2007), for example, discuss the issue of innovative forms of pedagogical content knowledge. A group of mathematics teachers while at college were asked to indicate their needs once they were in the field. Among many items, they listed a reform-consistent mathematics curriculum, technology integration, a linking of mathematics to other areas, mathematics content and how best to build mathematics concepts, and finally a bank of challenging and imaginative mathematics problems and lessons. It is possible to see that much of this needs to have a direct implication for pedagogical knowledge and skills. For example, it is difficult to discuss a reform-consistent curriculum without discussing possible methodologies which are applicable and in line with reform. Again, the link between areas is difficult to discuss without, for example, discussing mathematics as the language of physics. Even more important, building a bank of exemplary mathematics problems and lessons requires know-how on what to select and how to teach and learn. All these depend heavily on pedagogical choices.

In the discussion about teacher knowledge, Shulman (1986) raises the question of effective teachers and what distinguishes teachers with an excellent track record. Three areas were argued for. First, teachers need a deep understanding of content knowledge and, if they do not have it, they need to be given opportunities to learn. Second, teachers need pedagogical knowledge to enable them to understand the methods and strategies of teaching that allow them to continually develop and refine their own practice. Lastly, pedagogical content knowledge is needed to teach and learn specific subjects. In this case, the specific subject under discussion is MTE. The strong argument for general and specific pedagogy may justify teacher educators' ideas for giving it priority.

In several other studies (Ball, 1996; Cooney & Krainer, 1996; Lieberman and Mace, 2008; Loucks-Horsely, Love, Stiles, Mundry, Hewson, 2003) teachers indicated what they want is a new professional development model after realising that traditional models do not meet their needs. Interestingly, they wanted content, which is endorsed by research, and addresses both content and pedagogical knowledge within the context of teacher's learning experience. In addition, they wanted content which provides opportunities to access and discuss exemplary reform-based resources as well as allowing them to create and publish resources for new teaching practice. Again, pedagogical knowledge emerges as a critical component in teacher development ideas.

Concluding remarks on category A: MTE development as emphasis on pedagogical knowledge and skills

Essentially, it has been revealed by teacher educators that development of mathematics teacher education means an emphasis on pedagogical knowledge and skills. This refers to working towards improved classroom interactions, MTE as an activity-based approach to learning mathematics, and the use of appropriate teaching and learning resources, as well as team-teaching. Besides this, reflection following a MTE lesson is of prime importance for self-adjustment. There is a narrow view of emphasising pedagogy only as a means of better performance in examinations and not as a tool for better understanding. In the discussion about the failure of the traditional model of teachers' professional development, pedagogical knowledge and skills seem to be the option for turning things around. This issue is to be discussed at length in Chapter 6.

Category B: Development of MTE as a focus on subject matter knowledge and skills

In this category, teacher educators conceived MTE development as attaching special importance to subject matter knowledge and skills. For them, knowledge in MTE development refers to conceptual and procedural knowledge: understanding the concepts, the structure of mathematics and relationships between topics. Furthermore, they were of the view that it is more natural first to think of what content to teach than what method to use in a given subject matter. This formed the basis of their argument on the emphasis of content and reflects the aim of this study. The following illustration by Lea is a case in point to support their thinking:

> *"In thinking about development ideas in mathematics, I encourage student teachers to learn mathematics by solving real life problems mathematically (i.e. using problem-solving techniques, procedures, understanding of concepts). I normally try to find a problem and give it to students and ask them to solve it by using mathematical procedures. An example of a specific problem could be: children below 5 years of age are most affected by malaria, what specific age is the most affected by malaria? This is a social problem and I encourage students to come up with mathematical solutions. Through this action we were encouraging creativity in teaching and learning mathematics as well as solving specific mathematics problems first".* (Lea, December, 2007)

Lea is perhaps reminding us of the concept of mathematical knowledge which seems to constitute concepts and procedures. For example, odd, even and prime numbers could be concepts while at the same time thinking about operations (addition, subtraction, multiplication and division), while whole numbers could be left to procedures or what Lea calls techniques. As a case in point, Lea asks students to find out at what age children are most affected by malaria. This is not a question with a direct answer, but rather one which may need an investigation involving data collection or at least some variables, for example, age (or time) against number of children affected. This may be transformed into drawing graphs, perhaps of a specific type. This is a process which may start from numbers to drawing of graphs. It is the opposite of concepts, procedures and skills which constitute knowledge.

Enea's emphasis on subject matter is equally interesting. The following quote may be of help to portray his ideas on mathematics teacher education:

> *"..Some ideas which I have really implemented well as an individual teacher educator (tutor) or as part of a team are giving mathematical sets or any other motivating items to students who perform well in mathematics. This caused many students to like mathematics. Another idea is the use of extra time to solve challenging or difficult problems concerning mathematics with student teachers, and sharing ideas with them. For example, during the evening we can discuss with students and share different ideas and work out solutions to challenging problems".* (Enea, December, 2007)

Enea's thinking about how to develop mathematics teacher education emphasises subject content, but differently compared to Lea. In this case, Enea uses challenging problems as an approach to teaching and learning mathematics. I am made to understand that the challenge problems are non-routine problems in mathematics, that is, engaging student teachers to learn mathematics through the application of knowledge gained or simply via what is referred to as challenge problems. To be able to learn from solving challenge problems, one needs to have the required knowledge base in mathematics. I find this a necessary condition. All in all, Enea is to a large extent inclined towards subject matter knowledge and skills.

Some more cases of subject matter are also considered. Kally is a teacher educator, whose line of thinking advocates mathematics content in the course of teaching and learning. Kally's ideas are even more thought-provoking and look at the notion 'subject matter' from many sides. This is part of the entire expression:

> *"..I introduced the idea of relating one topic to another to help solve the problems of other topics… the idea of competencies in mathematics is also a way to develop mathematics teacher education. The problem is that some who teach mathematics are not mathematics teachers and, for this reason, students are discouraged. They often remark that if the teacher fails to solve a mathematical problem, what about us?".* (Kally, December, 2007)

Although Kally is shifting from one idea to another, for example from connections of mathematics topics to teacher educators' inadequate knowledge of mathematics, the message concerning the interest in MTE is about the resourceful relationships between concepts, topics and so on. If I may develop Kally's argument, the concept of equations, for example, could be defined in different ways depending on the level of learners. I still have good memories of my mathematics teacher educator in the mid-1970s, who used to say that any topic can be taught at any level. He would start, for example, by telling us to think of $2 + \square = 5$, $2+? = 5$, $2+ x = 5$, $y=2x$ and $3x+4=2y+6$ as well as $x^2-9=16$. All these equations are appropriate to different levels in learning mathematics.

In order to have a better understanding of mathematics teacher education as subject matter knowledge and skills, I need to review some literature to find out what other researchers are saying about this emphasis on subject content. First of all, I find it important to have a clear definition of subject matter knowledge before even discussing its importance in MTE. Shulman (1986) and Attorps (2006) consider subject matter knowledge as the amount and organisation of knowledge per se in the minds of the teacher. The reasons for emphasising subject matter knowledge are, however, diverse. It is not the purpose of this study to go into them, but it may be worth the effort if a few are brought to light. Many teachers lack a conceptual knowledge of many topics in the mathematical curriculum (Attorps, 2006; Shulman, 1986). For this reason it is important for teacher educators to address this issue right away. Secondly, the majority of student teachers would immediately resort to some algorithms upon encountering a mathematics problem or some other procedures before even thinking about the method needed to present the content. It has been said before that it is natural to think about what to teach before you can even think about the strategies.

In addition, Siddiqui (2004) asserts that in mathematics everything follows rules, and every rule rests on reason. What does this have to do with thinking more about subject matter knowledge than, say, pedagogical knowledge? Reasons, rules, procedures, postulation and conjectures are what preoccupy teachers in mathematics because they immediately provoke thoughts. Take, for example, the statement $5 \div \frac{1}{4}$ equals 20. The immediate thinking is what are the reasons or rules governing this kind of conclusion? It may be possible to reason that '¼ goes into 1 four times' and since there are 'five ones' the result is 20.

The stress laid on subject matter knowledge is not a new agenda in teacher education circles. A discussion about issues in MTE (by the American Mathematical Society, 2001), as well as by Sfard (1998), argues that there is only one object that can be called mathematics and anything that differs from this unique thinking cannot be given the same name. I am of the view that this is another emphasis of solid mathematics, without any attempt to water it down and give it a different name. The American Mathematical Society (2001) in the same vein put forward five recommendations as a way to emphasise subject matter knowledge in mathematics education. First, prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach. Second, there is a strong argument not only for the quality of prospective teachers but also for a sufficient amount of coursework or more weight given to content for elementary and middle grade as well as high school teachers. Third, courses on fundamental ideas about school mathematics should focus on thorough development of basic mathematical ideas. Along the same lines, the courses should enable prospective teachers to develop careful reasoning and mathematical common-sense in analysing conceptual relationships in solving problems. Fourth, building mathematical knowledge is of prime importance, but at the same time prospective teachers should develop the habits of a mathematical thinker and demonstrate flexible, interactive styles of teaching. It sounds convincing that, to a large extent, all these ideas from Sfard (1999) and The American Mathematical Society (2001) attach special importance to subject matter.

Concluding remarks on category B: development of MTE as a focus on subject matter knowledge

Teacher educators whose thinking attaches special importance to subject matter argued that content comes first before one could even think of how to present the material. The issue of subject matter, which is taken to mean the amount of knowledge and its organisation, may be complicated, especially when teacher knowledge is considered. The immediate question to put to advocates of subject matter knowledge could be: what teacher educator knowledge base is devoid of subject matter? There is no single answer to this question, but my personal view is that in any consideration of teacher educator knowledge base, both content and pedagogical knowledge are likely to surface. The two cannot be separated. This issue, together with the recommendations of subject matter development, will be discussed in Chapter 6

Category C: Development of MTE as a process integrating the results of assessment

In this category, teacher educators associated the development of mathematics teacher education with the need to take into account assessment in order to support learning. Thinking about how to incorporate assessment in what teacher educators do reflects well is inherently in the process understanding MTE development. This is one of the central tasks in this study. It is for these reason aspects like portfolio and continuous assessment, and formative evaluation emerging from teacher educators define assessment in a unique way to make it exclusive. Bahati was one of the teacher educators who responded and showed interest in the idea of assessment in the development of MTE. He said:

> *"In teaching mathematics we gave students weekly questions on methodology as part of assessment and, discussed a lot of questions in groups in order to gain knowledge in mathematics topics. Mathematics tutors invited each other into the departments to support teaching difficult topics in mathematics, set questions and made sure they were marked to get feedback, set standard marks in monthly tests at 50%. These encouraged students to work hard in mathematics…using various methods in teaching and learning mathematics, for example, by answering oral questions".* (Bahati, December, 2007)

Bahati's view about the meaning and purpose of assessment is worth following up. It is quite difficult to establish the link between assessment and learning. My interpretation of Bahati's views on assessment is that everything is tied to passing examinations. It reminds me of the work of drill-masters before the emergence of notions like 'problem-solving', 'investigation,' 'discovery' and more in mathematics. Further conversation with Bahati on assessment revealed that:

> *"Assessment is a way of determining student teachers' mathematics learning in relation to specified objectives and making decisions on the basis of what has been found".* (Bahati, December, 2007)

Again, assessment as a concept of interest among teacher educators remains narrow in meaning because its benefits are limited. For example, how does it help teacher educators? What are the gains for student teachers? I think assessment has many dimensions. To broaden the conversation on assessment as an integral part of mathematics development, Ndella was the next teacher educator to reveal his views about assessment and development in MTE.

> *"…following students' failure in mathematics, I decided every Wednesday to make time for mathematics club activities, and it was a success. I prepared some questions and posed them to my learners, who tried to solve them. When they failed, I assisted them…I planned for extra time in teaching. This came up after many learners failed mathematics and those who scored below 50% were requested to attend extra time teaching. It was during the evening after the meal every Monday. I repeated the teaching of topics which had not been understood well. These ideas helped many student teachers to cope with learning the subject. I also prepared and gave them some questions which they could discuss and come up with solutions. When they failed, I gave them some help. Thirdly, I introduced the idea of competition among classes. This was conducted once a week and the winning class was given a prize. This helped student teachers to study more in order to be awarded a prize".* (Ndella, December, 2007)

Ndella's ideas on how to develop mathematics teacher education is interesting for one main reason. There are a lot of efforts to keep MTE on track. What we are not sure of is whether there is growth in understanding mathematics concepts and procedures or whether it is just a matter of routine activities. Of course, there might be a lot of interaction in the process of remedial classes that she calls extra time lessons, mathematics club activities and competitive awards in mathematics examinations. Kezzy was the next person in the conversation about development, when he advocated an innovative assessment system based on portfolio assessment as an option for development of MTE. In his own words, he attended many of The Teacher Education Project programmes and later a university outside the country and, this is what he revealed:

> *"I learned using a system of student self-evaluation of learning commonly known as portfolio assessment. Introducing the use of portfolio to... teacher educators...and student teachers was a very tough task because some teaching staff and students did not like to change from using a normal exercise book to using a file, and did not want the*

task of segmenting the file and having a lot to write in each segment. Segments used by students in their mathematics portfolio were classroom notes, group work, exercises, quizzes, tests, and problematic questions, while others included teaching methods, warm-up, exercises rough work, reflection. Further on came discovery, mathematics formulae, and suggestions. The importance of the portfolio is to make students keep track of their regular learning through their own efforts, as well as providing a systematic feedback to both the teacher educator and the student. For care and monitoring of the system the portfolios are frequently checked, and through discussion conducted between tutors and individual students, a consensus is reached about different segments of these portfolios. Tutors must be very dedicated to doing it and checking student progress". (Kezzy, December, 2007)

Kezzy's ideas may form the basis for possible reform towards a more effective assessment system in Tanzania. With this in mind, I would like now to focus on some of the key ideas for discussion, namely portfolio assessment, the connection between assessment, teaching and learning, as well as assessment and performance. It is also important to take into consideration Kezzy's ideas on options for a more precise assessment system during the discussion chapter. At the moment, it may make sense to revisit some related research literature in order to establish a basis for discussion of the role of assessment, teaching and learning.

Although the issue of assessment is of intense interest to parents, pupils and teachers, for quite some time the issue has not exercised the minds of philosophers of education for lack of tradition. Philosophers were pre-occupied with traditions like rationalism, empiricism, pragmatism, sources of knowledge, and so on. There is, however, a wind of change following the introduction of competency-based education (Williams, 1998). Reasons for supporting assessment may come from different viewpoints, but the basic ones frequently argued for are first, human beings would normally be interested to know how well they have been doing or learning, and secondly, often teachers will also be interested to know how successful their lessons have been. This has to be through evaluation or assessment of some kind. Then would follow other reasons we might be aware of. Williams (1998) raises a concern on one of the important issues. This is about human ambitions, which tend to exceed the ability to learn. It is argued that assessment provides us with the relationship between our ambitions and abilities. One may question the manner in which assessment is made, but perhaps not the nature of learning through assessment. It is for this reason that concerned parties like teachers, students and parents would defend assessment, which is more comprehensive, detailed, precise and accurate (Carr, 1998).

In another development about the nature of assessment, Crockett (2007) introduces an interesting discussion on the significant difference between Japanese and American teachers. Crockett (2007) points out that teachers in the USA view teaching and learning as separate acts, rather than a constitutive activity. The latter view is said to be typically Japanese. This argument suggests that mathematical teaching problems cannot be attributed to lack of disciplinary knowledge and impoverished pedagogical knowledge only. It is the separation view which is a problem. If teaching and learning is constitutive (inseparable) in practice, the teacher´s role is to make student thinking pedagogically central. This implies the ability to assess student thinking. On this basis, and using the words of Crockett (2007), assessment is formative and involves gathering evidence that is used to monitor student progress, to inform moment by moment pedagogical decisions, and to mediate teaching and learning which are mutually constitutive and inseparable activities. This, of course, is said to be contrary to the views of teachers in the USA, where assessment is an activity which happens after teaching has been done. In line with this thinking, and from teacher educators' expressions, assessment is likely to continue exercising our minds however hard one tries to downplay it. I suggest concluding at this point, leaving room for a detailed discussion on assessment and learning at a later stage.

Concluding remarks on category C: Development of MTE as a process integrating the results of teaching, learning and assessment.

In this category, teacher educators conceptualised MTE as having a special relationship with assessment as a tool to enhance the relationship between teaching and learning. The reasons for assessment are quite open. More direct ones are that human beings naturally want to know how well they have been learning, and teachers very often want to know how successful their lessons have been. This has to be through evaluation or assessment of some kind. Assessment is formative and involves gathering evidence that is used to monitor student progress, to inform moment-by-moment pedagogical decisions, and to mediate teaching and learning, which taken together are mutually constitutive and inseparable activities. A few issues to take forward for a detailed discussion may therefore include the link between assessment, teaching and learning, as well as the possibility of developing a more effective assessment system.

Category D: Development of MTE as building relationships

In this category, teacher educators expressed ideas to develop MTE which involved building relationships among student teachers. How this relates to the purpose of the research task is important to note. Thinking about development of MTE could be in different ways, and building relationship in the process of learning is just one, as revealed by teacher educators. This is what relates the category of description and the research task. Aspects which make this category of description distinguishable have been expressed in a number of ways: first, interactions between student-tutor-learning materials (textbooks), next is the relationships within topics in MTE and between disciplines. In addition, building relationships within the college encourages the development of MTE as well as classroom motivation. Gama is a teacher educator, who conceived that the development of MTE should emphasise building relationships. When requested to respond and give ideas on development, he stated:

> *"…as a tutor I convinced my fellows in the department to initiate a mathematics club. I convinced some students to join the club, where they can benefit a lot from group discussions. In the mathematics clubs there is a lot of interaction, for example analysis of the mathematics teacher*

education syllabus in order to understand the topics and see connections between them, and preparation of model lessons in mathematics problematic areas. The college sometimes awards model lessons as a way to motivate students opting to choose mathematics…as their career subjects. We attempted even to video-record some of these lessons for others to see and learn". (Gama, December, 2007)

The central issue is building relationships between teacher educators and students by creating opportunities where students may learn effectively. This may be through the establishment of clubs or even associations in the case of teacher educators, as stated by Gama. Of course, one would say the purpose of establishing a club is more important than mere establishment. I expect a club is where people of similar professional orientation may meet and share routine and non-routine ideas, and in this case mathematics ideas. If the club meets this condition, then it may help to build relationships in teaching and learning mathematics. I am of the view that clubs, if nothing else, are expected to resolve the claim that mathematics or even MTE is a cold science rather than part of human experience. Safari is another teacher educator interested in building relationships, but with a different focus. His idea is based on a pedagogical approach and is illustrated below:

"..I encourage the use of participatory teaching methods in order to build thinking capacity, confidence, develop the habit of sharing knowledge in mathematics and try to practice myself.. I have been helping student teachers to use the internet as a source of information relevant to MTE. From the digital library we have developed soft copies in mathematics education and this has attracted students who have become interested in the subject…students were able to browse some of the websites which enabled them to..be aware of discussions of different topics around the world. In addition, they would post difficult topics on the internet so that interested persons could respond and give their views and opinions on the topic of concern. I introduced a club in order to improve the sharing of views and to conduct discussions on the issues of their competences.. This is where students and teacher educators present and discuss some of the issues demotivating teaching and learning of mathematics teacher education". (Safari, December, 2007)

Safari's conception of building relationships between teacher educator, student and curriculum material reminds me of an important experience. During my initial training as a teacher educator, my educator once said 'Any curriculum can be brought to life depending on the pedagogical approach'. This statement indicates the power of 'methods of teaching and learning' in getting mathematics concepts and procedures understood. This, of course, does not mean a wholesale employment of methods in MTE. The role of content in MTE is underscored. It only shows how the use of appropriate methods may help to build relationships in teaching and learning mathematics. Gama's idea on building relationships in MTE raises my interest, and perhaps that of others. The following quote may help to serve the purpose:

"I tried to introduce mathematics clubs in the college. Within the club, we solved different problems so that the students could appreciate a lot

of logic hidden in mathematics, power, precision, and pleasure in teaching mathematics education. We deal with a range of problems from ones focusing on usefulness in practical life to puzzles like the one to follow - How many triangles can be found in the figure?". (See Figure 13) (Gama, December, 2007)

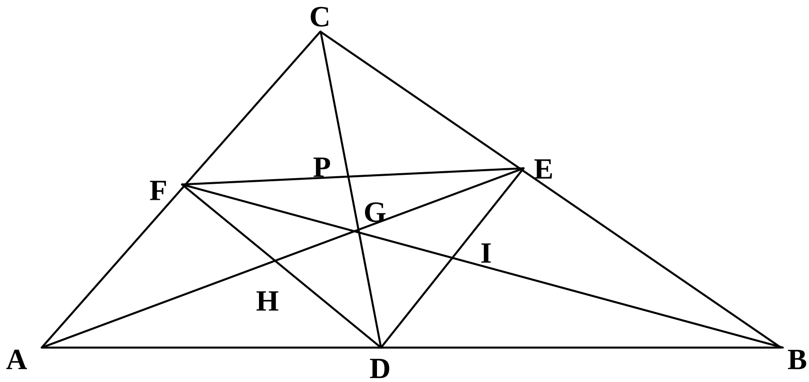

Figure 13. A challenge problem from a system of triangles

Gama's ideas on how to motivate student teachers stress the usefulness of mathematics in real life. I appreciate the initiative regarding puzzles, but he could go further by solving problems, for example. Besides Gama, Besta also hinted at building relationships in learning mathematics:

> *"In solving a considerable number of such problems together, students gradually became interested in studying mathematics after a time. I also tried to develop activities on preparation of teaching and learning materials. So far, we have a special room for demonstration of teaching and learning materials. These activities help students know how to prepare and use different teaching and learning materials in mathematics".* (Besta, December, 2007)

The central issue again is building relationships or motivation for doing mathematics. From personal experience, motivation in this case is taken to mean directed behaviour towards a certain phenomenon of interest. Both Gama and Besta taken attach special importance to motivation through various means. The use of mathematics clubs again is an option, but this time as a forum to discuss and appreciate the hidden logic of MTE and to witness the pleasure it gives. The use of puzzles is one way of indicating the richness of logic in MTE. At this point, I now find it necessary to revisit research literature on the role of building relationships in teaching and learning mathematics.

In a study on emotion, identity and teacher learning, Hodgen and Askew (2007) found that many primary teacher relationships with mathematics are filled with emotion. Much of the fear is related to negative experiences of school mathematics. It is the same experience for student teachers who regard mathematics as dull, boring and irrelevant. The central question is - how do we break the vicious circle? They suggest an urgent solution is to teach mathematics differently – with a style paying attention to the issue of building the relationships that are involved in teaching and learning mathematics. The next question is how to turn things round concerning a subject which is dull and out of the human context? Not easy to answer, but I think a starting point could be within the quest for teachers' professional development.

Equally important, Cirilo (2007) in a study on how to humanise calculus raises a concern on how some of the mathematics topics are presented today. Experience indicates that mathematics teacher and teacher educators present formulae and rules without taking time to talk about the evolution of mathematics as a human invention. It is very often forgotten that the history of mathematics may supply answers to the why where and how of many concepts in mathematics. The following remark by a student is living evidence:

> *"I know how to use the power rule to find derivatives, but I don't know where it came from or who made it up or why we have to use it - except that this is what the books and teachers want us to do".* (Calculus student, spring 2005, p.23)

This statement and many others of this type are common, and little effort is made to help bring mathematics teacher education to life. What if an exercise is given to student teachers to find out the genesis of calculus? Cirilo (2007), for example, suggests starting with stories about Isaac Newton (1642-1727) and the laws of motion, and Gottfried Leibniz (1646-1716) on differentials. They may come up with background stories about their present status, their family, where they grew, up their early life, and whether they discovered together or independently. The purpose of this definitely is to guide students to discover that knowledge (mathematics) was developed by human beings. These two examples signify the relationship between MTE and those studying it. The next example signifies the relationship between MTE and other disciplines.

In a study by Chaachou and Saglam (2006), the characteristic relationship between mathematics and physics is discussed. I am using mathematics in this case on the understanding that there is a strong relationship between mathematics and mathematics education, and in the same way MTE. It is argued that the origin of the relationship between mathematics and physics can be traced from philosophical and epistemological positions. It is argued that mathematics constitutes the language of physics and this explains the relationships between the two disciplines. In the same discussion it is asserted that laws are drawn from experience but, in order to state them, a special language is required. Ordinary language has been questioned in terms of expressing relationships which are so delicate. For this reason, physics cannot go without mathematics. However, there is a caution, as the selected language (mathematics) carries a reduction of the real system. Leverage is therefore important.

At this point, I would like to pull all the thoughts together in the form of a conclusion concerning this important issue of building relationships among those involved in MTE.

Concluding remarks on category D: Development of MTE as building relationships between learners, educators, contexts

It has been noted that some teacher educators viewed efforts to develop mathematics teacher education as activities involving building relationships of what is involved. Relationships have been expressed in terms of interactions between student-tutor-learning material relationships, within topics and between disciplines. Options to build relationships (or to motivate) in order to resolve the very often negative experiences have been suggested and they comprise strategies such as the use of mathematics clubs as avenues to share ideas and experiences between teacher educators and student teachers, exposure to more productive pedagogical options, and teacher educators' professional development. Teachers' professional development concerns coming to know and understand more about teaching and learning, which, if done appropriately, is constitutive. In regard to teacher professional learning, it needs to be acknowledged, among many other aspects, that learners are not equal. This brings us to the next category of description about research as an idea as it specially relates to mathematics teacher education.

Category E: Development of MTE as studying of teaching and learning contexts

In this category (E), teacher educators thought that the development of MTE could give special importance to researching teaching and learning situations. The results of research could help to find ways of improving what teacher educators do, as well as supporting student teachers. Seen from many sides, this category is related to my topic of study in the sense that the development of MTE involves research undertakings to provide solutions to teaching and learning problems. Aspects which constitute this category and therefore make it distinguishable from other categories are identifying problem areas as a starting point, concerns about teaching and learning MTE, and motives and conducting surveys. Asha is one of the teacher educators who expressed the development of MTE in terms of studying teaching and learning situations. To use her own words:

> *"...After the discussion we came up with the following challenges: lack of skills in preparing teaching/learning materials; lack of proper selection of teaching and learning materials; lack of a teachers' guide and failure to stimulate student's thinking as well as ability to conduct good action research in mathematics teaching and learning. We came to realise that the solutions to these challenges are...to use methods which will make all the students participate e.g. discussion groups, gallery walk, jigsaw, concept map etc to assign students; to open a portfolio... Also students came up with the idea that mathematics is a difficult subject and they prefer science or other subjects, so we had to motivate them about the advantages of mathematics, e.g. it is used everywhere in normal life. Also conduct some small-scale studies on teaching and learning mathematics to find out problem areas".* (Asha, December, 2007)

The type of challenges met by Asha in teaching and learning mathematics teacher education are not unique in initial mathematics teacher education programmes. This is to be discussed at the appropriate time. But a point of interest is the prescribed solutions, which seem not to match the identified problems. Many of the solutions identified may require teachers' professional development. Researching into teaching and learning mathematics situations is an idea which stands out as an aspect of development in the area. Salma was next among teacher educators who attached doing research and MTE. According to Salma, research is expected to enhance teacher educators' knowledge. Salmas' description went as follows:

> *"..Teaching and learning mathematics encounters a lot of problems, some of which include poor pedagogical skills among mathematics tutors, lack of teaching and learning materials/resources and/or facilities. The following are some mathematics development ideas I have implemented very well: establishment of a subject club; identification of problem areas in primary school mathematics syllabus using questionnaire (a study), techniques of data collection; discussions and conducting workshops at zone level. A sample of 66 mathematics teachers was selected, including the district school inspector. Problem areas were identified in geometry, concept of a number, coordinate geometry and algebra in general. The tutors conducted workshops for participants in these areas, whereby the knowledge gained helped the participants to change and be able to deal with the topics as well as improving their pedagogical skills"*. (Salma, December, 2007).

Salma's remarks indicate a sense of a systematic way of seeking solutions to problems in teaching and learning mathematics. A thorough reflection of Salma's statement is likely to reveal, first, that there is a need for research in MTE given the problems. Second, Salma's ultimate aim is to improve practice in MTE. Third, the dissemination of research results needs a mechanism to deliver the results, which could be workshops, seminars, and meetings, to mention but a few.

Is there anything we can learn from studies on mathematics teacher education as a field in order to inform practice? I would like again to talk about MTE through the eyes of mathematics education, or what Adler (2005) calls MTE in mathematics education – its closest equivalent, differing mostly in focus. Ernest (1998) questions the notion of 'mathematics education' itself and sees it as an ambiguous term. It is further argued that it signifies first, practice and in another sense a field of knowledge. By practice, it means teaching, for example elementary school children. It is a field of knowledge because of its nature of academic specialisation, with a career ladder from undergraduate to PhD level, a system of conferences, journals, and many more. It is the same for 'statistics', which refers to the field of study and interpretation of data, and making decisions on the basis of the data studied. Statistics is also a mathematical field of knowledge. The argument advanced by Ernest (1998) is that mathematics education is too broad a term. To share Ernest's (1998) views, it is just another way to acknowledge the complexity of the field - the dual existence of mathematics education (as practice and field of knowledge). This takes the term to new dimensions and covers the teaching and learning of mathematics at all levels (school and college), out-of-school teaching and learning, the study of mathematics education as a course in pre-service teacher education (all levels) and research in mathematics education at all levels. It may be fair to conclude that amidst this complex and layered meaning (Vithal, Adler, & Keithel, 2005), mathematics teacher education has strong foundations to exist as a distinct research site for teaching and learning. This is important, and we shall have the opportunity to discuss it more in Chapter 6.

Concluding remarks on category E: MTE as studying teaching and learning contexts

In choosing the words to conclude this category of description, MTE as research into the context of teaching and learning will be the starting point. But the literature has already indicated that this is not all, and gives a glimpse of a challenging situation in a field which is unfolding to inform the practice of learning mathematics through research. Despite the notions, for example, of 'ambiguity of mathematics teacher education´ and 'periphery and core components', mathematics teacher educators naturally have to respond to emerging issues in teaching and learning mathematics. I am of the view that this is the central issue, whatever label is given to the field. Research as part of MTE functions is inherent and seems to remain important. Teacher educators have indicated undertaking small scale action research aimed at improving their practice. Further, as part of their preferred activities, surveys have been conducted in order to provide possible solutions in teaching and learning.

4.3 Teacher educators' thoughts on MTE knowledge-sharing strategies

In order to gain an insight into teacher educators' thoughts on how to share knowledge and skills in mathematics teacher education, the following question was asked: In what ways do you share your development ideas on mathematics teacher education with colleagues? The interviewees were required to say exactly what they actually do, not what they think they could do (see Appendix III). This question arises out of research question two. The aim was to capture teacher educators' thoughts on how to put into action their thoughts, and actually examine their strategies for sharing knowledge and skills in MTE. From a pragmatic point of view it aimed at the ways and mechanisms of sharing knowledge and skills. This was the reason for requesting them to say what they actually do rather than what they think they could do. On the basis of this question, a set of thoughts about strategies or mechanisms of sharing ideas in MTE was developed. Table 12 is a presentation of the categories of descriptions of these strategies and their corresponding quotes.

Strategies of sharing knowledge and skills on MTE in short

Table 10 is a summary of two strategies of how teacher educators could share knowledge and skills in mathematics teacher education. In category A, for example, teacher educators thought they could use neighbourhood learning group sessions to share knowledge as part of professional development in MTE. To achieve this, suggested models which stand out as aspects of neighbourhood learning are seminars, workshops, in-house capacity building, mentorship, team teaching, lesson studies, subject panel discussions and assessing student work. In category B teacher educators' thoughts on sharing knowledge and skills pointed to networking and collaboration as the strategy. This is an extension of the same central idea about professional development in MTE. Their way of sharing knowledge and skills consider, among others, the following aspects: articles and modules and writing newsletters, website visits, e-mail exchange, television programmes and study tours. Table 13 indicates teacher educators' two strategies for sharing knowledge and skills in MTE. The two categories as a result of classification are neighbourhood learning groups and networking and collaboration.

To elaborate, Table 13 indicates teacher educators' distribution of thoughts on sharing knowledge and skills in MTE. Irrespective of a specific way, 21 out of 32 teacher educators in category A thought neighbourhood learning groups are professional development sessions and better options for sharing knowledge and skills in MTE. Within limits, the 'how' aspects of neighbourhood learning groups as revealed by teacher educators are subject seminars and workshops, in-house or college-based workshops, team-teaching, as well as mentorship. Only 11 of 32 teacher educators in category B thought that collaboration and networking was a strategy by which they could share knowledge and skills (ideas) in MTE. Next is an account of each category of description in detail. I will treat one at a time in order to maintain a logical sequence.

Table 12. Teacher educators' thoughts on MTE knowledge-sharing strategies

Category of description	**Aspects**	**Representative quotes**
A: Sharing knowledge and skills in MTE through *neighbourhood learning groups.*	Seminars, workshops, in-house capacity building, mentoring, team teaching, lesson studies, subject panel discussion, assessment, study tours	*"I started by conducting capacity-building workshops in neighbourhood schools. Through capacity building, we could introduce what we have in mathematics by sharing ideas with other nearby teachers, educators. I shared my ideas with colleagues within the college by presenting what I have learnt in seminars/workshops where many mathematics teachers/tutors gather".* (Salum)
B: Sharing knowledge and skills in MTE through *networking and collaboration*	Using ICT, articles and module writing, newsletters, e-mail exchange, website visits, radio programmes, mobile handsets, television programmes	*"...ideas could be shared with other colleagues through the use of new technology; since every college has computers, ideas can be sent to more than one tutor through e-mails. Networking written in magazine will reach tutors in many parts of the country, ideas shared through TV, radio broadcasts shared through exposing ideas to teacher trainees so that when they leave the college they will communicate the information to others".*(Anna)

Table 13. Teacher educators' variations of categories of description of MTE knowledge-sharing strategies

Category	Count	Educators' variations
A: Sharing knowledge and skills (ideas) in MTE through neighbourhood learning groups	21	21/32
B: Sharing knowledge and skills (ideas) in MTE *through distance collaboration and networking*	11	11/32
Total	**32**	**32/32**

Strategies of sharing knowledge and skills in MTE in detail

Category A: Sharing knowledge and skills (ideas) in MTE through neighbourhood learning groups

In category A, teacher educators thought of neighbourhood learning groups as a nucleus of professional development sessions and a sustainable strategy for sharing ideas. How this is related to my research task is important to note. MTE is essentially an interactive and knowledge sharing process. Here is a category of description with some expressions from teacher educators on how this could be carried out. This reflects part of the research task as well as the unique aspects which distinguish this category from the other one. The aspects are neighbourhood learning, ranging from seminars, workshops, and in-house capacity building to mentoring and team-teaching. Let us consider one thought at a time. Nesse (a given name) thinks that neighbourhood learning is part of professional development and seminars are a sound way of sharing ideas. This way of thinking about ways of sharing knowledge and skills is indicated by the following quote:

> *"..I did presentations in different seminars to neighbours also. Ideas, for example, division of fractions, multiplication of negative numbers within college and other colleges… by doing so, and access to internet, I have been sharing mathematics development ideas with my colleagues. Also when attending seminars as a mathematics teacher educator in Tanzania, where we discuss mathematics ideas. Through this, I have been able to disseminate the knowledge which I have gained and also get new ideas which I have to discuss with my colleagues in my college".* (Nesse, December, 2007).

According to Nesse, the starting point for professional development in MTE is the sharing of ideas, whether through seminars or other strategies like the internet. The teacher educator seems to have thoughts built around sharing mathematics knowledge through seminars, as indicated by most of the second part of the quote. Kia is a teacher educator who believes in knowledge gain through in-service education and training. Though there are questions surrounding this term, in a strict sense in-service education and training entails professional development. On this the educator said:

> *"…it has been difficult to get time to share knowledge in formal settings. This is because we do not have enough forums for mathematics tutors in Tanzania where tutors could share… However, I do share knowledge when we meet in workshops planned and organised by the Ministry of Education and Vocational Training, or when invited by our District Education Officer to help nearby primary school teachers in mathematics teaching techniques. I also share thoughts with other teacher educators close by when I come back from attending Mathematics Association of Tanzania annual meeting workshops. In such workshops any tutor who has an interesting method, game or technique related to teaching mathematics is free to share with other teachers or tutors. The activity could be an icebreaker or puzzle as a starter for a particular topic at the beginning of the workshop session".* (Kia, December, 2007)

One strategy for sharing knowledge and skills in MTE, as indicated by the results, is through teachers and teacher educators who live and perhaps work in close vicinity. Interpretation of the statement by the teacher educator reveals limited opportunities for both college-based as well as what the educator calls mathematics teacher education '*forums*'. Furthermore, the teacher educator seems to have in mind a formal system for sharing knowledge in MTE. Unlike Kia, Yambo, a teacher educator, has a different idea, but not an opposing one on how to share knowledge in MTE. I would like to come back to the central issue, which is how teacher educators can share some of the ideas they generate in practising as mathematics educators. Yambo demonstrates mentorship as another strategy within the neighbourhood conception of professional development. To make her point, she revealed the following:

> *"I think some ways which are good in sharing mathematics ideas with other colleagues are first indoor and outdoor seminars, and workshops concerning mathematics. Mentoring is next; through mentoring, you can see what other people do with mathematics ideas. Lastly, I could write or write as a group mathematics articles on different ideas and expose them to readers on the internet. Interested ones may read through them and pass them on to other people to read and give suggestions on how to improve them".* (Yambo, December 2007))

Yambo is suggesting a very specific way of sharing mathematics knowledge in MTE. She brings to light the issue of mentorship among other models or strategies for sharing ideas as part of professional development. But my immediate comment on mentorship as a model or strategy of professional development is that it is based on relationships, meaning the close relationship between the 'mentor' and the one to be 'mentored'. The assumption in this study is not that of a 'novice' and 'expert' teacher, which may not fit well in this context. But in distant regions one could appreciate nearby teacher educators sharing knowledge and skills. I will come back to this issue in Chapter 6. Lea contributed ideas on how to share what is known to her and at least practised by her. She is still in the area of professional development through neighbourhood learning groups, but with a difference in emphasis. Some of her ideas are shown in the following extract:

> *"..There are a number of ways I use to share mathematics development ideas with colleagues in Tanzania. Some of them are through seminars and workshops; we also share mathematics development ideas by discussing with other tutors. Apart from that, we are used to sharing mathematics development ideas by teaching as a team, which helps because if one of the tutors is not competent,… can get support. Moreover, inviting another tutor as a guest speaker is also a good way we use to share some mathematics development ideas. In topics in which I am not competent enough, a guest speaker can help me. In addition, we are used to sharing mathematics development ideas when marking examinations we get feedback.... This is also a good way of sharing ideas because we prepare even the marking schemes together".* (Lea, December, 2007)

In the second part of the quote, Lea is making a point with emphasis on team teaching and guest speakers as practical ways or strategies for sharing knowledge generated from best practices in MTE. Team teaching involves some discussion beforehand. It is all an issue of teacher educators who are close by taking advantage of supporting each other. Yambo, like Lea and other respondents we have seen so far seem to view seminars and workshops as a panacea. I have no serious question to raise at this point about their choice of strategies on how knowledge can be shared. Interestingly, Lea is suggesting that marking examinations as a group of teacher educators gives them an opportunity to get feedback. Assessment comes in with a different label in the name of examination marking panels. One serious doubt is that if this serves the ultimate purpose of sharing mathematics knowledge, then it has to meet at least one important criterion. The kind of assessment developed has to relate to teaching, learning, and assessment.

Up to this point, several ideas have been floated on how development ideas in mathematics teacher education could be shared. Among strategies of sharing knowledge and skills in MTE are neighbourhood learning groups. Key features of neighbourhood learning groups as revealed by teacher educators are seminars, workshops, team teaching within college mentorship, and so on. These groups can be regarded as vehicles of neighbourhood learning and point to teacher educator professional development.

Ideas closely related to this result have been discussed by Dooner and Mandzuk (2008). They argue for a strategy very similar to neighbourhood learning groups despite their challenges. Some of the immediate advantages of community learning as revealed by Dooner and Mandzuk (2008), as well as related ideas from teacher educators, are that beneficiaries who are situated in the same area and context can translate their knowledge to the benefit of a particular school on day-to-day practice. In addition, there is an ongoing interplay between the nature of community and its demand for shared perspective. Of course, all may not run smoothly in collaborative efforts because at times teachers lack trust, time and talent. In a similar vein, Collins (1998) is in support of communities of learners and learning circles as fundamental aspects of learning, which have been missing from current teacher development designs. Again, to some extent this is in line with teacher educators' thoughts on neighbourhood learning, except for the different contexts.

Concluding remarks on category A: Neighbourhood learning groups as a way of sharing ideas in MTE

In this category, teacher educators thought of neighbourhood learning groups as a strategy for sharing knowledge and skills, and part of professional development in MTE. Essentially, it means taking advantage of their physical closeness in exchanging ideas rather than waiting for the traditional top-down workshops and seminars organised by ministry officials, who are usually far away from the colleges and schools. One may question the manner in which the strategy is used, but in theory it remains a preferred option, and gives meaning to MTE development. The conceived strategies to carry out professional development are seminars, workshops, in-house capacity building, mentoring, and team teaching. Other strategies within neighbourhood learning are lesson studies, subject panel discussion, neighbourhood study groups, and joint assessment of students´ work.

Returning to collaboration and networking, it has been argued previously that this option has some immediate advantages because beneficiaries are very often situated in the same area and context, and can translate knowledge to a particular school's day-to-day practice. In addition, there is ongoing interplay between the nature of the community and its demand for shared perspective. Category B indicates teacher educators' thoughts opting for collaboration and networking as a mechanism for sharing ideas in mathematics teacher education. A careful examination of aspects in these categories indicates that they carry features of distance, collaboration and networking sharing of ideas in MTE. Examples are using ICT articles and module writing, newsletters, e-mails, websites, radio, mobile handsets, television, and many more.

Category B: Sharing knowledge and skills (ideas) through collaboration and networking

Let us remind ourselves that the central issue is how do teacher educators take forward development ideas in mathematics? In a strict sense, we are talking about professional development and strategies for making it happen. In this category teacher educators thought collaboration and networking at a distance is a strategy for sharing ideas in MTE, using ICT as a tool for teaching and learning, article newsletter and module writing, all with a focus on MTE. Again, the relationship between this category of description and the search for MTE development ideas seems to be based on strategies for teacher educator development. Teacher educator development uses a variety of strategies, and this is just one. Features of this category which distinguish it from the other one include seminars and workshops as major strategies for sharing knowledge and skills. In most cases, these strategies of professional development were dominated by face-to-face sessions in neighbourhood learning groups. Bahati's thoughts on how to put ideas into action attach special importance to the use of ICT. This is what she described when requested to respond:

> *"….I can use e-mails to send mathematics articles to other teachers concerning ideas on the subject. I can also prepare papers, articles and modules that can be used in teaching mathematics and sharing mathematics development ideas".* (Bahati, December, 2007)

Bahati's thoughts on the use of internet services for sharing ideas in mathematics teacher education cover a wide range, including exchanging articles, preparing papers and articles and using ICT as a tool to facilitate teaching and learning. George was the next person to speak about the importance of sharing ideas through networking and collaboration, and suggested writing articles about them. To emphasise his point, this is what he said:

> *"…communicating with colleagues through the internet about improving teaching and learning mathematics, by use of workshops and seminars like the one we are now conducting here at Morogoro Teacher College. This gives me a chance to explain what I know and exchange ideas on a face-to-face basis, as well as through distance communication. Last year one of the tutors in mathematics invited me to help her to teach about probability. She said she was not comfortable. I taught the lesson to student teachers while she was in the same class listening and adding some ideas. She is now good at that".* (George, December, 2007)

George's options on sharing mathematics ideas in teacher education are built around collaboration with colleagues (exchange of ideas, supporting where one is failing). To some extent this exhibits elements of conventional means, which very often include seminars and workshops, and so it is no wonder that they carry the same top-down features. From personal experience, I can say that apart from the use of guest speakers and the writing of articles, it is difficult to see a more systematic way of ensuring teacher professional development. A lot of ground work might need to be done if seminars and workshops are to be productive. But let us not forget that George is introducing quite a new phenomenon. This is an approach very similar to modelling. The lady teacher who initially was not able to teach received good support from George to the extent that she is now able to continue on her own. Lea extends and broadens the use of ICT to the level of mass media when she remarked:

> *"...Mathematics development ideas can be shared with other colleagues through the following ways: first, I shared ideas through seminars that can be organised regionally according to zones or national seminars for tutors; secondly by use of new technology, since every college has some computers, ideas can be sent to more than one college at the same time or to tutors through e-mail; thirdly, ideas can be written in newspapers or magazines and these ideas will reach tutors in many parts of the country. Fourthly, ideas could be shared through TV and radio broadcasts. Lastly, ideas could be shared through exposing them to the trainees so that when they leave the college they will pass the information on to other parts of Tanzania".* (Lea, December, 2007).

Lea's idea about the use of mass media as a strategy for sharing knowledge could be effective if the capacity to prepare ICT-based mathematics teacher education programmes is in place. Collaboration and networking as opportunities for sharing ideas could be made broader and teacher educators could have a wider choice. Before I conclude this part, it would be good to have a look at similar ideas closely related to the results.

At the heart of collaboration and networking is mathematics teacher educator professional development. This is conducted in face-to-face sessions as a means of sharing ideas. Connected with this result is the question of why teachers think of collaboration and networking as a strategy in the first place, as it has been reported that learning mathematics is threatening to most teachers (Dalgarno & Colgan, 2007). Even worse, professional development programmes for graduating teachers come when it is already too late. By the time programmes enter educational institutions teachers are already instilled with the traditional image of teaching and learning, and have already shaped their conceptions of mathematics during their stay in practice. For this reason, teachers are more likely to teach as they have been taught throughout their schooling rather than in the way they have been taught in teacher colleges. When the results are reflected against ideas from Dalgarno and Colgan (2007) and others, some difficulties appear to prevent having much more productive strategies for the development of MTE. What are these difficulties? There is no single answer, but teacher educators seem to depend a lot on traditional models and strategies of professional development which have often been challenged. Consider seminars and workshops with a top-down structure as a case in point. It seems that this traditional approach, and the strategies used to share ideas in MTE do not work. However, the interesting point is the great emphasis on collaboration and networking.

Concluding remarks for category B: Collaboration and networking as mechanisms for sharing MTE

In Category A, teacher educators thought that collaboration and networking was an option for sharing knowledge and skills. Key features or aspects which are associated with this strategy for sharing ideas are seminars, workshops, in-house capacity building, mentoring, team teaching, subject panel discussions, and assessing students work. These aspects have characteristics related to face-to-face sessions. It is important to note that there are arguments as to whether these traditional approaches to teacher professionalism could have the desired impact. While teacher educators view workshops (one-shot workshops) seminars, training-of-trainers, and guest speakers as positive initiatives, the downside is that these approaches are not meeting the demands of teachers in terms of knowledge construction (Jenlink & Kunnucan-Welsch, 1999). What are the options? The positive side supports new teacher educators' professional development strategies, the content of which is endorsed by research addressing both content and pedagogical knowledge in the context of teacher's learning experiences and provides opportunities to access and discuss exemplary reform-based resources. Even more important is content which allows them to create and publish resources for new teaching practice (Cooney & Krainer, 1996; Lieberman, 1995; Loucks-Horsely et al, 2003; Sykes, 1996).

Although the results seem to support seminars, workshops, mentoring and others, research by Dalgono and Colgan (2007) strongly questions this approach. A shift away from transmission or telling to a model for acquiring more knowledge is important. The new strategy, in the name of collaboration and networking needs to take into account that consultation, problem solving and programme development can be taken by teacher educators into their own hands. The traditional model is likely to provide neither the content nor the opportunities teacher educators view as essential for their professional development.

All in all, it is interesting to note that some of the 'how to share ideas´ (knowledge and skills) as revealed by teacher educators form the basis of this study's implications for teacher education. This discussion is likely to provide a point of departure in Chapter 7. On the basis of the conceptions revealed by teacher educators of MTE and the thoughts generated for development of mathematics teacher education, what conclusions can be drawn? A discussion of this is my next task in the following section.

4.4 Summary of research results and conclusions

In many pieces of research involving the use of phenomenography as the research design, the development of categories of description originate from a variety of sources, for example, the creation of labels from the researcher's own invention, from a pool of constructs of the literature review, from the words of the informants and from observations made by the researcher. In this study, the categories have been generated from the statements of the teacher educators and hence constitute the results of this study. The following are the main results in order of the two research questions.

Research question one: what are teacher educators' conceptions of mathematics teacher education?

Data analysis indicates that there are sharp conceptual variations among teacher educators, which in turn could have an influence on their actions in relation to MTE. It may be important to note that one of the reasons for studying conceptions is the fact that, teaching is essentially a pedagogical decision making process and, pedagogical choices are made on the basis of conceptions. With that in mind, teacher educators conceived MTE in seven qualitatively different categories of descriptions, and these are:

Category A: *MTE as a process of learning through investigation;*

Category B: *MTE as a process of learning through inspiration;*

Category C: *MTE as a process of learning with a focus on problem solving;*

Category D: *MTE as a process of developing student teachers' pedagogical knowledge and skills;*

Category E: *MTE as subject didactics;*

Category F: *MTE as a strategy for teaching with a focus on subject matter knowledge; and*

Category G: *MTE as a process of learning with emphasis on the integration between subject matter and pedagogical knowledge.*

On the basis of these results the following conclusions are drawn:

a) Teacher educators experience sharp conceptual variations concerning mathematics teacher education in qualitatively different ways. This is because individual teacher educators carry their own experiences or have diverse mathematical histories, and this may have influence on the conceptions they reveal. The conceptual variations (conceptions) revealed are linked to their pedagogical knowledge gained from previous experiences during mathematics project implementation. This is supported by expressions like *'I had an opportunity to learn it from outside the country during our mathematics upgrading programme'*. (Freddy, December, 2006).

b) The sharp conceptual variations are possible grounds for differences in making methodological choices or preferences, and at the same time for telling about MTE as it is. This is evident from their expressions, for example:

> *"I am not teaching mathematics as taught in secondary or primary schools, especially after attending some project-financed in-service training"*. (Aden, December, 2006)

c) Some mathematics teacher educators see mathematics as unquestionable (absolute). At the same time they appear to argue in defence of what they know about MTE. They seem not to accept the pressures placed on them to teach mathematics teaching methods instead of mathematics. Take Idd's statement as a case in hand:

> "*But I think it is more or all about content or mastery of subject matter, it is a subject guided by rules and formulae. Why deviate? In mathematics we are certain, sure, it follows a logical order, it is solid knowledge and difficult to challenge, guided by universal acceptance. Very often you are diluting the subject"*. (Iddi, December, 2007)

d) The integration of subject matter and pedagogical knowledge is what divides (or inspires) most teacher educators regarding conceptions of MTE, and also that some teacher educators transfer acquired experiences (conceptions) from school mathematics to teacher education in the name of mathematics teacher education. This is evident from expressions like:

> *"….mathematics teacher education implies demonstration of a high standard of knowledge of subject matter, principles, rules, structures and relationships between topics as key"*. (Hamad, December, 2006).

Research question two: What are teacher educators' thoughts on the development of MTE?

Teacher educators revealed five qualitatively different thoughts on the development of MTE, and two qualitatively different ways of sharing knowledge and skills of the phenomenon of interest, these are:

Category A: *Development of MTE as an emphasis on pedagogical knowledge and skills;*

Category B: *Development of MTE as a focus on subject matter knowledge and skills;*

Category C: D*evelopment of MTE as a process integrating the results of teaching, learning and assessment;*

Category D: D*evelopment of MTE as building subject-learner-educator relationships, and*

Category E: *Development of MTE as studying teaching and learning.*

Teacher educators also revealed thoughts on strategies for sharing knowledge and skills.

Category A: *Sharing knowledge and skills in MTE through neighbourhood learning groups, and*

Category B: *Sharing knowledge and skills in MTE through collaboration and network learning*

Further reflections on the categories of descriptions in relation to research question two guided me to draw a few general conclusions. On these grounds, these results enabled me to generate the following conclusions:

a) Teacher educators' thoughts on the development of MTE are sometimes influenced by diverse historical background ideas, for example, shifts away from teaching (telling) to learning, and the use of ICT as a tool of teaching and learning, and also from the traditional way of assessing to portfolio assessment.

b) Teacher educators' thoughts on how to share knowledge and skills in MTE indicate various strategies in use, and are grounded in teacher educators' professional development needs: first, through college-based face-to-face sessions, and second, through distance collaboration and networking.

c) Teacher educators' ideas about development in MTE indicate that attention needs to be paid to building relationships between educator-student teachers, and educational material in use as a way to address stigmatisation, phobias, low self-esteem and apathy in mathematics education.

d) Teacher educators thought MTE as a ‘tool’ of research in teaching and learning potentially exists but is yet to be fully recognised.

The basis of the differences is in seeing the same phenomenon in a varied way appears to point to the differences in focus of the MTE aspects which form the basis of the categories. To elaborate this, Kezzy’s statements on portfolio assessment infer making choices, and at the same time taking positions:

> *“Introducing the use of portfolio to staff and student teachers was a very tough task because: i) Students did not like changing from using a normal exercise book to using a file (a compilation of A4 papers) (ii) and teacher educators though it not appropriate for mathematics, better to use the normal way…students did not want the task of segmenting a file and having a lot to write in each segment. Normally in an exercise book they write notes, quizzes and exercises in mathematics”*. (Kezzy, December, 2007)

5 A critical reflection of the research methodology

This chapter deals with a critical reflection of the methodological question with a focus on the qualitative research approach, and justification of phenomenography as a methodological solution. I also make a self assessment of the trustworthiness of the research results by looking at how the issue of validity and reliability have been addressed. This involves looking at how validity (credibility) and reliability (dependability) of the research results were addressed. Finally, I draw some conclusions regarding the methodological solutions taken without losing touch of ethical considerations. The purpose of making a critical reflection on the qualitative research approach, as well as the reasons for choosing phenomenography, is simultaneously a justification of the methodology and at the same time a way of addressing the credibility and dependability of the research findings.

5.1 Critical reflection on the methodological approach

The issue of whether the research approach should be largely qualitative or quantitative or taking either route was discussed in Chapter 3. I find it important to take the discussion one step further as it has some implications for the results of this study. Taking Niglas´s (2004) findings as an example, the researcher addressed several questions concerning the combined use of the quantitative and qualitative approach in educational research. As a contribution to this discussion, Niglas (2004) conducted a systematic analysis of 48 research papers with regard to different features commonly used in qualitative and quantitative approaches. The findings showed that more than a third of all studies combined qualitative and quantitative aspects and /or features of inquiry in different phases of the study.

Furthermore, even the aims of the studies were not fundamentally divided along the lines of quantitative or qualitative in a dichotomous way. Interestingly, even the authors of studies that claim to follow a qualitative or quantitative approach show no clear point where a line can be drawn to separate the approaches. This supports the argument that it is the concrete research problem or aim rather than a fixed position which determines the study approach. That is, depending on the nature and complexity of the problem, the approach can be either qualitative or quantitative, or a combination of both. My interpretation is that the division into qualitative-quantitative exists mainly at the philosophical level, but in practice the combined use is the norm. It is for this reason in this study that some elements commonly found in the quantitative research approach have been used to help provide a more detailed description where needed - not for the generalisation of ideas. This is my modest way of viewing the qualitative vs. quantitative discussion. Figure 14 provides a summary of the qualitative vs. quantitative discussion, and shows how the approaches tend to merge from the methodological level to data collection.

As an epilogue to this qualitative-quantitative divide, one could state that, in many essential aspects, both qualitative and quantitative approaches have much in common and can work in combination. They both have similarities in argumentation and in drawing conclusions. Thus, the main principle of selecting a research approach is the need for the approach to reflect the research questions. Indeed, this is what I kept in mind.

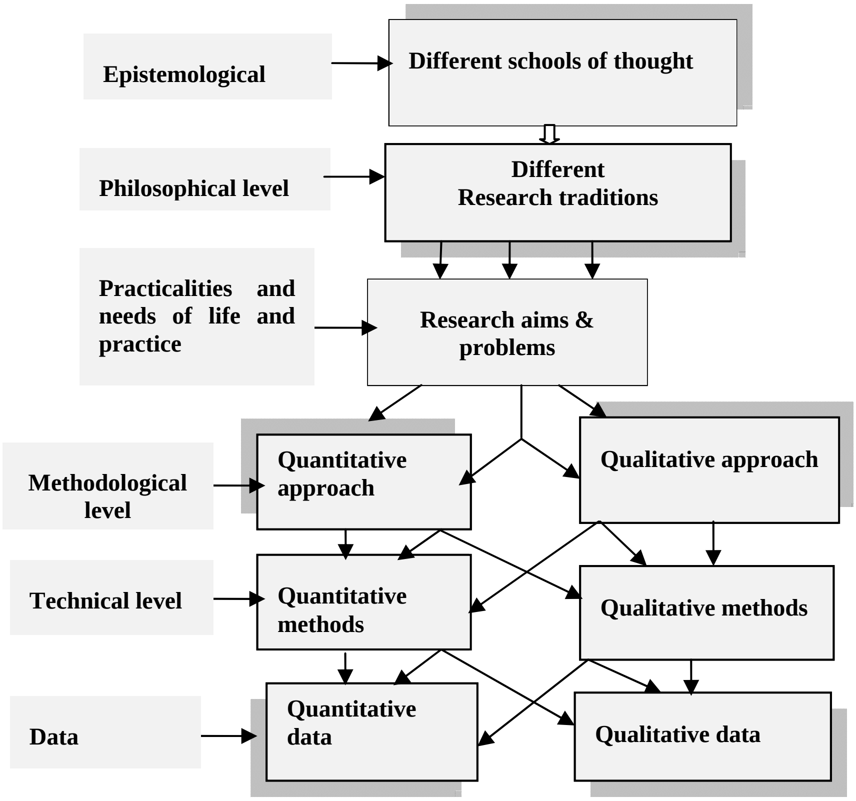

Figure 14. Levels of research in practice and the qualitative-quantitative divide (Niglas, 2004)

In Chapter 3 there is an attempt to give a description of the phenomenographic research approach and its 'associate', the notion 'conception'. This has been approached from a very positivistic point of view. This makes it necessary to unveil some of the criticisms directed against phenomenography. Säljö (1994), for instance, came out strongly and criticised phenomenographers for decontextualising human actions from the concrete practices which trigger them. Because of the interest in conceptions, phenomenographers have been reducing participants' utterances to isolated statements. The interest has been to 'siphon off' conceptions without taking into account the contexts in which they have been constructed. The immediate response from proponents of phenomenography, specifically Marton (1995), refuted this, and argued that learning and thinking are situated practices, or context-based. I find Marton's argument strong, and it is challenging to think of a learning situation which is devoid of context or setting. One can really work hard to find such a setting. In this study care was taken to avoid this criticism by consideration of natural settings of the teacher educators who were interviewed. This made it possible for them to express themselves in the environment they had good control over, including the use of mathematics teaching and learning materials posted on their office walls.

Further, sometimes critics of phenomenography easily ignore the fundamental aims and outcomes of phenomenography as a research approach. This has been stated in many circles: for example, Åkerlind (2005) reminds us that phenomenographic research aims to explore a range of meanings within a sample group, as a group, not the range of meanings for each individual within the group. That is to say that interview statements, for example, cannot be understood in isolation from others. This appeared to me as a very fundamental principle very often misunderstood by the critics.

The second criticism casts doubt on whether the conceptions or categories of description really reflect the content of the interviews (Francis, 1993). Of course, this is a validity issue, and there is a way to address it. An appropriate starting point would be to think of a searching question in that direction. How much interpretation of teacher educators' collected statements needs to be done to avoid distortion? To answer this question, Strauss and Corbin (1990) point out two views on data treatment. One view is not to have data analysed per se, the role of the researcher being to gather data and let the data speak for itself. Other researchers are concerned with accurate descriptions, and given the difficulties in using all the interview research material, it becomes necessary to reduce and order, which actually involves selection and interpretation. This is at the heart of qualitative studies: what kind of a researcher would in fact be interested, for example, with redundant information, irrelevancy and so on? I think this is what guides the principle of reduction. My immediate reaction is to ask what kind of scientific investigation that has no interest in precision? I am of the view that generation of conceptions, handling interview words and the research material can be managed with great skill to minimise some of the doubts and contradictions being claimed. In this study, for example, it was seen that, without going deeply into what each expression meant, it was difficult to make a dividing line between categories of descriptions. Otherwise the entire coding process and data analysis may not be realistic. One may think of the level of difficult in handling, for example, the whole statement approach. As indicated in the beginning of this paragraph, there is a possibility of reducing the clarity of key aspects of interest in a given study.

The third criticism is related to researcher competence in phenomenographic studies (Burns, 1994). There is a challenge that most phenomenographic researchers work individually and are confined during their data analysis. There is an opportunity to open up very late towards the end of the study. There is an argument for bringing in additional researchers for a number of reasons. Above all, it is a way of making the data collected open for challenge from the very beginning rather than wait until the end. The second reason is the potential to make an even better outcome space because of the greater open-mindedness and awareness of alternative perspectives (Åkerlind, 2005). Åkerlind argues further that an individual researcher can make a substantial contribution to the understanding of a phenomenon, but must not forget that group research might take that understanding further. This is a valid challenge as it may imply limitations in making a reliable interpretation for lack of open mindedness and alternative perspectives in the area under study (not necessarily subject matter area). However, I would say that this criticism applies not only to the phenomenographic research, but to other approaches as well. Again, it is not only the issue of researcher competence in making a valid interpretation, but also in making an unbiased interpretation.

It is for this reason I engaged former practising mathematics teacher educators (two of them fellow doctoral students or colleagues, and the other did his master's thesis in mathematics education) in the development of the draft and agreed categories of descriptions. In my view, the chances are that the credibility of engaging others at least during the coding and analysis stage solves many problems likely to appear at a later stage of study evaluation.

5.2 Addressing validity, reliability and self-critique of the research results

Research findings, like conceptions in MTE and thoughts about development, need to conform to certain criteria of evaluation. Marshall and Rossman (1999), for example, raise concerns about the credibility of research findings and the type of criteria that can be used to judge them. Questions to address the extent of truth and consistency could be built to find out the possibility of replication of the study if conducted with the same participants and the same context. Some of the issues being raised are then certainly problematic, for example 'same participant', 'same context'. It is known that over time these are not the same in any given timeframe. Like many others, Ary, Jacobs, and Razavieh (2002), and also Gay and Airasian (2003) emphasise validity and reliability as criteria for research trustworthiness or credibility. That is to say, conceptions and revealed thoughts about the development of MTE have to undergo validity and reliability judgement. The criteria against which research findings can be judged have many dimensions because there are different types of validity and reliability.

Validity can be demonstrated or addressed by the degree of honesty, depth of coverage of data and the extent of triangulation, to mention only a few approaches. Viewed in this way, validity appears to have many dimensions. For the purpose of this study a distinction between internal and external validity is considered in judging the credibility of categories of MTE and thoughts about MTE development. Internal validity primarily concerns the internal logic, coherence of the study, and logic of the arguments from premises to conclusions. On the other hand, external validity expresses to what degree the findings correctly describe conceptions of MTE. A better explanation is to say that the stated findings (identified categories) are considered valid if they correspond to the actual state of affairs of teacher educators' conceptions- if there is a fit between the recorded data and the interpretation and, for example, actual conceptions of mathematics teacher education on the ground. It concerns both correspondence and coherence of the findings and what is being studied.

Another aspect of external validity is the degree to which the results may be generalised to a wider population. Generalisation may be problematic again, especially in qualitative studies because of the context-specific criterion. Generalisation of the findings is really not the primary purpose of this study. There are some difficulties in doing so because it is like an attempt to generalise truth. This again raises epistemological concerns. As a matter of emphasis, LeCompte, Millroy & Preissle (1992) raised concerns against generalisation because human behaviour is infinitely complex, irreducible, socially situated and unique. And, in the words of social constructivism, truth is constructed by the social processes and constantly shaped, not fixed, which makes it difficult to generalise about. This view is also shared by Schoenfield (1993) in his cautionary word about the use of external validity. One possible question is, in the absence of generalisation, what is the use of research findings? The same researchers mentioned in the preceding paragraphs seem to support that findings can still be useful for understanding other similar situations and not primarily for generalisation and replication. One general way of addressing external validity could be by conducting a discussion on the categories and thoughts about further development of MTE with strategic external participants. In this study, teacher educators are strategic participants. Other ways of dealing with issues of validity and reliability are to come in the course of this discussion.

One aspect of reliability of research is the extent of consistency. Reliability as a measure of accuracy and consistency in qualitative studies is not simply for generalisation and replication of the research findings. For qualitative research like this one, attempting to identify variations in conceptions, consistency is particularly a strong requirement. In this investigation, reliability was improved by engaging colleagues with a similar exposure to and theoretical background in MTE during the process of coding (see coding in Chapter 3). Increased reliability was also achieved by involving a peer (research assistant) in the examination and addressing how well the established categories of descriptions of MTE converge. This was done by using conceptual labels (names). It is important to note that reliability expresses the extent to which the instruments used tend to generate the same categories of descriptions, regardless of who uses them. It also relates to the way the questions and the tasks are formulated – if they lead the thoughts in specific directions or if respondents are truly open and free to answer in a variety of ways.

I stand on the theoretical consideration of judging research findings against validity and reliability as important principles. But now I need to move from theoretical considerations to practicalities and actually show how I addressed methodological questions. Making a description of theoretical procedure does not spare me from the demands of a detailed description of how validity and reliability were actually addressed in this study.

I prefer to discuss in detail the issue of validity and reliability as credibility and dependability respectively because these terms are now frequently used among qualitative researchers, and in particular the phenomenographic approach (Ary, 2002, Jacobs & Razavieh, 2002; Gay, & Airasian, 2003; Åkerlind, 2005). This does not mean to move away from the fundamental meaning of validity as truthfulness, honesty and reliability as consistency or replicability of research findings. Many of the researchers cited in this paragraph take the view that credibility in qualitative research concerns the truthfulness of the research findings. This involves the researcher's ability to show evidence of the credibility of the findings based on research design, participants and the context. I would like now to deal with evidence of credibility followed by research dependability or reliability. Having read Ary, Jacob and Razavieh (2002), as well as Åkerlind (2005), I will deal mainly with triangulation, agreement or consensus, richness of data, control of bias, and transferability as strategies or evidence of how credibility in this study was addressed. I will first discuss evidence of validity in terms of credibility, and then move to reliability in terms of dependability towards the end of this section. Finally, there is a discussion to conclude this section.

In this study credibility or internal validity has been enhanced in a number of ways. The first evidence is connected to data triangulation. Results are increasingly trustworthy if they originate from a variety of sources of research materials (Denzin, 1970). In order to have a rich source of data dealing with the same phenomenon, different data collection techniques were used. The act of employing multiple instruments of data collection techniques, for example recorded interviews, and at the same time taking notes focusing on key aspects for research question one, made it possible to cross check the two ways of taking statements. Equally important, the open-ended questionnaire for research question two was supported by an immediate follow-up interview of a few teacher educators (5) in order to crosscheck the authenticity of the data by triangulation – to clean up, clarify some areas, look at miscommunication, and so on. In terms of open-ended questionnaires as used in this study, I need to relate to Hannan's (2007) views on the issue. One may ask teacher educators, for example, to indicate how strongly they feel about emphasising 'subject matter' as opposed to methods of teaching mathematics. Depending on the specific structure of the question, it is possible to end up with quantification or content analysis. On the other hand, the interest may be in posing a similar question and allowing teacher educators to formulate their own written expressions. Open-ended questionnaires allowed teacher educators to state their ideas in ways not pre-selected by any interested person. By using an open-ended questionnaire, it was possible to discover hidden ideas or what is sometimes termed unsuspected answers. Further, an open-ended questionnaire enabled the respondents to challenge some of the ideas taken normally for granted.

The challenges in using open-ended questionnaires in this study were in the difficulty of coding and analysis. I experienced a challenging situation in the course of category generation, despite working with a team of three assistants for the purpose of encouraging open-mindedness and minimising bias. It was not uncommon to face difficulties in dealing with open-ended questionnaires in the process of category development. The expressions would sometimes fall short of focussed ideas despite the open opportunity to respond. It was difficult to establish the facts from the products of the open-ended questionnaire, and the interview responses. The problem was further compounded when the language switched from Kiswahili (the national language) to English. My critical view of the entire process of generating categories of descriptions, even with pre-testing the instruments, is that some of the statements were not sharp enough to be easily categorised. Some ideas appeared extremely questionable and between boundaries, ambiguous, and contradictory. By saying this, I do not mean I expected something clinically clean - it is telling the narratives as they are. Yet, in some cases, I found reasonable connections between the results and the theoretical framework. In all, there was a general agreement between the different techniques of data collection, and hence the first evidence of internal validity.

The second way of addressing internal validity or credibility was based on agreement or consensus between my interpretation and those of my colleagues in the coding and analysis stage. This has been addressed in two ways. First, during the coding process I engaged three research assistants and independently coded half of the teacher educator statements on their understanding of MTE, discussed to agree on the categories of descriptions, and then I continued the process to complete the rest of the statements. The purpose was to make the data open for challenge from the beginning, rather than wait to some stage at the end. The second level involved the use of a colleague with a similar exposure to the study but not in the strict sense of a co-judge. The colleague helped to critically examine and validate the categories, creatively put the categories in a more distinct way, rather than the right or wrong matching between my interpretations and a would-be co-judge in phenomenography. The reason for doing this was based on the realisation that the coding and categorisation process has been a long and demanding exercise from the time I started. Some factors might have intervened, for example in the writing process. As a solution to this problem I found it important to engage a peer with a similar kind of exposure to the subject of the study apart from colleagues who participated during the coding and data analysis in earlier stages of the process. The procedure worked with a high level of agreement, and differences, for example, in wording were creatively shaped. The numerical value overemphasised in the use of a co-judge was therefore not my first priority. Of course, this is like triangulation between researchers, and it stood to be my second way of addressing internal validity or credibility.

The third way of addressing internal validity was through a careful development of the categories of description with matching aspects and teacher educators' statements on MTE and their thoughts on the development of MTE. This is portrayed in Table 8, Table 10 and Table 12. Equally important are the elaborative quotations from the teacher educators about their conceptions of MTE, their points of view on mathematics teacher education development, as well as strategies for sharing knowledge and skills. It is reasonably convincing that my interpretation of the teacher educator expressions was carried out reasonably well. The requirement is the fit between my interpretations and teacher educators' statements, which are indicated as stated in Tables 8, 10 and 12 in Chapter 4.

The fourth strategy for addressing internal validity was in terms of how I controlled the bias of the research results. Sometimes this is referred to as neutrality (confirmability) in qualitative research and is taken to mean the extent to which the researcher is free of bias in the procedures and interpretation of results (Ary, Jacob & Razanieh, 2002). Essentially, this has been explained under addressing agreement of the research results. The use of research assistants made it possible to critique the data, while at the same time developing the categories. In that way, they helped to make the data open to challenge and therefore minimised the possibility of bias in the whole process of coding and analysis. The other strategy of avoiding bias and attaining neutrality was to constantly reflect on my own actions in the process in order to avoid any influence in the study results: for example, establishing the principle of selecting educators as respondents (see Chapter 3) from different geographical locations to increase variability. In the words of Ary, Jacob and Razavieh (2002), bias is controlled mainly through reflection of the entire research process. Constant reflection on how the study was conducted is evidenced by piloting of the data collection instruments before going to scale (see Chapter 3). Piloting of the data collection instruments suggested relevant revision of instruments in order to ensure that the data captured was what was intended by the study.

I also addressed external validity or transferability as is the case in qualitative studies. Many researchers in qualitative research are cautious about the transfer of research results to other contexts, and there is even more caution in phenomenography (Ary, 2002, Jacobs & Razavieh, 2002; Gay, & Airasian, 2003; Åkerlind, 2005). External validity in this case refers to the possibility of generalisation of the research findings; in other words, if the findings are applicable in other situations. This is, however, problematic because of the differences in contexts. For phenomenography the discussion is about transferability of the category outcome space. Transferability involves taking into account the qualitative approach and by asking whether the generated categories of descriptions of MTE are applicable in other contexts. For example, if another researcher is given the same data concerning conceptions of MTE, would he/she report the same categories of descriptions or outcome space? It can be stated that some of the categories of descriptions are time and context sensitive, while some can be used to explain other situations. In a specific way, conceptions of MTE and thoughts on development of MTE seem to reflect teacher educators' diverse mathematical histories. It is important to note that some of the constructs or conceptions may be unique to specific contexts under investigation. The unique historical experiences of participants like teacher educators may be the cause of limited comparison. I now turn to the issue of reliability.

Addressing reliability or dependability was important for judging the results of this study. In quantitative research reliability has to do with consistency of the research results. That is the extent to which data and findings would be similar if the study was repeated or replicated. In qualitative research this is difficult because of changing conditions; 'same participants' and 'same conditions' are difficult to attain. Human behaviour keeps on changing over time. Researchers in qualitative studies use a variety of strategies to address reliability or dependability. A range of these strategies are the audit trail, replication logic stepwise replication, inter-rater comparison, and triangulation. Some of these notions might not directly explain how dependability (reliability) has been addressed, but I need to explain how one could build confidence on what has been found. First, there is a complete description of the study conduct as indicated by the study tasks from Phase I of testing the instruments in August 2006, and data collection for research question one in December of the same year. This was followed by pilot-testing of the instrument for research question two in March 2007, and again data collection in December 2007. This sequence of research tasks combined with the expressed data collection procedures is the first stage to explain the dependability of the research results. The principles of the selection of the participants have been explained in Chapter 3 and are also important in terms of the dependability of the research findings. In a strict sense, all these form part of what qualitative researchers call audit trail.

The dependability of the research results has been addressed by the inclusion of participants from different geographical locations for the purpose of increasing the variability of ideas. To achieve this, I collected data from a heterogeneity sample of teacher educators working in diploma and certificate teacher colleges. Diploma teacher colleges are staffed by graduate mathematics teacher educators, while there is a combination of graduate and diploma staff in certificate teacher colleges. Done in this way, the variation within the sample reflects the variation in the desired population, although not in a proportionate way. The remaining issue is how these results can be applied to other mathematics teacher educators.

Dependability was also addressed by engaging research assistants. In this study, three colleagues familiar with the study independently coded and analysed a sample of the data to begin with. The involvement of colleagues in the process of coding and data analysis to the level of mutual agreement on the categories of description not only enhanced the dependability of the study, but also minimised the possibility of bias. However, the process was not all that straightforward, because reaching a professional consensus was often very difficult. Arriving at draft categories of descriptions involved a serious debate among the colleagues. Some ground rules had to be agreed on and these included respecting of each others' opinions unless a scientifically superior one was given. For these reasons, I find the study to have generally demonstrated not only the reliability issue but also validity in terms of credibility, authenticity, transferability in terms of outcome space, and conformability. The traditional approach is that of a confined researcher working individually during coding and data analysis. Some authors argue for bringing in additional researchers for two reasons: to make the data open to criticism from the start rather than do it in the end; and second, there is a possibility of improving the outcome space given the open-mindedness and awareness of alternative perspectives (Åkerlind, 2005).

In addition, it is important to end the discussion on reliability of the research results in the light of the data collection instruments. The long and elaborative statements from both the interviews and the open-ended questionnaires signify the reliability of the data collected. To achieve this, the interviews were conducted in teacher educators' natural settings by playing down the usual high-level interview formality. In the Tanzanian context, the high-level approach to interview sessions sometimes creates tensions for interviewees (teacher educators) because they regard the exercise as a personal assessment or an inspection. To create trust, and an atmosphere of balanced discussion, it was necessary to assume a low profile without losing sight of the data collection mission. In this regard, the ultimate aim of the study was kept open, and was used to motivate the subjects to come forward and give their views. The route taken by the open-ended questionnaire did not follow the traditional posting of the questionnaire and waiting for the postal service to deliver the responses. In this method returns are normally poor unless there is strict follow-up of the respondents. To come to the final stage of coding and analysis was not a 'once and for all activity'. It consumed a lot of resources in the sense of time. There was a process of coding and undoing at some stages because of new thinking or new perspective. Establishing 7 categories for research question one, 5 for the first part of research question two and 2 for the remaining has to be trusted given the amount of reflection and care accorded to the process.

As mentioned previously, the purposeful selection of subjects of the study was not for generalisation. Purposeful selection of respondents in this case means teacher educators who met the intentions of the study and were chosen according to principles. The representativeness is, however, satisfactory for some generalisation within Tanzania. Generalisation or transferability of the research findings outside Tanzania have been addressed by a careful description of the national and local conditions, and by the quotes that exemplify the conceptions and thoughts in Tanzania. Some of the research results will no doubt correspond to situations in other countries with similar problems in MTE. Other issues are purely flavoured by the conditions in Tanzania.

Losing touch with ethical considerations in a study like this one is a serious omission. As noted in the beginning of the chapter, the main purpose of engaging in a critical reflection of the research methodology is twofold. On one side it is partly a process of methodological justification and at the same time a part of ethical consideration. The intention of the chapter is therefore to discuss the research methodology without losing touch of ethical considerations. This brings us to the use of given names for the purpose of confidentiality; the consent of participants during the interview was attained through rapport sessions. Equally important was an approval by the education ministry to visit, interview and collect teacher educators´ written statements in order to code and analyse them.

Concluding thoughts on the methodological critical reflection

Despite the systematic and reliable approach in studying conceptions using phenomenography, the same could also be studied by applying other research methods, for example case study and ethnography (Black & Atkin, 1996; Thompson, 1992). Certainly, the ethnographic approach would have compelled the researcher to stay with mathematics teacher educators for a much longer time. For this reason, and given the available time, ethnography was not seen as an appropriate methodological solution. How would the results have been affected if I had allowed myself to see teacher educators in teaching sessions instead of only identifying conceptions and thoughts using the statements given? I am aware that sometimes physical actions and the way mathematics lessons are presented in actual classroom situations may allow certain interpretations. For example, it may be possible to see in action what is claimed as a focus in mathematical investigations, inspiration and/or emphasis on pedagogy. This is a missed opportunity.

The inability to see educators in actual classroom action cannot alone amount to affecting the credibility or validity of the research results. But seeing educators in action would have made it possible to confirm the strong positions teacher educators take when claiming to be advocates of problem solving, subject matter, and so on. This shortfall is sometimes capitalised on by critics of phenomenography. Säljö's (1994) criticisms have been pointed out in the beginning of this chapter and discussed.

Furthermore, phenomenography is about studying how people experience the same phenomena. Mathematics teacher educators' innate experiences of MTE indicate that teacher educators were exposed to the same phenomena of interest, and yet they had variations in focus on the same object, in this case MTE. Why did this happen? One possible explanation is that they conceived it in qualitatively different ways because of their varied backgrounds in MTE. Teacher educators have a different subjective history in terms of subject matter and pedagogical and content knowledge. A more recent explanation of the variations view is based on the differences in focus on what phenomenographers call 'critical factors'. It is argued that people focus on the critical aspects of a phenomenon differently, and this creates variations (Pang, 2003). It is also important to note that preconceptions influence present conceptions because people carry with them their subjective history; in other words, learning and thinking are situated practices (Marton, 1995). All these have an influence on what they had to reveal as conceptions, and thoughts for further development, as well as thoughts on strategies for sharing knowledge and skills in MTE. Therefore, it sounds logical to argue that MTE is composed of different critical factors or aspects which are also focussed on differently by mathematics teacher educators.

In this study, and on the basis of its credibility or validity and dependability or reliability as explained in the preceding paragraphs, the choice of phenomenography as a research approach enabled me to identify conceptions and provide a description of their variations in terms of categories of descriptions. Interestingly, the categories of descriptions which form the main results seem to have a direct relationship with MTE as seen by practitioners in the Tanzanian context. Peers with whom I have worked helped to minimise bias from the very beginning. A colleague managed to critically examine and discuss with me the fourteen categories generated as a result of coding and analysis for the two research questions. The engagement of a colleague with a critical mind in the final stage (not strictly in the sense of a co-judge), together with peers (colleagues engaged in the initial part of categorisation) in the coding process was applied to address the issue of trustworthiness (validity and reliability in a strict sense). In addition, the categories of descriptions are distinct, except for the unexpected hybrid category labelled as organised integration of subject matter and pedagogy, which is an integration of subject matter and methods of teaching and learning mathematics. I am of the view that there are practitioners who are open and conceive phenomena from a holistic perspective or homogeneous way, rather than having fragmented ideas. I found this to be realistic. In all the categories of descriptions the main findings stand the test of quality criteria (see Chapter 3 on features of quality criteria) and really reflect content from the interviewees.

In summary, this chapter has dealt with the credibility or validity and dependability or reliability of the research results and the attempt to provide strategies on how to address the issue of trustworthiness of the research results. This is when the issues of data collection techniques and triangulation, colleagues engagement and reaching agreement during coding and data analysis, control of bias. To pull all these important research tasks together, a rich description of the study conduct has been done from the inception of the study in August, 2006 to the time of coding and analysis of data completion. The next chapter now attempts to find the connection between the research results and new theories.

6 Discussion of research results

This chapter comprises discussion of the research results. A detailed account is presented in the order of the research questions. The discussion begins with a summary of the research results followed by overall remarks on the empirical study results, and within this, variations of thinking about MTE is covered. Next is a discussion about the variety of thoughts about development of MTE. Finally, there is a discussion of the variations in thinking about strategies for sharing knowledge and skills among mathematics teacher educators.

The discussion of the research empirical results is more than a routine activity of researchers. It is rather a task based on valid reasons which may be based on a number of factors. To begin with, the attempt to understand MTE from a second-order perspective on the basis of the fundamental principles of phenomenography challenged my knowledge of the approach in an interesting way. I encountered a range of challenging moments. For example, some of the ideas identified appeared apparently new in the context of teaching and learning mathematics in Tanzania, some ambiguous, some contradictory, and so on. Yet in some cases, I found a reasonable connection between mathematics teacher educators' ideas about what they think and do and for what purpose, and the theoretical framework. These are some of the reasons for the attempt to make comments on the study results. I would say that each of the major results in terms of categories of descriptions had at least one outstanding issue with the potential to stimulate a mathematical discussion. For this reason, it might be extremely difficult to discuss each and every single aspect of the study results. With that in mind, the results are sometimes discussed in combination, because in practice they are inseparable.

6.1 Summary and overall remarks on the main research results

As a reminder of the research questions I would like to raise them once more. The questions of interest were: a) what are teacher educators' conceptions of MTE, and b) what are teacher educators' thoughts on the development of mathematics teacher education? The results indicate that teacher educators conceived both (of the two research questions) phenomena in qualitatively different ways. That is, MTE with a focus on mathematical investigations and a process of inspiration; MTE with a focus on problem-solving; and then MTE with emphasis on pedagogical knowledge and skills. Furthermore, MTE was qualitatively conceived as a focus on subject didactics, subject matter knowledge; and even further, thought of as an organised integration of subject matter and pedagogical knowledge and skills. Regarding the second research question, teacher educators´ thoughts on development of MTE focussed on pedagogical knowledge and skills, subject matter knowledge and skills, a process of integrating assessment, teaching and learning, as building relationships, and as studying about teaching and learning. In addition, teacher educator's conception of sharing knowledge and skills focussed on neighbourhood learning groups as well as collaboration and networking among themselves.

It is important to make a few general observations regarding the research results before engaging in a detailed discussion. This is because some of my reflections run through the entire study results.

My first observation (remark) is that teacher educators think in a variety of ways about MTE, the development of MTE, and strategies for sharing knowledge and skills. That is, there are sharp variations in the way they experience the same phenomenon, MTE is seen and experienced in qualitatively different ways. How teacher educators came to conceive MTE and related developments in different qualitative ways is linked to their own diverse histories or previous experience, which also feature a lot of differences in focus, beliefs, and conjectures about MTE. They are likely to be a product of different theoretical orientations or routes of knowledge gained up to the time they were interviewed in this study. Many studies link prior knowledge and learning, for example the longstanding pedagogy by Ausubel (1978) in which there is a strong argument that preconceptions influence present learning. Teacher educators who were asked to reveal their insights about MTE and thoughts on development are, in the final analysis, learners, and therefore not unique in this particular case. Evidence exists to support this. Consider statements from teacher educators like 'I learnt portfolio assessment from an institution outside Tanzania', or 'I attended a seminar and workshop on inspiration mathematics', which are clear indication of carrying personal previous experience in the form of ideas into mathematics teacher education. Perhaps this may be a better response to critics who argue that phenomenography is interested in siphoning-off conceptions with a total disregard of context. In this case, we have respondents revealing the source and context of conceptions and thoughts on development of MTE, and are a clear indication of situated learning. Hence, the variations in thinking about MTE.

The second overall observation on the results is based on the permanence of the conceptions of MTE, thoughts on development and strategies for sharing knowledge and skills. The conceptions of MTE, thoughts about development, and in the same way, strategies for sharing knowledge reflect what temporarily dominated during the time of the study. I am of the view that conceptions can be context and time sensitive. A number of studies have been carried out to bring this argument to light. Marton's (1995) view on learning as a situated practice is a logical argument. For this reason, it may not be plausible to generalise much of the research results, as the findings are situational. But they may be used to learn about other situations with similar issues regarding teaching and learning mathematics.

The third overall remark rests on the status of teacher educators' conceptions of MTE. The results indicate differences in views not strictly 'narrow' or 'digging deep', but rather a situation of elaborative and compact responses. This is in relation to conceptions of MTE, and thoughts for development and strategies for sharing knowledge and skills. Viewed in that light, the ambition from the onset was not to test how knowledgeable mathematics teacher educators are, but rather to study the conceptions through their eyes and be able to provide a description of the variations. This has been achieved, and I have realised that they are at different levels of reflections on what MTE is. This in turn forms the basis of variations in focus.

The fourth major remark is a reflection on the connection between the results and the existing theoretical framework (See Chapter 2 for theoretical framework and the previous chapter for research results). The results indicate that one has to work harder at establishing shared conceptions of MTE than at seeing the variations or the complex nature of MTE (Adler, 2005). The same situation exists for mathematics education (Bass, 2005; Ernest, 1998; Lerman, 2001; Lester & Lambdin, 1999; Mura, 1998; Niss, 1998; Wittmann, 1998) given their close relationship built around teaching and learning mathematics as well as being lively research sites. However, in this study the recognition of MTE as a research field seems to be less than, for example, its recognition as pedagogy. MTE when considered as part of teacher education has an important additional role, very often overlooked. I have in mind the process of making a teacher with emphasis on 'teaching about teaching' and 'learning about teaching' in the sense of Loughran (2006).

Finally, mathematics teacher educators have their own preferences and aspects of emphasis, sometimes taking pedagogical positions in MTE, thoughts on options for development, and ways of sharing knowledge and skills. The results show that the revealed conceptions on MTE are the basis for making pedagogical choices, and the positions taken on what mathematics teacher educators do. Making choices between pedagogical options in MTE has been seen to be challenging, which in turn leads to taking positions as a result of pressures from the prescribed mathematics teacher education curriculum. This conflicting situation in the process finally leads to sharp conceptual variations in MTE.

6.2 Various ways of thinking about MTE

As with many studies using the phenomenographic approach, the interest is to identify ways of experiencing the same phenomenon (conceptions) from a second-order perspective and provide descriptions of variations. I am aware that phenomenography is an ever changing and growing specialisation, as seen in Pang (2003). Although it is inappropriate to change the rules of the discussion in the middle of the research task, I am obliged at least to recognise recent developments in phenomenography. This is important in order to establish the line of argument in the discussion of the study results. In phenomenography, there is currently a shift away from how the different ways of experiencing a phenomenon can be captured methodologically towards the theoretical nature of the differences or variations. That is, early phenomenography is descriptive and methodologically-oriented, while recent phenomenography is theoretically-oriented and attempts to look at the nature of variations (Marton & Pang, 2006; Pang & Marton, 2007; Pang, 2006; Pang, 2003). The discussion of the study results is likely to fall mostly in the former, for one main reason: this study was created and shaped in between the transition, which is not yet over.

The findings indicate that teacher educators conceive MTE in qualitatively different ways, with a focus on MTE as mathematical investigation, a process of inspiration, a focus on problem-solving, as well as an emphasis on pedagogical knowledge and skills. Furthermore, MTE is qualitatively conceived as a focus on subject didactics, subject matter knowledge, and even further, it is thought of as an organised integration of subject matter and pedagogical knowledge and skills.

Each of the conceptions highlighted above has a pattern of its own, here referred to as a category of description. The first research task, 'what are teacher educators' conceptions of MTE', points to some critical features of the objective. It makes sense to say that these features or aspects are the building blocks of the broad concept or what makes the phenomenon (refer to earlier part of this chapter on presentation of results). To be more precise, teacher educators focus on these critical aspects differently and thus variations become the mode. I sometimes use the analogy of two people watching the same picture and the same conditions on a screen, but seeing different things. This is what may be called the conceptual foci of MTE.

Category A: focus on investigation

In this category of description, the discussion centres on the concept of MTE as a process of learning with a focus on mathematical investigations. Before that, it is important to raise a very pertinent question. What precise meaning is assigned to investigation by teacher educators? From a very general point of view, the term investigation may mean different things to different teacher educators. Judging from teacher educators' expressions, there are two possible meanings of the notion. First, they understood it as a pedagogical approach to teaching and learning. This is partly supported by statements like, 'It is pedagogy or as others say it is education about teaching methods', or 'a particular way of managing the interaction process in the classroom'. These statements, and many like this, were made by teacher educators. This strongly implies that investigation is viewed as a pedagogical approach. In other words, one would say it is mathematics teaching methodology or mathematical classroom instruction. In a similar way, Ernest (1991) made a case for investigation as a pedagogical approach and compared it with guided-discovery and problem solving. Of course, it is difficult to differentiate between them unless a criterion is established. It is worth pointing out their similarities and differences, as shown in Table 14. The criterion to distinguish the term 'investigation' from the rest is based on the role of the student and that of the teacher.

Table 14 presents comparative views of what may be called an inquiry-based pedagogy, indicating the place of investigation as a pedagogical approach. Between the three pedagogical approaches, the respective role of the students and the teacher educator differentiates the terms. Ernest's (1991) theoretical approach to clearly set out the difference is logically convincing. One concern is its functioning in the actual classroom situation, and location of the dividing line for practitioners. On this basis, mathematical investigation is seen as a pedagogical approach or as a method of teaching and learning mathematics. It is my view that guided-discovery, problem-solving and investigation are, in the final analysis, processes of getting knowledge. This focus could have been influenced by some innovative ideas about mathematics teaching knowledge being implemented at the time of study.

Table 14. Comparative views of inquiry-based pedagogy for MTE (Ernest, 1991)

Pedagogical approach	**Role of teacher educator**	**Students' role/activity**
Guided-discovery	Presents a relevant problem in context with goal in mind. Guides student teachers in the process towards a solution.	Prepare, do the activities by following instructions.
Problem-solving	Presents a relevant problem; leaves method of solution open.	Work independently to solve the problem.
I*nvestigation*	*Chooses a relevant case, event, situation in consultation with students.*	*Define own problem within situation.* *Independently find own solution.*

In another dimension of MTE the broad sense of investigation points to doing research. Some ideas from Anna's (teacher educator) statement will be used here to put this 'other' view of the term investigation in perspective. Anna's statement gave an interesting hint when she said it is 'research or inquiry or activity-based, for MTE involves investigation into teaching and learning and in the process problems are identified'. This view of describing the term 'investigation' refers to studying about teaching and learning and is different from investigation as a pedagogical approach. One may question this view, but here already is unsuspected response from the teacher educators' statements.

What do the two generic terms mean in MTE? It is important to differentiate the context in which the term 'investigation' is used. To make the two foci of investigation understood, consider the case of 'mama Ntilie' (a local food vendor in Tanzania), who increased her price of buns, and as a result her sales went down drastically. To find out a mathematical solution, student teachers may be required to investigate as a project what prices may be more acceptable to customers and hence increase daily sales. This may be seen both as a pedagogical approach to learning about the drawing of graphs and at the same time studying about some business problem regarding pricing. Students may be required to collect samples of a number of customers over a period of time, draw graphs, and finally come up with some mathematical advice to the vendor. But, to find out why Aden and other students cannot solve quadratic equations may warrant an investigation in the sense of research. This is the basis of teacher educators linking the notion 'investigation' and research.

For very important reasons, I would not let the issue of investigation pass without a remark. To equate the terms 'investigation' and 'guided-discovery' with research is stretching it too far. Yes, they are close and it is quite difficult to draw a line between them. Guided discovery, for example, is as old as the natural sciences. It has strongly established itself there because of the quantitative approach in both learning and research. Should, for example, a journalist investigating and reporting on students' mathematics-avoidance syndrome, stigmatisation, and phobias be said to be doing research work? Perhaps not, unless it is supported by a sound theory raising interesting research questions or hypotheses and has a clear and systematic way of reporting the scientific findings. Methodologically, while some teacher educators conceived MTE as a focus on mathematical investigation, others viewed it as an inspirational process of teaching and learning mathematics. This is another interesting area to penetrate and will be discussed next.

Category B: focus on inspiration

In this part of the study, the intention is first to discuss the meaning assigned to the concept 'inspiration' and then try to qualify it with a collection of examples on how inspiration may happen in a typical MTE learning situation. MTE as a process of learning with a focus on inspiration invites discussion. It was rather complex to establish its concept from educators' expressions, because it brought in a number of other closely related constructs, examples being stimulation, amusement, enterprise, motivation, and more. The central issue perhaps was what meaning is assigned to inspiration and how can teacher educators make it happen within the context of MTE? This appeared to be a relevant question because it has a direct impact on ways of addressing issues of low self-esteem, stigmatisation, and mathematics-avoidance syndrome (shrinking interest). This is also prompted by the systemic problem of making students interested in MTE. The main task throughout was how teacher educators can make inspiration happen. The bottom line is that teacher educators are expected, for example, to take on a new role by being a rich source of inspiration to mathematics student teachers rather than transmitting knowledge (Lee, 2008). Experience indicates this to be a difficult task among teacher educators.

From the results, it is possible to see critical aspects or features leading towards inspiration. These features of inspiration are 'attracting students', 'building relationships', 'love for the subject'; 'creativity in the subject'; 'motivation'; 'stimulating'; 'enterprising'; 'puzzles', amongst others (see Chapter five for details). How to make inspiration part of MTE is equally difficult, but perhaps drawing from teacher educators' statements, current MTE research literature and my personal experience as a mathematics teacher educator, trainer in teacher/educator development and advisor on curriculum matters, it is possible to suggest at least a point of departure, though perhaps not the end. Even the starting point is wide open. I would suggest that the source of inspiration in MTE could come from, for example, the use of relevant and context-based puzzles, building relationships between teacher educators and students, as well reader-friendly teaching and learning materials. The options for promoting inspiration are again broad and may involve many mathematics activities.

Puzzles, for example, are very powerful tools, but often neglected, even though teacher educators know full well their role in attracting students' interest. I have in mind a few examples as a way of emphasising the point. Puzzles have been used to start lessons or as an exercise for students to reflect on. But perhaps they have other important roles also: consider the following examples:

Example 1: "*In Sukuma land lived a man with many cows. Before his death he said that half of his cows could be divided between his four sons and a quarter between his 6 grandsons, and another quarter of the cows between his 8 daughters. What proportion did each get in the family?"*. (Mathematics lesson, context within Tanzania)

A process towards finding a solution would normally start with: let the number of cows be x, then the division among sons, grandsons and daughters is respectively 1/8, 1/24, and 1/32 of x. The proportions for each can be shown as 12:4:3 to sons, grandsons and daughters, respectively. An example like this one could be used to stimulate a lesson in arithmetic proportion as it is based not only on some activity of human interest but is also in the context of learners. In addition, it could be converted into a number of properties including shares, savings accounts, and so on.

In a study about 'which mathematics problems do teachers consider beautiful?' Karp (2008) insists that what has been established as the main source of motivation to study the subject is its beauty and elegance. I find it difficult to enter into another discussion on beauty, as it is difficult to define in MTE or mathematics for that matter. The author, however, relates it to "a clear and compelling presentation of results", "harmony of numbers and forms", and "seeing the hidden order in the seemingly confusing picture". All in all, and perhaps to pick only what is relevant to explain 'inspiration' in relation to MTE, talking about beauty and pleasure in mathematics and MTE is not a feeling that arises spontaneously, and there may be a failure on the part of mathematics to cultivate or inspire such a feeling in students. This brings us to the issue of the beauty of mathematics problems as a source of inspiration. To further substantiate this discussion, consider the following example:

Example 2. *"Perform the following operation on your calculator: (a) enter the first three digits of your telephone number; (b) multiply the number you obtain by 80; (c) add 1; (d) multiply by 250; (e) add the number formed by the last four digits of your phone number; (f) add this number again; (g) subtract 250; (h) divide by 2. You will obtain your phone number"*. (math lesson, experience from a USA teacher- Karp, 2008 pp.36-43).

The process towards finding the solution can be summarised as: let the telephone number formed by the first 3 digits be x, and let the telephone number formed by the last 4 digits be y. Following the same sequence of the problem, the numbers are: 80x; 80x + 1; 250 (80x + 1); 20000x + 2500 + y; 20000x +2500 + 2y; 20000x +2y and finally the personal phone number is 10000x + y.

Example 1 and 2 may be considered as typical cases where teacher educators are expected to adapt and shift from being seen as mere transmitters of mathematics knowledge to sources for student inspiration. Whether they can act as a source of inspiration or not is an issue of discussion. Karp (2008) went further by developing a collection of USA and Russian experiences regarding mathematics problems considered to be inspirational. His findings indicate mathematics problems built around real life situations seem to inspire not only learners but teachers, too. It is also plausible to argue the same for teacher educators because of the connection between mathematics education and school mathematics.

There is an interesting connection between 'inspiration' as a research result and the experiences as exposed by Karp (2008) with regard to the nature of the problems and how they can promote inspiration in MTE. The examples are different and all affect the human senses differently. They all involve some algebraic manipulation. The first is simple, but connected to the real world in the Tanzanian context. The second problem in a strict sense has no meaningful link except for the phone number as a real world object. Apart from these examples, one may go a step further by thinking of problems which are purely mathematical. For example, solutions to quadratic, simultaneous equations, simplifying radicals designed only to test the use of knowledge of rules and procedures. Consider the following two examples taken from Hagman and Mitts (1988), which may be common in a variety of school and mathematics teacher education learning experiences:

Example 4. Find the value of x and make a comment on your final solution in the equation: $(2x - 3)(3x + 5) + (x - 1)(4x + 3) = 36$; and

Example 5. Simplify the following radicals:

$$(2\sqrt{3} + 5\sqrt{2})^2 - (2\sqrt{3} - 5\sqrt{2})^2 + (2\sqrt{3} - 5\sqrt{2})(2\sqrt{3} + 5\sqrt{2})$$

In example four it is possible to see that upon opening the brackets, and adding and subtracting like terms, one is likely to end up with $10x^2 - 18 = 36$, which leads to the value of $x = \pm\sqrt{5.4}$. An equally important final step is to reflect on the solution and check whether it satisfies the original equation. Along the same argument of dealing with purely mathematical problems unconnected to direct life situations from face value as in example 1 and 2, example 5 is not different. The process of simplification involves the squaring of terms, opening brackets, adding and subtracting like terms, and after several steps gets

$$12 + 20\sqrt{6} + 50 – (12 - 20\sqrt{6} + 50) + 12 + 10\sqrt{6} - 10\sqrt{6} - 50,$$

the final expression is $40\sqrt{6} – 38$.

These examples seen in the light of Karp (2008) involve different interpretations, despite the insistence on mathematical problems bearing direct relationship with human activities. In a world where teacher preparation is increasingly in the spotlight and the importance of attracting student interest in MTE is on the agenda, these examples tell a story which either serves the purpose of inspiration or may not do so. Puzzles in particular are thought provoking and therefore better positioned to serve this purpose. Karp (2008) supports inspirational problems as a way to motivate learning mathematics and as a point of emphasis. I have nothing against Karp (2008), except for the acknowledgement as argued by Hudson et al (1999), that mathematics, like any other discipline, is a matter of student interest, and is hated and at the same time loved, said to be bad looking and beautiful, clear and unclear to others, and finally negatively frustrating, and positively challenging. I may add, one has to be strong with it. This is the wisdom of practice, and I think Hudson et al (1999) stand to be supported regarding this position. Table 15 shows the distribution of mathematics problems which are considered inspirational by Russian and American teachers, respectively.

Table 15. Distribution of problems by topic as submitted by American and Russian teachers

Category of topic	Number of problems and originating country	
	USA	Russia
Algebra and number theory	11	40
Logic, probability and combinations	9	1
Analysis	8	1
Geometry	9	11
Real life situations and applications	12	9
Puzzles	8	1
Trigonometry	-	9

Source: (Karp, 2008)

It may be questionable if the Russian example on algebra and number theory is really inspirational as opposed to real life situations and applications, as very little is known of the study sampling procedures. Of course, human beings under normal conditions are likely to be attracted by mathematics problems which touch their own life in some way. According to Karp (2008), the differences between American and Russian problems are typical and raise questions as to whether the interest is in critical features to promote inspiration. It can be further argued that the formulation of the problems needs to make reference to real-world situations if they are to be considered \`beautiful´ problems. Table 15 gives a glimpse of what kinds of problems teachers in the respective countries consider to be \`beautiful´ in the words of the researcher. What is considered a \`beautiful\` problem is likely to be either “real-life situations or applications” or ‘puzzles’. On the basis of the three examples, the criteria for inspiring problems are also linked to these two clusters. The issue now is to what extent teacher educators can strike a balance in order to address inspiration as a component of MTE. The key message from mathematics teacher educators is to design and use problems which inspire student teachers. The extent to which teacher educators can strike a balance in order to address the issue of inspiration may warrant a different study. To reach this point, I started the discussion by looking at the first two categories of descriptions: investigation, followed by inspiration. In order to keep a logical sequence in the variety of ways of thinking about MTE, problem-solving comes next. I am of the view that all are concerned with how teacher educators can continue to reject acting as mere transmitters of knowledge to becoming sources of better instructional strategies.

Category C: focus on problem-solving

In this category of description I would like to discuss problem-solving in two ways: the first, problem-solving as conceived by teacher educators, and secondly, in connection with other researchers. Key aspects or features used by teacher educators to discern problem-solving are the application of mathematics knowledge to solve life situation problems, which is a standard approach in solving mathematics problems. Among others, they include understanding the problem to be solved, deciding on procedures towards finding a solution, and finally reflecting on the solution. This way of understanding problem-solving focuses on only two characteristics of problem-solving, that is, problem-solving as a vehicle (application or transferability), and as practice to reinforce skills (Stanic & Kilpatrick, 1998). To some extent, justification for the curriculum includes problems connected with real life. My experience and perhaps that of many others in problem-solving is that it started like a 'catch-phrase'. If understood at all, then it was in its very traditional sense, that is, as a way of justifying mathematics to be part of the teacher education curriculum, and as practice to reinforce skills. That is all, and not as strongly argued as a focus on motivation, recreation, and vehicle of learning, application or transferability mechanism (Charles, 1989).

Another way of talking about problem-solving is based on how to differentiate it from other frequently used terms, for example, investigation and guided discovery. It was shown at the beginning that problem-solving also means a focus on application. This brings in the issue of transferability, which is taken for granted to be automatic. Experience indicates that problem-posing and problem-solving, guided discovery and investigation, which have been discussed previously, go together, and very often they are taken to mean one and the same thing. But what are their differences? Let us start with how teacher educators described investigation. For them, the notion makes sense in at least two ways. The first sense is the pedagogical approach. The second sense refers to a small-scale study about a mathematical problem. In a strict sense, it brings in the notion of research. The problem of equating the term 'investigation' with research has been discussed in the early part of this chapter.

Problem-solving as a philosophy to guide teaching and learning mathematics is increasingly becoming strong in teaching and learning mathematics, given the amount of research work on it (Ernest, 1991; Lester, 1998; Noddings, 1998; O'Connor, 2000; Shoenfeld, 1998; Stanic & Kilpatrick, 1998). Ernest (1991) went further to distinguish between investigation and problem-solving. I have two comments to make on this. One is that it is challenging to make a distinction between the two, though both relate to mathematical inquiry -the process of getting knowledge in its simplest form. Further, if the interest is to distinguish between the two, as Ernest (1991) argued, one then needs to establish a criterion. The distinction criterion has been suggested before and thus gives a summary of their differences (see Table14 for further clarification). All in all, problem-solving in view of the teacher educators' statements refers first to a pedagogical approach, and second as the application of mathematical knowledge. A comparison of the research results and the studies cited seem to agree in principle that problem-solving is used as a justification for teaching and learning mathematics, motivating learners, recreation, a vehicle for application and a focus on practice.

In this case, the suggestion is to consider 'relevant problems' for problem-solving, even if we are operating with the view of problem-solving as practice or justification. Experience indicates cases of problems being posed just to keep student teachers busy rather than as a critical factor of MTE. The key message in this case is that problem-solving is regarded as a mathematics teaching approach and is a process. In this study, teacher educators conceived problem-solving as the application of pedagogical knowledge and skills in teaching. Of course, this is looking at just one aspect and omits what has been described earlier. The issue of application still has a lot of pending questions because transfer of knowledge and skills is not automatic. In line with this uncertainty, let me extend the discussion on reasons for focusing on problem-solving. About two decades ago, Lave, Smith, and Butler (1989) made a strong statement, which still makes a lot of sense today. Problem-solving came to the surface as a reaction to past characterisation of mathematics, including MTE as a set of facts, and algorithmic procedures. In addition, and for the same purpose, allow me to add closely related conceptual foci like inspiration and investigation. In contrast to the new views, mathematics has been regarded as knowledge to be mastered by rote or mental exercise. Problem-solving as a philosophy operates with an open mind to the extent that there is a shift towards learners being more active, with problems less precisely and narrowly defined and problem-solving as coordination of several levels of activity at once.

I have discussed the various ways of thinking about MTE. The bottom lines of variations are a result of teacher educators' differences in focus on the same phenomenon - MTE. Despite the differences, mathematical investigation, inspiration, and problem-solving all carry elements of pedagogy as a way of ensuring student teachers master the necessary knowledge and teaching skills. The next category of description to be discussed involves viewing MTE as a focus on pedagogical knowledge and skills. This is an important link because it is likely to provide an opportunity to learn more about the inside of pedagogical knowledge and skills.

Categories D and F: focus on pedagogy and subject matter discussed together

In this section, both pedagogical and subject matter as ways of experiencing MTE will be discussed simultaneously, for important reasons. Though theoretically considered as separate domains, in practice they are inseparable. One may not be able to separate the two sides of the same coin or the head and the body and continue to leave an active life. It is a duality to be discussed at a later stage. Two guiding questions serve this purpose. First, what are the critical attributes of both pedagogical and subject matter knowledge? And second, where do the two meet? Pedagogical knowledge and skills were found to be one of teacher educators' strong ways of viewing MTE. Subject matter was found to be its natural match. It has been argued before that the basis of variations in looking at MTE lies in teacher educators' differences in focus on the critical aspects or attributes of MTE. What is considered a critical aspect of the phenomenon forms the foundation of the resulting conception and the basis of variation. For example, I am convinced that a sound mastery of mathematics content is a critical factor in terms of subject matter. In the same way, knowledge of instructional strategies is critical in making pedagogical decisions. Therefore, teacher educators are likely to reveal different conceptions of MTE depending on their focus.

Teacher educators identified themselves as exponents of pedagogy in the sense of presenting methods of teaching as the priority. Others supported the idea of focusing on subject matter in the course of mathematics teacher education programmes. The outstanding critical features to support their idea of focusing on pedagogy pointed to strategies of teaching and learning mathematics. Let us first take the pedagogical way of thinking about MTE as revealed by teacher educators. I would like to link this view with Kalder's (2007) discussion about mathematics teacher preparation programmes in the USA. The following quote is a relevant theoretical basis to qualify this variation in thinking about the place of methods of teaching and learning mathematics. It is stated:

> *"The majority of people who become mathematics teachers have a successful background in the subject but one which is a result of producing correct answers – application of correct algorithmic procedures. That is, they know the procedures but they do not know the concepts that underlie the procedures"*. (Kalder, 2007 p.146-149).

To some extent, this is like pedagogical breaking news. It is possible to draw out two issues in the quote provided above. The first, important in mathematics, is what I may call over-emphasis on the correct answer, with little attention paid to how one arrived at the solution. On this issue I have seen cases, especially during teaching practice of student teachers pressing hard on learners to strictly 'underline the answer'. Very often this is done at the expense of mathematical argument. The second issue is a pedagogical support of the correct answer. Kalder (2007) then continued to cite some examples, which have regularly generated interesting pedagogical discussion among teachers and researchers. Take, for example the division of fractions:

$3\frac{1}{4} \div \frac{1}{2}$, in which the teacher quickly jumps to $6\frac{1}{2}$.

The teacher would say the following are the fast rules to learn. First, keep the first fraction the same, flip the second fraction and use the reciprocal to multiply. Surprisingly, when student teachers were asked to create a story to illustrate the problem, what followed was disappointing. A possible story could be to think of three and a quarter kilograms of sugar to fill half kilogram containers, then proceed to find how many such half kilograms containers will be needed. Strong evidence of pedagogical gaps to support teacher educators' basis of emphasis is further brought to light in the next few paragraphs.

In a study involving a total of 397 pre-service teachers from James Cook and La Trobe Universities in Australia, Tobias and Itter (2007) conducted a competency test with the said sample of prospective teachers. The competency test comprised 40 short answers and multiple choice questions designed to assess understanding of procedures and concepts covering the primary and lower secondary mathematics curriculum. The test items were sourced from questions appropriate for students in year 8 because it was thought reasonable for pre-service teachers to have mastery of the mathematics content at that level. The items were previously identified by the researchers in the field as problematic. The collated test items focussed on place value, fractions, decimal understanding, along with process and problem-solving skills appropriate for lower secondary students - the actual level they were going to teach. Use of calculators was not allowed. A more descriptive data analysis was conducted. The results achieved by the entire sample of 397 pre-service teachers on the competency test ranged between 3 and 38 out of a possible maximum score of 40 points. For the purpose of this discussion, I will show a sample of detailed results showing particular gaps in understanding procedures. Table 16 indicates the responses to items probing the understanding of basic arithmetical procedures by pre-service teachers at the two universities (Tobias & Itter, 2007).

Table 16. Responses to understanding of arithmetical procedures by pre-service teachers at James Cook and La Trobe University

Type of operation	Item	Correct %	Incorrect %	No attempt %
Long division	$27\overline{)805}=?$	14.7	85.3	48.4
Decimal multiplication	$0.3\times0.3=?$	19.2	80.8	2.2
Division	$1\frac{3}{4}\div\frac{1}{2}=?$	17.5	82.5	30.7
Order of operation	$3^3+4(8-5)\div6=?$	17.2	82.8	18.7

The correct and incorrect figures in Table 16 refer to those who attempted the test. These examples were selected to test understanding of mathematical procedures and concepts. This is fine, but there could be some pedagogical issues as well. This is supported by the large numbers of prospective teachers who cannot perform basic arithmetical operations. The issue constraining understanding of division has been dealt with in the previous paragraph. What about multiplication of decimals, for example, 0.3 x 0.3? Is it the same as multiplication of counting numbers, where students may say it is the same as repeated addition? Perhaps not, so what about long division, for example 805 divided by 27? Is it possible to start with chunking to introduce the meaning? The same for the item demanding relevant instructional strategies about the order of operations, starting with brackets, of, division, multiplication, addition and finally addition. These are just a few examples where pedagogical knowledge and skills could play an important part, but are ignored for no reason.

I would say that cases like these are common in mathematics teacher education in Tanzania.

I now turn to subject matter knowledge as the natural partner of pedagogical knowledge and skills. The critical features or aspects focussed on by teacher educators are mastery of subject matter, structures, principles, and rules in mathematics teaching. The question about what is subject matter knowledge has been dealt with before. Subject matter knowledge, viewed from many sides, refers to an understanding of mathematical concepts, procedures, algorithms, principles, proofs, rules, and computational skills (Attorps, 2006; Tobias & Itter, 2007). Much has been written on the teacher knowledge base, but we all know that the development of knowledge in mathematics is a process of coming to terms with dilemmas in teaching and learning mathematics which are ever-emerging. A number of studies have been performed, some of which are regarded as cornerstones of both teachers´ subject matter and pedagogical knowledge. It must, however, be acknowledged that the teacher educator knowledge base and teacher knowledge base are two different domains, despite having teaching and learning mathematics as a common underlying factor. Their main difference rests on the fact that teacher educators focus on teaching about teaching (teaching teachers). Specifically, the teacher educator deals with students who are in the process of becoming mathematics teachers. The purpose of all this is to arrive at an understanding of the background of teacher educators and why they focus on subject matter knowledge. It is like searching for their prior knowledge, which indeed is the immediate influence of present concepts.

The discussion about teacher educators' emphasis on subject matter knowledge is interesting if research work elsewhere is considered. I have in mind several related findings (Attorps, 2006; Bass, 2005; Ernest, 1989; Kalder, 2007; Shulman, 1986). The effort to discuss a possible teacher educator knowledge base is motivated by the subject matter dimension of MTE. I will call this 'subject matter features' or 'aspects' as a starting point. Table 17 gives a summary of what some of the prominent models showing features or aspects of a sound teacher knowledge base. They will be used to suggest a similar knowledge base for mathematics teacher educators.

The duality of subject matter and pedagogical knowledge as categories of MTE

Table17 provides further evidence of the emphasis on subject matter knowledge with regard to teaching and learning mathematics. As indicated in Table17, subject matter and pedagogical knowledge form a common thread running through Shulman's (1986) work to more recent work by Kalder (2007). I call this the subject matter - pedagogical knowledge duality of thinking about MTE. The source of emphasis on both subject matter and pedagogical content knowledge in mathematics education is associated with learning experiences, teacher education programmes and teacher practice (Ernest, 1989). This is similar to the model of understanding MTE suggested in Chapter 2. In a similar way, teacher educators seem to have sourced their knowledge about MTE from their diverse history of learning experiences. Hence, subject matter knowledge and pedagogical knowledge and skills are considered as categories of descriptions with dual existence under discussion now.

Table 17. Features of teachers' knowledge base in teaching and learning mathematics

Shulman (1986)	**Ernest (1989)**	**Attorps (2006)**	**Kalder (2007)**
Mastery of subject matter. Pedagogical content knowledge. General pedagogical knowledge.	Knowledge of procedures. History of Maths. Links with other subject Knowledge about maths.	Knowledge of substantive structures. Knowledge of content, organisation of learning Knowledge about maths principles, canons, proofs, rules. Knowledge of student understanding, curricular	Deep understanding of the structures of maths. Working knowledge of how basic maths knowledge is developed. Mastery of subject matter.

It was stated earlier that being a teacher educator requires an understanding that goes beyond being just an effective subject matter teacher. What does this mean for a mathematics teacher educator, and what accounts for the difference from other teachers? First, and very relevant, is the need to be able to theorise practice in such a way that one is able to translate it into what to teach in MTE (subject matter issue), how to teach MTE (pedagogical issue), and why teach this way or that way (justification for activities of MTE). All these tasks may involve creating, researching, disseminating and using new knowledge. Figure15 is a proposed model to conceptualise not only the critical aspects or features as shown in Table17 but also teacher educator knowledge. This is an attempt to recognise the importance of both subject matter and pedagogical content knowledge without compromising either. The inclusion of elements of teacher educators to be initiators of research in their own subject areas has gained wide acceptance (Reis – Jorge, 2007) and is part of the requirements for a teacher educator to match the expanded role in MTE.

Secondly, there is a need for teacher educators to challenge stereotyped ideas, for example, teaching as telling or transmission of subject matter. Thirdly, MTE is an arena where important research findings on MKT, as well as subject matter are applied, and constantly questioned and tested. All these are expanded experiences of seeing MTE within the subject matter - pedagogy duality. I have brought in to the discussion the subject and pedagogical knowledge base in relation to teacher educators, for one main reason. Their conception of MTE as a process and a focus on subject matter and pedagogical knowledge and skills has roots, and the roots can be found in their subject and pedagogical knowledge base.

I would like now to bring in some ideas from Wu (2005), when he raised a very fundamental question - must content dictate pedagogy in mathematics education? This issue comes amidst some teacher educators' indication of their conception of to what extent pedagogical knowledge and skills count in MTE. Just as teacher educators have generated variations in thinking about MTE, and subject matter is one of the variations, so research work has produced the same result (Attorps, 2006; Bass, 2005; CBMS, 2001; Crawford, Gordon, Nicholas & Prosser, 1994; Ernest, 1989; Kalder, 2007; Lester & Lambdin, 1999; Shulman, 1986; Wu, 2005).

The views of the many researchers cited above recognise the importance of content in mathematics education, though they might differ in focus. Wu (2005), for example, holds a strong position that content has to move to the front and centre of mathematics education. Bass (2005) moderates it by being impressed how mathematicians have been drawn into mathematics education and conducted studies to enhance the domain as a field of research. The author attributes this to the so-called 'mathematics wars', a term used in the US to describe the conflict between mathematicians and educators, mainly over content, goals, and pedagogy of the curriculum. Bass (2005) continues to argue that, although these wars attracted a great deal of attention, the involvement of mathematicians has a much longer history in the teaching profession. Wu's (2005) question is very provocative, not only to advocates of pedagogical content knowledge, but also to concerned teachers, teacher educators, university lecturers, curriculum developers and others.

The main argument by Wu (2005) and CBMS (2001) is that sound pedagogical decisions can only be based on sound content knowledge. It is argued that experience seems to show that, as a result of the mathematics education mainstream in the past fifteen years or so, there is an distinctive trend, which may be called the mathematics-avoidance syndrome. Certainly, mathematics avoidance-syndrome may be taken on the same basis as low self-esteem, negative attitude or even harsher words like stigmatisation of the subject. Experience indicates that this situation is not uncommon in Tanzania.

What is the solution? A number of initiatives have been taken, for example, improvement of the curriculum, pedagogy and assessment. However, the above authors seem to agree that these have remained mere words. There is very often a difference between talking and action. The failure of mathematics education to place the importance of content to the front and centre is alleged to be a shortcoming. It is further argued that to bring mathematics to the forefront means dealing with content knowledge relevant to classroom teachers, i.e. the kind of mathematics teachers teach in the classroom. I would say this is the climax of the arguments of Wu (2005) and to some extent of Attorps (2006) as a way of addressing the profound question: must subject matter knowledge dictate pedagogy? It should be noted that though the discussion is based on mathematics education, this has strong implications for mathematics teacher education. One is reminded of their close relationship.

Furthermore, I have read Sahlberg and Berry (2003), Lester (1989) and Wu (2005) with a reasonable awareness of their account of how pedagogical approaches like investigation, problem-solving, inquiry-based, small group discussion and lesson study entered the classroom. In a way, they are a reaction to past experiences of focusing too much on mathematics as a body of knowledge, procedures, rules for memorisation and mental exercise. While some appear strong and sustainable, others appear to have missed the point, and a lot of resources have been spent in the name of mathematics education improvement. Wu (2005) cites the lesson study as an example where teachers are involved in activities of refining lesson plans at the expense of the mathematics they teach in the classroom. This is a concern for all those who value content being given priority.

This argument is very impressive in the sense that teachers sometimes lack what is really relevant in the classroom in terms of subject matter knowledge and not an overemphasis on pedagogical content knowledge. It is a luxury to spend time on pedagogical content knowledge instead of teachers' content knowledge by directly teaching them the mathematics they need in the classroom. The author makes out a case to demonstrate why content must dictate pedagogy. His identification of problem areas in teaching and learning mathematics is not confined to elementary mathematics teachers, but is across levels (Kalder, 2007). Regarding this point, I can see the concerns of emphasising content as a direction of reform. But the examples given to support 'content first' have a lot of pedagogical implications and are not unique in the case of Tanzania. Consider the following examples:

> Example 1: Students' ignorance to the point of not knowing that 0.09 is smaller than 0.2;

> Example 2: Failure to realise that $0.45 < 0.6$.

The two examples perhaps indicate the challenges in teaching concepts like 'place value' by using the language of tenths, hundredths, thousandths and more. Advocates of 'content first' may fall short in recognising that 'place value' is a concept. Another way of viewing 'content first' is to say sound subject matter knowledge serves as the basis of critical pedagogical decisions. I have also worked with teacher educators on misconceptions in multiplication, addition, and division of fractions. The following examples provide further reflection:

> Example 3: Addition and multiplication make numbers bigger, while subtraction and division make numbers smaller.

> Example 4: Division of fractions for example $6^1/_2 \div ^1/_4$ and how to arrive at the solution of 26.

In example 3, and depending on the grades learners are in, student teachers may try $(-3) + (-3) = -6$, and $(-3) + (-4) = -7$ to check the misconception that addition makes numbers bigger. For multiplication, which is also generalised as repeated addition, students would try $2 \times 3 = 2 + 2 + 2$ or $3 \times 2 = 3 + 3$. What about 0.75×0.75? The story is different, as repeated addition no longer applies as learners would normally expect. In example 4, mathematics teachers sometimes quickly run into the famous statement many of us passed through*: you maintain the first fraction the same, then flip or apply reciprocal multiplication to obtain the answer.*

It seems that the real issue in the examples given above is not the content part, but how to create stories for learners which might help to drive the point home. The argument put forward, especially by Wu (2005), and also the CBMS (2001), on content to be brought to the forefront is an idea to be welcomed. One serious comment is the oversimplification that pedagogy and content are separable. In reality, where do you locate the demarcation line between the two? As Wu (2005) finally agrees, separation does not exist. What happens in practice is usually a focus on one or the other, just as teacher educators in a variety of ways think about MTE. This is a recurring dilemma - a dual experience of MTE by mathematics teacher educators.

The essential aspects of subject matter, pedagogical knowledge and skills expressed by educators have been highlighted. The term ‘subject matter’ according to Attorps (2006) refers to the amount of mathematics knowledge a teacher has in mind and the manner in which this knowledge is organised. This also covers knowledge of key concepts or definitions and algorithms, principles, rules, procedures, proofs, and of domains and the relationship between them. Even more important is the understanding of substantive structures of the subject. Pedagogical subject matter knowledge, on the other hand, refers to knowledge of student understanding, curricula, instructional strategies, contexts, and organisation of learning (Attorps, 2006). In Figure 15 I attempt to consolidate and synchronise the discussion about the teacher educator knowledge base in terms of subject and pedagogical knowledge and skills.

Guided by such thinking, the entire sum of knowledge and skills (K & SK) possessed by the mathematics teacher educator is the sum of the parts of subject matter knowledge (SM), professional development and research (PD-R) capacity, teacher education and conceptions of teaching and learning (Co-t/l). Here it should be noted that I am not trying to create a mathematical equation, but rather pointing out that subject matter and pedagogical knowledge and skills are eclectic and possessed by the same individual - the mathematics teacher educator. I am of the view that this is the meeting point, and is done through teacher education in the broad sense.

One can make reference to the educational studies and subject specialisation that mathematics student teachers would normally undergo. The assumption is that since they are possessed by the same individual, theoretically one expects them to meet at a certain point. In practice, this has been more difficult than ever (Kansanen & Meri 2009). Beane, cited in Kansanen and Meri (1995), presents some arguments against what others may term protecting factors for the continued separate approach to academic and professional studies. In the first place, there is a network of academic elites with symbolic relationships, which includes academics and teacher educators, test and text publishers, and subject area associations, whose identity and advantages are behind some subjects.

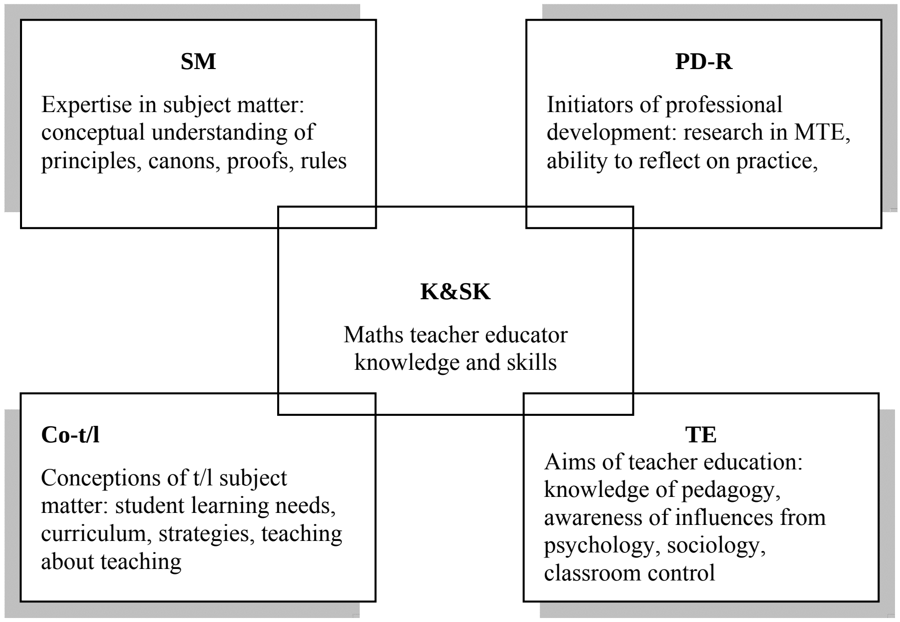

Figure 15. Model to conceptualise mathematics teacher educator knowledge and skills

Secondly, there is a strong tradition among parents, and any effort to integrate appears to be a great risk, sometimes backed by conservatism. Important also is that teachers would normally identify themselves with the subjects they have studied and are teaching. In my view, to achieve a solution for this needs a complete vision of teacher education as whole rather than a piecemeal one.

To conclude the discussion about pedagogical and subject matter knowledge as two different categories of description in thinking about MTE, it is worth mentioning the level of conception. Teacher educators' conceptions of MTE, first as a process and second as a focus on pedagogical and subject matter knowledge, were somewhat wide and lacked depth. This is because pedagogy was conceived as a method of teaching, while, to a large extent, it goes beyond teaching only. It includes also teacher–student–curriculum material relationships. The same is true for subject matter knowledge, where a deep understanding of concepts or structural meaning is essential and not procedural knowledge alone. I now turn to subject didactics, a notion which I seriously considered for the first time in the course of this study. I say the first time, meaning substantial purposeful reflection on subject didactics. In other instances, it has been mentioned only in passing.

Category E: MTE as subject didactics

My first serious encounter with the term 'didactics' was during the interviews connected to this study. Before that I had experienced the use of the word according to the borrowed British tradition, which simply meant to 'instruct' or 'methods of teaching'. This new experience threw me into a challenging situation. During the interview there appeared aspects which defined didactics as teaching, studying about teaching, learning, what to teach (content selection), how and why, very often focusing on mathematics teaching methods. This is what makes didactics vary from other conceptions of MTE. As the rules of the qualitative research approach require, I took it without further reduction. I reserved it as a task in order to find out its meaning in connection with other studies.

As one reads different literature regarding didactics, the immediate impression is a link with Germany's system of education (Hudson, 1999; Kansanen, 1999; Niss, 1998; Seel, 1999; Westbury, 2000). Attempts to find the roots of the notion 'didactics' was challenging, but I did not leave it there. Seel (1999), for example, associates the German word 'Bildung' with 'didactics'. To me this is questionable, for it is only a translation implying 'formation' or 'erudition', which means an initiation or the process of learning or developing into a certain culture. Extending this argument with a cautionary interpretation, didactics in its simplest form would imply the process of adapting to a culture - the culture of a teacher or teaching and learning. The Concise Oxford Dictionary (Fowler, & Fowler, 1995) further describes 'erudition' as 'instruction', 'training'. The culture of 'instruction' is again very similar to teaching and learning, and differs only in terms of power in the classroom. Seen more precisely, didactics is regarded as a process of learning in order to qualify to be a member of a certain group. To be a mathematics teacher, you have to undergo a didactical process to gain that culture of mathematics teachers, and so also with teacher education. At this point, I would say that when teacher educators expressed MTE as subject didactics to mean teaching mathematics, I find no serious contradiction in using the German view only as departure point. In addition, Kansanen (1999) views didactics as an applied translation from Greek, which means both teaching and learning or the art of teaching. The current meaning is therefore rather broad to encompass research on teaching and learning.

A further review of literature revealed that didactics has two components. The first component deals with solutions and provision of cultural components in terms of goals and content of learning, and the second system supports the learning process. In my view, the second component makes the role of mathematics teachers educators important. They also have to undergo the same didactical process in order to be members of a certain culture of teachers – that of mathematics educators. Since didactics is a very wide term, I have reason to switch to a more specific kind of didactics which fits well with educators' understanding of MTE as subject didactics. Again, Seel (1999), Hudson (1999) and Westbury (2000) all argue for subject didactics (not meaning general didactics). This gives the impression that subject didactics refers to knowledge for teaching a specific subject, for example mathematics.

This view may be extended to cover general didactics as a science with focus on planned support of learning to acquire 'formation' or 'reduction'. This may allow special didactics to be seen in terms of challenges of teaching in different types of school: for example, schools dealing with children with learning difficulties. These may need their own special didactics and also their own teachers. The same goes for a particular age group, and their teachers have to undergo their own didactical process. More important also is a consideration of specific domains of content, for example MTE, which is our main concern. It should also be noted that didactics as a theory of teaching in schools has to deal with problems of content and procedures of teaching in the classroom. Interpretation of Seel (1999), Hudson (1999), as well as the work of Hopman and Requarts (2007) on didactics may be summarised into three basic questions. What content or learning areas should be legitimised? How can this be done? (strategies of teaching). And why are selected areas or options in MTE taken as the strategies for teaching?

My view on the selection of content and the strategies for teaching is that what is selected has to support the learning processes of students, which have to be based on what Seel (1999) asserts as active learning. In this way, the students' learning process may be conceived as a problem-solving process, which has to be evoked and guided. (See previous treatment of problem-solving). It has been brought to our attention in the preceding paragraph that one of the major types of specialisation of didactics is subject didactics or selected content and related strategies. In a strict sense, this is what teacher educators brought up and hence the reason for this discussion.

What does subject content mean in the eyes of subject didacticians? Borrowing from Hudson (1999), subject content may be seen as selected knowledge and skills. If this is interpreted in MTE, it refers to fields of concentration on methodological work. According to Hudson (1999), subject didactics, like the didactics of mathematics education, represents a scientific discipline, which involves what he calls tasks. The tasks form a long list, but in the interests of better elaboration, I will try to categorise them in accordance with what is sometimes viewed as the fundamental questions which guide didactics. The questions are what content to select and why? How to support the learning process and why? I am of the view that the last question refers to why the focus is on certain critical approaches to learning. With the help of the different research work cited in the last two paragraphs, Table18 is my personal interpretation of the whole idea of subject didactics in relation to the phenomenon of interest - MTE. I do not intend to make any oversimplification either, but rather it may be a better basis to express the term didactics in relation to subject content.

At this point, I would like to summarise the notion of didactics in relation to the category of description under discussion. Teacher educators conceptualised MTE as the didactics of mathematics or the teaching of mathematics. On the basis of what has been discussed so far, didactics is beyond the single task of teaching. It includes the selection of content in mathematics education, the reasons behind it, reflection on the teaching process and even more important, research into the contexts of teaching and learning.

Furthermore, when 'subject didactics' is compared to the American 'curriculum and instruction' tradition, and using the role of the teacher as a criterion to differentiate them, the two traditions differ significantly. Overall, there is more freedom of action and autonomy for the subject teacher under a didactical system of teacher education than the American 'curriculum and instruction' tradition (Westbury, 2000). It is further argued by Westbury (2000) that the didactical system emphasises teacher self-assessment of the lesson and making decisions accordingly.

I would also add, that this is reflected by the didactical fundamental principles of what content to teach and how or what strategies to use, supported by reasons in each of the steps. This is the basis of the autonomy being claimed. However, it is also important to note that Kansanen and Meri (2009) remind us that Shulman (1987) introduced the term pedagogical content knowledge, and it has been noted to resemble subject didactics.

Table 18. Features of MTE didactical tasks

Didactical tasks under what	Didactical tasks under how	Didactical tasks under why
1. Selection of MTE relevant content and its structuring.	1. Development of MTE specific procedures for teaching and learning. 2. Evaluation and securing of results of MTE learning in the context of results of didactics. 3. Assurance of the standard of MTE academic quality assessment.	1. Constitution and legitimisation of MTE as a contribution to achieving the general goal of teacher education

Category G: MTE as integration of content and pedagogical knowledge

Some teacher educators conceived mathematics teacher education in a more comprehensive or holistic way that is not only a collection of parts but rather a consideration of an organised combination of content and pedagogical knowledge to form MTE. They regarded it as a skilful integration between pedagogy and content. The immediate question is what is the proportion of subject matter to pedagogical knowledge? Generally, knowledge of subject matter includes conceptual knowledge and procedures, which also take on board skills, principles, proofs, rules, and perhaps more. Pedagogical knowledge, on the other hand, aims at knowledge of what content to teach, how to teach and why. This extends to knowledge of learners, curriculum, methods of teaching and learning, the context, as well as organisation of the learning situation. The question of what proportion or what areas to concentrate on is complex and implies difficult pedagogical decisions. Despite the challenges, the intention is just to set the scene on the integration or skilful blending of content and pedagogical knowledge.

Integration of content and pedagogical knowledge is not a strange idea. Van Dijk and Kattmann (2007) and Hudson, Burberger and Kansanen and Seel (1999) had a novel idea on how to go about it. In their argument, they value the influence of subject didactics on the subject discipline providing academic knowledge relevant to the teaching of the particular subject. Seel points out emphatically that:

“The influence seems to be necessary so that the needs and expectations of initial teacher education (e.g. Selection of topics and methods) can be met adequately by the programmes of the academic disciplines. If the other academic disciplines are not in a position to provide adequate programmes, they have to be developed by ‘Fachdidaktik’ (subject didactics) additionally and independently”. (Seel, 1999 p.85)

This quotation makes the point that one cannot exist without the other and neither dictates to the other. This is to assert that if pedagogical difficulties are not addressed in learning mathematics, then MTE is meaningless. I would still say the reverse is also true, and tackling the question remains a hard and complex task.

Unlike Wu (2005), who strongly advocated for content to dictate pedagogy, Bass (2005) strongly supports the question of enhancing the subject matter and pedagogy relationship. He is for the integration of mathematical ideas rather than a collection of parts. The critical issue is what teachers need to know. I would say he is referring to content, but in what ways? Perhaps an emphasis on the strategies of knowledge gained, but for what purpose? This is to insist on being clear about what knowledge is to be gained. All is in one package and not in isolated parts.

The discussion on the possibility of merging subject matter and pedagogical knowledge seems to provide a solution to the longstanding debate between subject content and pedagogy. This is exemplified by the longstanding history of mathematicians who have been serving in the mathematics education field (Bass, 2005). The results of this study indicate a sharp divide between pedagogy and a blended approach to subject matter and pedagogy. That is, the major clusters of teacher educators accept either pedagogy or the integration of subject matter methods of teaching mathematics. This approach as stated in the beginning may raise another question based on relative weight. It was the issue of relative weight which made it important to examine previous practice in MTE in the case of Tanzania. I also think that the variations (proportions) of subject matter to pedagogy are likely to be context-bound and not an international prescription.

At this stage, it is relevant to bring forward a few ideas about subject matter and MKT as part of the conclusion. First, Bass (2005), and Lester and Lambdin **(**1999) proposed a practice-based subject matter and pedagogy approach for better results. This is important in order to bridge the gap between theoretical mathematics and MKT at school level. Within limits, I also share the same view expressed in Charles’s (1989) discussion on teacher knowledge base and its relation to teaching problem-solving. In his remarkable observations, I came to the conclusion that knowing a great deal about the content to be taught seemed necessary but not sufficient to effectively direct a practice session with the entire class. Second, knowledge needed for teaching mathematics is different from that needed for other professions where mathematics is used. Third, mathematics through mathematicians has a long and honourable tradition of involvement in mathematics education. I would also add that the reverse seems logical. Fourth, eminent mathematicians around the world, for example Klein and Freudenthal

(1973), have exemplified the dual existence of content and pedagogy. Fifth, practitioners of mathematics and research mathematicians can bring valuable knowledge, perspectives, and resources to the work of MTE which has been in the tradition of continued development and support. The last point reflects part of the results of this study in research question two.

6.3 Various ways of thinking about MTE development

The first part of research question two requested teacher educators to describe activities related to the development of MTE that they had implemented well as individual or with colleagues. The revealed thoughts indicated theoretical and practical development options in MTE. The second part of the research question requested them to describe strategic ideas on how development ideas in MTE could be shared. In a strict sense, it was searching for thoughts on how they share knowledge and skills. The order of the discussion is based on the results as presented earlier in this chapter. I will discuss the results in turn unless explained otherwise.

Category A and B: focus on pedagogy and content knowledge discussed together

Teacher educators' descriptions of MTE development activities revealed thoughts with a focus on pedagogical knowledge and skills. Critical aspects which distinguish this category from other categories of descriptions are classroom interactions, preparation and use of teaching and learning materials, team teaching, pedagogical reflection, activity-based teaching and learning, and application of ICT in learning mathematics. Aspects which reflect a focus on content emphasise the mastery of mathematics concepts, procedures, problem-solving, and connection of ideas between topics and disciplines.

The two notions, pedagogical knowledge and skills, as well as subject matter have been discussed before when dealing with the results of the first research question. The emergence of the two notions seems to confirm how critical they are in the discussion of MTE development. I suggest keeping reference to what has been discussed in research question one in connection with pedagogical and subject matter knowledge and skills (see discussion under category D and F in this chapter for details). This is necessary in order to avoid losing track of these mainstream ideas.

It is important to note the connection between teacher educators' conceptions of MTE and what they consider as development of the same. From the very beginning, this study intended to go beyond the study of conceptions in order to add dynamics to the results. This could be achieved by a consideration of a study involving thoughts for development of MTE. Naturally this is forward looking. For this reason, the connection between teacher educators' conceptions and their thoughts on what they would like to be done as a way forward was investigated. Interestingly, the results seem to tell us that there is a connection between the meaning they assign (research question one) to MTE and thoughts on areas of focus in the course of MTE development (research question two). This has been indicated by the categories of description of pedagogical as well as subject matter knowledge and skills. They are given recognition as areas of focus in the question of MTE enhancement. This suggests that teacher educators would like to see their conceptions put into practice. That is to say individual classroom actions likely reflect their understanding of MTE. Unlike in research question one, subject matter and pedagogical knowledge are thought of in unison in research question two, and with regard to thoughts on the way forward they are thought of as two fields apart in terms of emphasis.

The implication for both subject matter and pedagogical knowledge and skills is for teacher education to consider their relative emphasis, as the interplay between them in the classroom is what makes learning happen. As introduced earlier, subject content and pedagogical knowledge have previously been discussed in detail under relative categories of description (D and F). This covered the meanings assigned to the terms, their constitution, their purpose, as well as the links between them. That being said, let me now take the rest of the categories of descriptions, and discuss them one at a time, starting with MTE and the emphasis of enhancing the relationship between teaching, learning and assessment.

Category C: process of integrating the results of teaching, learning and assessment

Within this category of description, teacher educators shared the common goal of enhancing students' learning, and how to teach and assess student learning. Though there were differences in how this could be done, great consideration was given to the relationship between teaching, learning and assessment. This view is in line with recent and past research work with a focus on aims, modes or types of assessment (Crokett, 2007; Howson, 1993; Koca, Jin-Hea, 1998; Niss, 1993 & Williams, 1998). It may be of interest to note that teacher educators held the view that assessment is an integral part of MTE. This was a surprise, because my personal experience in working with teacher educators suggests assessment would normally be taken for granted. More striking is that portfolio assessment, for example, though known about for some time by teacher educators, was nevertheless difficult to introduce. The resistance by teacher educators to introduce portfolio assessment in their respective teacher colleges is an indication of the challenge. The resistance to use portfolio assessment is not unique to Tanzania. Koretz, Stecher, Klein and McCaffrey (1994), writing from a USA perspective, raise issues of time intensiveness, questionable standards and being subjective in grading, among other criticisms.

This study revealed several aspects within the notion 'assessment'. The critical aspects which make this category of description distinct are use of portfolio assessment and continuous assessment. One should not reach any clear-cut conclusions on portfolio assessment as claimed by the study results. Portfolio assessment is very often labelled as evidence of learning on the grounds that it shows cases of students' best work as chosen by both the teacher and student, and it represents student interest and growth. Even more important, it is taken as evidence of self-assessment based on a technically sound sample of work (Wiggins, 1998). Despite the positive sides of portfolio assessment, research results indicate also a downside to portfolio assessment, for example, resistance to using it. There are also questions in connection with reliability, the assessment criteria used, as well as problems of standards. Angelo and Cross (1993), for example, raise questions over the academic seriousness of portfolio assessment, especially if the approach is not carefully integrated into the mathematics teacher education course. I was impressed by the level of teacher educators' understanding of this mode of assessment and how it can be used to enhance the present system, which, to a large extent, rests on written tests and examinations. In the real sense of assessment, examinations and tests may be regarded as tools rather than assessment per se. That said, I want now to discuss assessment in MTE from at least three viewpoints, while at the same time keeping an eye on the study results.

First, statements from the study results fall short on the rationale for assessment in MTE despite the attempt to relate it with teaching and learning. I regard this as a missing point. For someone engaged in a task involving the search for the meaning assigned to MTE by teacher educators, the issue of assessment may be seen as a distant objective in the first place. This is what happened from the initial stage save for some signs of it during the shaping and creating of this study: specifically, the issue as to why assessment in MTE first appeared in the discussion about the mismatch of what is examined and the MTE curriculum intentions in Tanzania. Instead of a focus or at least a balance between pedagogical knowledge and skills and subject matter in MTE examinations, there was a leaning towards the latter. I am of the view that this kind of assessment has an implication for the meaning assigned to MTE. Second, the issue of assessment is discussed in relation to this study about MTE conceptions because of the close relationship between what teacher educators teach, learn and what is assessed. The claim that teacher educators teach and learn what is more likely to be assessed, especially in systems involving external examination boards, is not new. At the same time, there is the experience that student teachers are more likely to concentrate on what stands a good chance of being assessed (Cooney, Brown, Dossey, Schrage & Wittman, 1996). This, too, is likely to influence what teacher educators designate as the meaning and purpose of MTE, and thoughts on its development. This has been the basis of including and relating assessment to the topic of study.

I would like to start by recalling William's (1998) argument that, for quite a long time, the issue of assessment and the reasons behind it did not exercise the minds of educational philosophers for reasons not very apparent. Gradually, it has now become an area of interest after the introduction of competency-based education. The notion of a 'competency-based' teacher education curriculum might mean different things to different teacher educators, as also in MTE. William (1998) discusses assessment in education because there are questions of accuracy, reliability, fairness and ethics. On the same grounds, a recent report by the TIE in Tanzania voices educators' views on the assessment of the two-tier diploma course (TIE, 2008). The report calls for a review of the rationale and purpose of the assessment system in teacher education if the objectives are to be attained (this includes mathematics teacher education). On the rationale for assessment, Niss (1993) challenges contemporary roles, functions and modes because they are not well understood. To emphasise this, Niss (1993), Howson (1993) and Koca, Asli-Lee and Hea-Jin (1998) all reported a compelling case for why assessment should be done in a competent way. Niss's case is more striking and there is a point in seeing it again:

> *"The current assessment modes and practices involve conflicting interests, divergent aims, and unintended or undesired side-effects. In particular it is difficult to devise assessment modes which at the same time (a) allow us to, in a valid and reliable way, the knowledge, insights, abilities, and skills related to understanding and mastering of mathematics in its essential aspects (b) provide genuine assistance to the individual learner in monitoring and improving his or her acquisition of mathematical insight and power (c) help the individual teacher in monitoring and improving his or her teaching, guidance supervision, counselling (d) assist curriculum planners, textbook authors, in-service teacher trainers in adequately shaping the framework for mathematics instruction".* (Niss, 1993 p.1-30)

Again, this statement is not only a good reminder of the rationale for a better assessment system, but also the implied meaning of mathematics knowledge for teaching (see, for example, parts (c) and (d) of the quote). For example, the failure to assess what is expected, inability to provide systematic feedback for both the teacher educator and the student, and failure to support curriculum developers and textbook authors in a meaningful way all support the need for a revitalised assessment system.

Second, I would also like to discuss the purpose of assessment, which, to a certain extent, is revealed in the teacher educators' statements. The purpose of assessment in MTE is multidimensional, though sometimes viewed with a questioning mind, extending William's (1998) discussion that assessment by its very nature appears negative to some people because it is an encroachment and not exciting free activity. Despite the challenges, the author concedes that one may raise questions about the manner of assessment, but not the nature of the teaching and learning associated with it. It is possible to see here how strongly assessment is associated with actions of teaching and learning, and possibly the way teacher educators may understand, for example, MTE. On this basis, teachers, as well as students, would want to know how well they have been doing. It is known that the ambitions of human beings tend to exceed their ability and therefore assessment is a tool which provides us with the possibility to know the relationship between ability and ambitions as well as between one individual and another (William, 1998). What does this mean for teacher education? Under normal circumstances, students' ambitions are high, but their ability might not be as high. In a general way, assessment is a tool to provide a picture of success or failure of the relationship between ambition and ability.

As one reads different literature, for example Niss (1993), William (1998), Howson (1993), Crockett (2007) and Koca, Asli-Lee, & Hea-Jin (1998), the purpose of assessment is central to the whole discussion. There is therefore a focus on assessment for formal and informal reporting for those in charge, for the purpose of pay, obtaining a position, selection for further schooling and a licence, for example, to teach in some countries. It is possible to conclude from the purpose of assessment that it exists, is likely to stay, and serves a variety of purposes not necessarily compatible and sometimes connected with controversy.

The main functions of assessment are also multidimensional and include provision of information for establishing the basis for information and decision-making, and more important for shaping social reality (Niss, 1993). If the functions of assessment are multidimensional, the chances are that it will carry different meanings and hence add to its complexity.

Third, and as a conclusion, it is important also to say a few words about assessment modes. To highlight some of the common modes of assessment, Koca & Hea-Jin (1998) made a case for supporting open-ended questions, reports of group projects, book reviews, mathematical research problems posed by students, and work on complex problems among others. I have no serious objection to this, but only a consideration of the context of assessment. As it appears here, the modes of assessment suggested demand high level reflection and give a possibility of probing deeply into the meaning of the subject being assessed, whether in MTE or another field of interest.

Apart from the modes of assessment, it is important to be aware of the different roles and objects of assessment. We may be aware that any assessment activity needs to specify who is to be assessed, what is the focus and items, and what are the procedures and circumstances of assessment before making any judgement on the results of assessment. It is also important to know well in advance about the style of reporting for public knowledge. For example, will it be some kind of league table? In Tanzania, for example, up to the time of this study, there has been comparison of the performance of regions and districts, with students completing the various levels of education. What has not been clear to me is the comparison of schools, districts, and regions of very diverse economic and social differences. I will take time to understand it. What are the effects? What about ethical issues in releasing the assessment results? I am of the view that solutions regarding these questions need to be reflected on well in advance.

While many of the researchers cited before discuss assessment in terms of rationale, aim, and modes of assessment, the issue of challenges inherent in portfolio assessment, for example, has not been dealt with adequately. It has been shown briefly how assessment modes can influence the level of understanding of the subject on which assessment is being carried out. It is the same for the aims and depends on whether the demand for assessment is only comprehension, knowledge application or analysis. As stated before, this is another way of relating assessment and the topic of this study. It is important now to bring forward a few cases of challenges in assessment. Angelo and Cross (1993), for example, revealed a number of concerns about portfolio assessment. First, teachers who have been using this system mentioned that portfolio assessment imposes substantial burdens on them, such as the demands on time to plan and administer it. Second, there is an issue of accurate recording, scoring and comparability of students' performance. It also seems that portfolio assessment is more subjective than traditional testing, and at the same time reliability and validity become more questionable. Furthermore, maintaining portfolios can be problematic and time-consuming. In addition, it is further complicated because decisions on the content of a portfolio are hard to reach because of the lack of guidelines that exist in a traditional system. But all in all, and like other modes of assessment, portfolio assessment provides systematic feedback to both the learner and to the mathematics teacher educator.

Much has been said about assessment at the global level. I have an obligation to contextualise this issue and reflect on what is happening on the ground regarding assessment in mathematics teacher education in Tanzania. During the build-up to this study I had an opportunity to conduct a focussed discussion with teacher educators on this subject. Our discussion reflected much more on the format used by the NECTA to prepare mathematics teacher examinations. This format is based on the famous Bloom (1954) domains of learning. Bloom's taxonomy considers assessment of knowledge, understanding, application, analysis, synthesis, and finally evaluation. Despite the potential criticisms from many sides, let us see the distribution or coverage by weight of the examination items. I am aware that examinations and tests are only tools in the broad sense of assessment. Table 19 and Table 20 represent a distribution of MTE examination items by using the cognitive level criteria (NECTA, 2002). It is a result of the analysis of the examination items to see where each would fit in the much preferred cognitive levels criteria. These quantitative data are not intended for drawing any conclusions, but rather to enrich the discussion about the relationship between teaching, learning, and assessment.

GATCE stands for 'Grade A teacher certificate examination'. Essentially, it is a summative assessment tool for student teachers who have completed a two-year teacher education course. On completion, they are expected to teach in elementary schools. The entry qualification is a successful completion of 'Ordinary Level' of secondary education. In the same vein, DEE refers to 'Diploma in education examinations'. It is a summative assessment tool for student teachers who have completed a two-year teacher education course. They are expected to teach in lower secondary schools. The minimum entry qualification is successful completion of 'Advanced Level' of secondary education. It is important to note that assessment influences how student teachers learn and how teacher educators teach. Both the mathematics teacher educator and the student teacher find themselves in the same boat, wanting to focus on what is required in the examination and therefore 'what is required to know'. I have a few concerns about this premise because it is central to the whole issue of enhancing the relationship between teaching, learning and assessment.

Regarding the first concern, Table 19 and Table 20 seem to indicate that there is an emphasis on MTE assessment of knowledge, understanding and application for both courses. This may fit well with large-scale summative assessment, but is lacking in real tasks which actually model a mathematics teacher.

One could expect greater weight to be given to college-based formative assessment, but this is far from realisation (TIE, 2008). One possible explanation is the lack of a professional mindset in that the assessment of teacher education is done by the same institution assessing school candidates.

The second concern is that, if assessment of MTE is a tool to determine the extent to which students have practically mastered MKT and skills, it is difficult to tell from Table 19, for example, how this is achieved.

Table 19. Distribution of MTE examination items based on the cognitive levels criteria for Grade A teacher education certificate course

Year	LEVELS					
	knowledge	comprehension	application	analysis	synthesis	evaluation
2002	4(15)	5(15)	2(15)	3(15)	1(15)	
2003	4(15)	4(15)	6(15)	1(15)		
2004	5(15)	3(15)	6(15)	1(15)		
2005	6(15)	3(15)	1(15)	4(15)	1(15)	
2006	5(15)	3(15)	5(15)	2(15)		

Table 20. Distribution of MTE examination items based on the cognitive levels criteria for Diploma in teacher education course

Year	LEVELS					
	knowledge	comprehension	application	analysis	synthesis	evaluation
2002	4(15)	4(15)	3(15)	4(15)	None	None
2003	1(15)	3(15)	4(15)	5(15)	1(15)	1(15)
2004	5(15)	3(15)	4(15)	3(15)		None
2005	5(15)	1(15)	3(15)	2(15)	4(15)	
2006	5(15)	2(15)	3(15)	2(15)	3(15)	

Some would argue that student teachers have a few weeks of teaching practice before their exit into their teaching career. One important question is how can we then use the planned teaching practice, independent studies, and mathematical research problems posed by students to revitalise mathematics teaching? The cognitive levels assessed as shown in Table 19 and Table 20 bring us to the same problem of educational studies and subject didactics failing to integrate in a productive way. This is a serious concern.

In the previous paragraph, I tried to point out some of the ideas worth spending time on and be given a chance in Tanzania. I have a few sample questions which are common in the examination rooms. During May, 2004 diploma student teachers were asked a range of questions, a few of which are of interest in this discussion. The questions were (1) List four characteristics of a good mathematics teacher, and (2) Every mathematics teacher at whatever level is called upon to test his/her pupils. What is the significance of testing pupils (NECTA, 2004)? These are typical examination questions. It may be easy to see that both questions test knowledge, and at most they involve listing the characteristics of a good teacher and the important points on testing. It is necessary to caution on the difference between assessment and testing, as the two do not mean the same thing.

The issue of assessment has been well investigated here; in particular, its relationship and role in support of teaching and learning. All in all, it is still possible to maintain that the benefits outweigh the challenges, given the rationale for assessment, and the purpose, modes and different roles of those involved in assessment. It is therefore of prime importance to address these challenges, and still use assessment in MTE as a way to enhance the relationship between teaching and learning mathematics.

Category D: focus on building relationships

The starting point for the discussion about building relationships may appropriately be positioned in the teaching and learning process. In view of teacher educators' expressions, it involves the interplay of three main parts: the MTE curriculum, the student teacher, and above all the teacher educator. In this case, the MTE curriculum includes a wide range of teaching and learning materials, from mathematics syllabus to textbooks and non-text materials. According to the study results, there is a significant focus on building relationships in the course of teaching and learning. The reasons behind this are not clear, but I may associate it with teacher educators' attempts to address the problems of mathematics stigmatisation, avoidance, low self-esteem and negative attitudes. The issue of building relationships between teacher educators and students has been viewed as an important category of thought in the development of MTE. Aspects which have been associated with building relationship are motivation, inspiration, and attraction to learn mathematics, and so on. With reference to the research results, teacher educators thought that development of MTE involves building relationships in the process of teaching and learning. This process involves the interplay of key players and the materials used. They have in mind student teachers, teacher educators, and learning materials in MTE.

In Figure 16, T-S represents both pedagogical interaction (more on organisational factors) between the teacher educator and student, S-L stands for student teacher interactions with learning materials in the form of texts and non-texts, and T-L represents teacher educator interactions with curriculum teaching and learning materials which match the identified teaching strategies and available materials. What is referred to as mathematics learning materials represents an organised mathematics education curriculum. In order to have a productive teacher education programme, teacher educators' work first needs to reflect the relevant curriculum materials, with the educator being part of the curriculum in the sense of a facilitator. This is the relationship T-L brought forward by the teacher educators. A similar argument holds for the interaction between student teachers and learning material (S-L). The interactions S-L is didactical under the guidance of the teacher educator, and expected to cater for student needs, stimulate thinking, and motivate and attract them in the field of MTE. Uljens (1997) constructed this relationship by saying that the activities of a teacher in this case is teaching, and that of a student teacher with learning material is studying which is what is visible, and a prerequisite of learning (which is latent). What the teacher can do is not to control for what is theoretically possible but rather only guide (Kansanen & Meri, 2009).

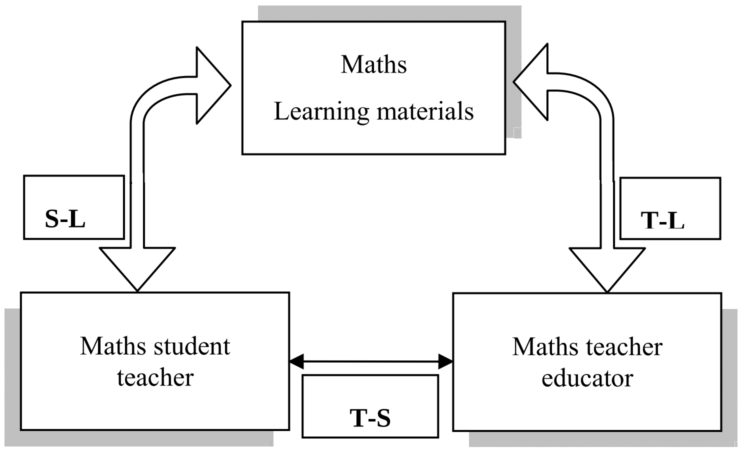

Figure 16. Educator-student relationships and interactions

Building relationships in the course of teaching and learning mathematics is important in many ways. Teacher educators' statements appeared to be against unacceptable behaviours among them when teaching mathematics. They also showed the use of harsh language as having a negative effect on MTE. The notion 'building relationships' is difficult to explain without taking into consideration of the critical aspects which illuminates the term for example: motivation; inspiration and in all, a positive learning environment between the teacher educator and student teachers. Literature search reveals interesting evidence on the role of building relationship in teaching and learning mathematics. A few examples are cited below in order to only illuminate teacher educators' thoughts of building relationship. Wu (2005), for example, pointed out the mathematics–avoidance syndrome in relation to the system's (teachers, teacher educators, learners) failure to enhance teacher-student relationships in teaching and learning mathematics. In another setting, Cirrilo (2007) cited a case about the failure to humanise calculus in which students are suddenly thrown into using the power rule to find, for example, derivatives without basic knowledge about it. Dalgarno and Colgan (2007) revealed a case indicating how mathematics is threatening to most teachers with limited experience. Chaachou and Saglam (2006) report on the characteristic relationship between mathematics and physics, which can be traced from a philosophical and epistemological point of view. This range of lived experiences is evidence of an important message concerning the building of relationships between learners, educators, and their interactions with relevant MTE materials.

Let us return to Figure16 for a moment. The figure indicates that learning revolves around interactions between the student teacher, the teacher educator and the learning material interactions. Let us go one step ahead and assume that there is interaction or communication in teaching and learning between the three participants in a given lesson. The interaction is indicated as educator to student (T-S), educator to learning material (T-L) and student to learning material (S-L).

I am aware of the claim that the S-L relationship does not exist because of what are called preconceptions. Hudson, Ongstad, Braathe, Hans-J and Pepin (1999), however, used the three-corner model. Assuming that the pedagogy has been productive, learning occurs or there is a change in conception. They argue that spoken words are the content or subject matter which reaches students through interaction or communication. They originate from the mathematicians of the past, textbook writers, and from people entrusted with managing education such as ministries, school authorities, teachers and a variety of facilitators. Spoken words carry with them the context of teaching and learning which is also loaded with values of relationships, each with its own weight of influence. In this case, transmission is the model, and understanding is likely to depend on what one focuses on or considers as important. The chances are that the focus is on a few critical factors, depending on the context.

The spoken words of the teacher can also be received as a directive to act or perform. Take, for example, the teacher educator who asked students to find the relationships of the three sides of a right-angled triangle. Consider Figure 17 and the students’ interest in finding the value of x. Immediately, a thinking process may start, and with the knowledge of Pythagoras´ theorem, students may be able to establish the first statement:

$$x^2 + (21 - x)^2 = 15$$

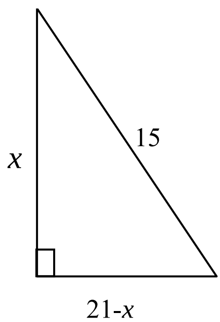

Figure 17. Right-angled triangle: meaning assigned following educator utterance

Previously it has been stated that understanding will depend on the orientation or context. A student with an orientation to problem-solving or ethno-mathematics, for example, may start asking ‘Why do this mathematics?’, ‘What use does it have for me?’, ‘Is it necessary to do it?’, and a series of similar questions pointing to the relevance of doing it or simply the relationship with the context. All these are grounded on the relationship between the student and the teacher educator who gave the instruction to do it.

Spoken words have a third dimension in addition to conveying content through transmission and action-based on relevance. Spoken words in mathematics and MTE can also be seen as aesthetic. This is a forgotten part of the role of relationships and sometimes is not even mentioned. Aestheticism is the art of appreciating the beauty, power, and precision of mathematics, together with its elegant principles (Hudson, Ongstad, Braathe, Hans-J & Pepin, 1999). This brings us to a very important point in contrast to what is orchestrated very often. From the perspective of aesthetics, and keeping aside some of the wholesale generalisations, for example the mathematics-avoidance syndrome, it can be seen by some that:

> *"Mathematics is hated and loved, it is awful and beautiful, it is clear and unclear, negatively frustrating and positively challenging".* (Hudson, Ongstad, Braathe & Pepin, 1999 p.147)

This statement may serve as a reminder of issues related to building relationships in MTE. On the basis of this statement, whole sale generalisation that there is total apathy towards mathematics is highly questionable. There is evidence of situations where mathematics is hated, but the starting point to addressing issues of frustration, negativity or stigmatisation in mathematics education is that it is loved by some.

All in all, the educator-student-learning material discussion suggests that the spoken words of the mathematics teacher educator may be seen in three qualitatively different ways. First, they can be seen as content which carries the context with it, a conception which depends greatly on critical aspects of focus. Second, they can be seen as a directed action or activity, and recognition or acceptance, depending on seeing the relevance of the instruction. Finally, they can be seen as expressing an aesthetic conception which depends on the appreciation of the beauty, power and precision of mathematics.

In order to enhance learning relationships, teacher educators focus on the critical aspects, namely motivation, raising students' interest, inspiration, and so on. The idea of building relationships gives a sense of how to make MTE part of life among mathematics student teachers. Of course, a practical and sustainable way to ensure learning relationships is to emphasise problem-solving and application of mathematical knowledge. It is also important to consider using motivating examples in order to build relationships in teaching and learning mathematics. Mathematics teacher educators are aware of the difficulties they face when introducing mathematics topics which involve investigation, problem solving or guided discovery. Let us take a couple of examples:

Example 1: A security guard has a team of thirty-six fierce dogs which are very unfriendly to each other. The task is how can a convenient shelter be constructed to save the dogs from biting each other?

Certainly there are a number of options to arrive at a reasonable solution. One possible solution is to start with a circular structure of 36 equal segments, and other options of the puzzle could follow.

In a lesson to introduce proof by discovery, Vernon (2005) introduced the lesson by using a puzzle closely related to what learners do in everyday life.

Example 2: A Frog presently occupies square A of a 6 square arrangement seen in Figure 18. All the squares are frog rests to save it from being eaten by hungry frog-eating alligators. Show how to get from square A to B in accordance with two rules ordered by the life saving wizard. Only horizontal and vertical leaps are allowed to an adjacent square. Each square can only be visited once.

Students are likely to make several attempts, and some might succeed immediately; some might not. Suppose the puzzle is solved, how many leaps is the frog going to make? Following this puzzle, a checkerboard is introduced, of which they are aware. They might now make a rush to try again. The checkerboard is related to their life, and it may motivate them to make another attempt.

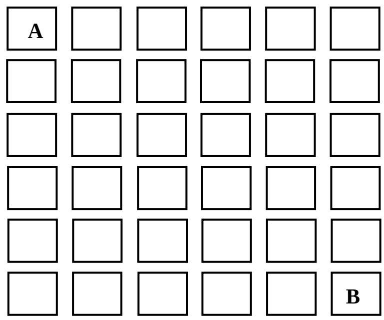

Figure 18. The plight of a frog to escape death (Adapted from Vernon, 2005 p.179-180).

To conclude this part, it would seem to me that the student teacher-educator-learning material pedagogical or didactical relationship is increasingly becoming an area of concern. The two examples given above are just a glimpse and a reminder that MTE is seen as a process of building pedagogical relationships. Teaching and learning mathematics is not only about content reception. It is about how learners are touched, motivated, inspired and attracted. What we see as mathematics-avoidance syndrome, negative attitude or stigmatization of the subject are reflections of the importance of building relationships as a focus in the development of MTE. I am of the view that there is still more to study in this area, above all along the lines of aestheticism. Indeed, the next category of thoughts on the development of MTE points to recognition of research in the domain.

Category E: focus on research of teaching and learning contexts

From a general viewpoint of the results, the recognition of MTE as a research field is lower than, for example, its acceptance as pedagogy, referred to as a method of teaching and learning. Conversely, pedagogy is a research field on knowledge of teaching. Even more striking is that few teacher educators recognise MTE as a scientific field. Despite the low recognition, teacher educators think the development of MTE has to focus on studying the context of teaching and learning in order to inform practice, and in the same way practice has to enrich theory. I intend to discuss this category of thoughts on the development of MTE by first revisiting the meaning assigned to the notion of MTE. This is to be followed by a discussion of MTE as a research field and the possible areas of focus. Third, the aim of studying MTE is briefly examined, followed by my concluding remarks.

The term MTE is a composite of many influences, as discussed in the theoretical framework (see Section 2.2). One of the critical considerations is the focus on ‘the process of becoming a mathematics teacher’. The process of becoming a teacher is influenced by teacher education, mathematics education, curriculum development, and strategies of teaching and learning. The influences of psychology, philosophy and sociology have been discussed before.

Mathematics teacher education, although a different domain from mathematics education, has much in common with it. Both mathematics education and MTE deal with teaching and learning mathematics. In addition, the need to know more about teaching and learning through research is a common thread running through both. One major difference is that MTE focuses on teaching about teaching (preparing prospective teachers), given the broad coverage of mathematics education. The relatedness is what convinced me not to work with a different theoretical framework. It was inspiring to use the same framework as in mathematics education - but this does not rule out other perspectives completely.

The amount of research about MTE has grown substantially in the past ten to twenty years, with the recognition of the influence of the teacher on children’s learning (Fou-lai, Cooney, 2001; Gravemeijer, 1994; Lerman, 2001; Putnam, Borko, 2000; Reis-Jorge, 2007; Sierpinska & Kilpatrick, 1998; Sutherland, 2007). It is further reported by these studies that there are limited theoretical perspectives. This is enough evidence of how much there is to say and how much there is still to learn (Lerman, 2001; Romberg, 1998; Schoenfeld, 1989). In the same vein, mathematics education as a theoretical field has (Westbury, 2000) its roots in mathematics classrooms, from nursery classes to those in universities. It has drawn on a range of theoretical resources in developing its own theory to account for what goes on in settings in relation to teaching and learning. Interestingly, what I often see missing in many of the exponents of research-based teacher education is why the emphasis on research in MTE, for example? More often than not, this is taken for granted. To state it briefly, Jakku-Sihvonen (2006) argues that teaching is a pedagogical decision-making process, and when teachers are engaged in researching about their work they are in a better position to make these decisions, or simply have better reflections. In the same vein, Carrillo (2001) points out the great dilemma between research and teaching. Teachers see researchers’ work as being theoretical, while theirs is practical. But what about a situation where the researcher and the teacher is the same person? This situation is likely to minimise the drifting apart between researchers regarded as being theoretical, and teachers/ teacher educators seen as being practical.

In his argument, Carrillo (2001) supports research as part of teachers’ professional development (MTE included). The research-based approach to teacher education creates a reflective teacher. This is because a researcher in action is constantly subjected to reflection. Second, it is further argued that processes that aim to integrate theory (research) and practice (teaching) contribute to forming a connection between initial and in-service teacher education. Indeed, they offer a frame of reference for future teachers to conduct research into their own practice, and in this way they establish the basis of role model as future professionals in teaching. Although I side with their analysis, I need to reflect a bit more on Jakku-Sihvonen and Niemi (2006), as well as Carrillo (2001), about their conclusions. Teaching and researching looks like a noble idea, but is it a one-size-fits all? I may be wrong in saying that research-based teacher education is context-specific, needing expert supervision. I can imagine a situation where the demand and supply of teachers is very high. In my view, research-based teacher education may run in parallel with other modes as a start.

In relation to the results of this study, research issues in MTE could be approached or formulated and focused on in a number of areas. First, it has been shown how challenging it is to work with student teachers, in mathematics clubs, for example, in discussing areas which are problematic to them. Another approach could be through teacher educator development when colleagues from neighbouring teacher colleges meet to discuss ways of enhancing assessment. A third approach could be through curriculum innovation, when they discuss how to cope with new mathematics education syllabuses or the reviewed curriculum. To round it off, the points of entry for research work could be built around mathematics teaching and learning problems in pre-service teacher education, mathematics teacher development or teacher change and mathematics curriculum innovations.

One of the aims of including research in MTE is thus to enhance practice. The results indicate that there are issues related to inappropriate pedagogical approaches. The results show that teacher educators have been doing some research work. This is indicated by expressions like ‘conduct surveys in order to identify areas of concerns in teaching and learning mathematics education’. Secondly, the aim is to generate knowledge. To ensure relevance, the aim need not focus on general trends in MTE, but on issues around teaching and learning contexts. Sierpinska and Kilpatrick (1998), for instance, insist on impact and suggest the need to focus on teaching and learning situations, didactical situations, the relationship between teaching and learning, and mathematical knowledge, as well as the reality of mathematics classes, societal views about mathematics and its teaching, and the system of education itself.

I would like to conclude with the fact that teacher educators have been doing some kind of research, which means that they have some experience, and it is possible to take advantage of this basic foundation in research and improve on it. Certainly, this is an added advantage and may provide a sound starting point if done in a constructive manner.

6.4 Thoughts on strategies for sharing knowledge and skills in MTE

From conceptions to knowledge-sharing strategies

Teacher educators tended to think and focus differently on strategies of sharing knowledge and skills. The discussion of the research results rests on two strategic thoughts on sharing knowledge and skills. Predominantly, teacher educators thought neighbourhood learning groups are a productive strategy for sharing knowledge and skills. Their other way of sharing knowledge and skills pointed to professional development through distance collaboration and networking. The two will be discussed together in the interest of reducing compartmentalisation of ideas without losing a grip on the logical sequence.

In this study, it all started with identification of teacher educator's conceptions of MTE in research question one. The next task was to identify thoughts which could take the conceptions forward in the event of development needs (research question two). The third stage is the present task, to deal with a discussion of strategies for sharing knowledge and skills so developed. This is based on the identified best practices by mathematics teacher educators. Expressed in another way, this chronology of the thinking process represents a trajectory of ideas from conceptions of MTE to pragmatic ideas worth focusing on. This in turn gave rise to how to share the knowledge and skills. With that in mind, I prefer to discuss neighbourhood learning groups as a strategy for professional development for mathematics teacher educators. But before that, let me briefly come back to the notion of professionalism. Professionalism is central in the discussion of neighbourhood learning groups, as well as distant networking and collaboration among teacher educators.

Very often notions like teacher education (the process of becoming a teacher) and teacher development are associated with professional development. But what is professional development? I find it rather difficult to answer this question. For the purpose of clarity, and to match the second part of the research question, I suggest talking about what teachers and/or educators do in professional development rather than providing a definition. Professional development has been discussed thoroughly in Chapter 2. As a reminder, Loucks-Horsely et al (2003), for example, prefer to discuss professional development in terms of activities built around lesson study, curriculum alignment, and selection of instructional materials, demonstration lessons, coaching and mentoring. Further reflection on these notions is likely to show that these activities are not based on the practitioner's wisdom but rather on a planned process leading towards growth in understanding. Professional development is therefore a process of continuing to learn after qualification - a continued deepening of mathematics teacher educators' knowledge with professionalism as a vision. On this basis, professionalism is a broad term and has certain key features. Among important qualities and features of professionalism, Eraut (1994) considers internship, enrolment in a professional college, passing examinations from a qualifying institution, recognised training and a collection of evidence of practical competencies. In addition, any discussion about professionalism needs to demonstrate high ethical standards of practice, engage in professional development activities, and demonstrate ability to work as a reflective practitioner, show and work with colleagues as well as parents in order to support the learning environment (Olulube, 2006). These are the basic features of professionals, and those who regard themselves as professionals could well gauge themselves against these criteria.

Whether teachers and teacher educators are professionals or not is an issue of intense discussion. But there is a widespread lack of confidence and perception that teachers are not up to the task of enhancing teaching and solving educational problems (Stigler & Hiebert, 1999). The lack of confidence covers teacher educators, too. In the words of the researchers, a popular solution is to 'professionalise teachers" because they are not professionals. It is further argued that students' failure to perform is unfairly directed at teachers and teacher educators. In some cases, poverty, demographic changes, erosion of values and breakdown of supportive families rank high among the reasons. What then is standing in the way of teachers being professionals? I would say the answers are varied. Stigler and Hiebert (1999) argue that teachers are not regarded as professionals because society thinks that anyone can be a teacher, and that formal expertise is not a necessary and sufficient condition. The other explanation is that teachers are not fully appreciated for what they do and thus become vulnerable to criticism. Next comes the issue of varied certification requirements, which are sometimes uncoordinated. There is little evidence of teachers themselves taking responsibility and living as models in what they do, and there is a lack of a defined career ladder. One would also find rare occasions for example, to see peer reviews of what teacher educators do based on some laid-down professional principles. Furthermore, there is no deliberate move to train teachers as researchers and encourage them to set the standards for entrance into their profession. All these shortcomings work against the possibilities of professionalising teachers and teacher educators. I am of the view that professionalism cannot be instituted by arbitrarily assigning professional labels to practising teacher educators. They can neither be created by certificates nor censures, but perhaps by the existence of a professional body of knowledge and ways of improving it. The issue of teacher educator development as a process towards professionalism is to be pursued further in the next section.

Category A: focus on neighbourhood learning groups for knowledge sharing

Teacher educators identified neighbourhood learning as one of the two options of knowledge sharing. Critical aspects which created the basis of this variation include seminars and workshops involving nearby educators, in-house capacity building in the sense of taking advantage of the expertise possessed by a teacher educator living and working close to someone. Other aspects detailing knowledge sharing options are mentoring, team teaching, lesson studies, subjects and assessment panel discussions, as well as study tours. These models have a long-standing history in serving the purpose of teacher development. I am discussing this in relation to neighbourhood learning groups as revealed by teacher educators. The emphasis and resources given to seminars, workshops, subjects and assessment panels is well known. Sometimes it is taken for granted that they stand for professionalism, while evidence shows it is not always the case. What is the purpose and why is there mathematics teacher educator development? I will start with the second part of the question. Teacher development in whatever form is said to be the point of entry for teacher professionalism. On the basis of the results, the neighbourhood learning group stands out as an important category of description. It has the features of a professional development model similar, for example, to community of learners. In Chapter 2 I spent time discussing the strengths as well as the challenges of these professional development models or strategies, and community of learners was one them. In view of the aspects related to the neighbourhood learning group, it seems to be unique in the sense of the physical location of mathematics teacher educators/teachers, and its smaller size than a community of learners. Take, for example, in-house capacity building, mentoring, team teaching, lesson studies, and study tours. All these imply immediate face-to-face sharing and support of mathematics ideas. This is what qualifies it to stand on its own. Despite the potential difference, it is closely related to the well known community of learners, which has helped many teachers/teacher educators around the world to grow professionally. I read a similar case in the UK, which appears to emphasize its worthiness.

At one time the United Kingdom was faced with problematic situations in which about a quarter of the teachers after qualification were inappropriately trained for their classroom work (Newby, 2007). It is further reported that when the same teachers were asked what they thought of their initial teacher education programmes and what solutions exist, they revealed a striking experience, some of it similar to the neighbourhood or community of learners' theory. First of all, they criticised what is happening in teacher education classrooms and pointed the finger at philosophy, psychology, history and sociology of education. These were labelled dry, dusty, book-bound subjects that did not tell them how they could continue learning after qualification, given the possibility of reality shock, transition shock and praxis shock, as argued by some researchers like Lindgren (2003). This series of issues which came from prospective teachers was an attempt to find solutions on how best they could benefit from working together in communities of learning. The UK experience is an appropriate match to teachers' neighbourhood learning groups and the latter seems to be its hybrid with potentials of credibility if nurtured and allowed to grow.

Having tackled the rationale for teacher educator development, I now turn to the aim of teacher development with a focus on neighbourhood learning groups. Teacher educators indicated very specific aims if they were to enhance MTE. Evidence exists to show that routine models (workshops, train-the-trainer, speaker series) that have primarily relied on transmitting new ideas of teaching and learning as suggested by teacher educators do not seem to work. Worse still, the top-down hierarchical structures suffer the same weakness (Dalgarno & Colgan, 2007). It is known from research reports that teacher educators or teachers would profit more from knowledge and insights developed personally by themselves through activities, discourse, reflection, inquiry and application, rather than prescriptions which give them very little flexibility (Dalgarno &

Colgan, 2007; Fou-lai & Cooney, 2001; Gravani, 2008; Hodgen & Askew, 2007; Krull, Oras & Sisask, 2007; Macrae, & Nessoro, 2006; Wang, 2007).

According to the literature cited above, traditional approaches to teacher development (following classical laid-down rules) have made it more difficult than ever. For one thing, the study of mathematics teaching and learning occurs in highly formalised, programmed activities, very often outside the context of the teacher educator's work. Furthermore, the target beneficiaries have little control over the focus of mathematics subject matter and pedagogy. To make the situation even worse, the sessions are too often scheduled at inappropriate times. To tell it correctly, the sessions are held at the convenience of workshop organisers.

What should the new models of teacher educator development entail? Fou-lai and Cooney (2001), for example, talk about three approaches to bringing teachers into the reform process. To ensure the required conceptual growth, discussion with teachers or teacher educators about their beliefs and practices in MTE is imperative. Following this, it is important to guide them cognitively in the learning process and even more important to work intensively with teacher educators. The way I see it is that activities in neighbourhood learning groups need to reflect ideas originating from them. That is, professional learning is expected to provide opportunities to access and discuss what teacher educators consider as their own best practice. Next, the kind of mathematics they learn is expected to address both subject matter and pedagogical knowledge, bearing in mind their context of teaching and learning experiences. All in all, professional learning should allow them to create and prepare written resources which they can share as practitioners. This idea of neighbourhood learning groups as revealed by teacher educators has similar features to the famous community of learners now dominating training venues, bookshelves and websites.

Category B: focus on collaboration and networking for knowledge sharing

Collaboration and networking was another strategy conceived by mathematics teacher educators. Critical aspects which make this category distinct from neighbourhood learning groups is the application of ICT for teaching and learning, shared articles (including research material), module and newsletter writing, e-mail exchange, website visits, and listening to radio and television mathematics programmes. This strategy is also intended to address the quest for sharing knowledge and skills. Like neighbourhood learning groups, it is directed towards teacher development with a different focus on how to share knowledge and skills. Teacher educators' thoughts expressed how they could take advantage of the richness of ICT now available in their institutions to enhance professional learning.

With this knowledge-sharing strategy, teacher educators thought collaboration and networking among themselves was an effective way of sharing knowledge and skills in MTE. I would like to discuss the issue of collaboration and networking by using two related examples of different contexts. Let me start with the experience of the Japanese lesson study and compare it with an account of the USA teachers' professional development approach. Collaboration and networking is taken to mean teacher educators cooperating from within and with others outside by whatever means (e-mails, websites) for the purpose of knowledge and skills sharing. Lesson study, in the Japanese context, is an account of what teachers do in the classroom to enhance teaching and learning. Support of teachers' efforts takes place by directly refining the lesson plan and dealing with the mathematics they need in the classroom. This is different from the USA approach, where the emphasis is given to teachers' development of pedagogical content knowledge not necessarily through networking and collaboration. The Japanese strategy of professional development starts in the classroom and ends in the classroom, spreading to a network of other schools. The USA approach starts outside the classroom and may end in a network of many classrooms. In real sense, it has features of imposition. Crocket (2007) is an advocate of the systematic approach of collaboration and networking linked to specific school teaching and learning goals. The situation in the USA falls short of an orderly arrangement and is dominated by overlapping innovations or projects where teachers may not be from the same school and not linked to school teaching and learning goals.

Stated another way, the Japanese system of professional development rests with the school, and each school is linked country-wide to form a national network of collaboration. The situation in the USA is structured in such a way that professional development is conducted outside actual mathematics lessons, and expects teachers to apply the pedagogical knowledge and skills later in their classrooms. Immediate reaction to this approach concerns its complication because of the distance from the classroom in terms of interpretation, application or transferability. I may call this a first-order complication, possibly followed by another one during actual lesson presentation.

Within collaboration and networking learning, teacher educators described a series of MTE activities. In the long list we have mathematics club meetings, seminars, workshops, student remedial lessons, college-based orientations, identification of mathematics difficult topics, and preparation of teaching and learning materials. Other important activities which have been mentioned are article writing and editing, searching for mathematics education scientific papers from websites, and many more. It is possible to visualise this as an attempt and a process to get teacher educators involved in real mathematical tasks. If this is managed by the head of department, for example, then a vision has to be developed collectively, issues identified, objectives as to what is to be done set, and in the end an implementation plan drawn up. Monitoring the daily tasks is essential and is part and parcel of the entire process of collaboration and networking.

These are ideas across continents, from the USA to Japan and elsewhere. Collaboration and network learning imply a serious shift in relation to how teacher educators could learn. It appears to be the case that the days of learning by transmission and telling are gradually receding, being replaced by more reflective strategies of knowledge sharing among educators. There is a lot of interaction with texts depicting mathematics teacher education as they collaborate and network. Collaboration and network learning is a notion coined by teacher educators to mean formal knowledge and skills sharing as well as day-to-day consultations among teacher educators. Depending on the focus, this is the equivalent of the more globally recognised teacher development models such as communities of learners, learning circles and others. Greeno and Goldman (1998) cites Riel (1998) on this important issue regarding collaboration and network learning communities, as follows:

> "*Group work provides a context for the externalisation of thinking. It allows for the discussion of multiple perspectives and helps all the participants realise that each person creates one of the many perspectives on a topic or problem. Learning to see from the perspective of others helps create a more complex understanding of situations. Learning how to use distributed expertise as a resource and organise a team of people to accomplish a task are some of the lessons that have been missing from the cultural transmission approach to teaching and learning*". (Riel, 1998 p. 373)

The above quotation is not a rejection of traditional transmission, but rather evidence of its serious shortcomings if used by teacher educators as the sole approach to teaching and learning mathematics. Mathematics teacher educators need to use the approach with an open mind and think of accommodating other strategies for the purpose of analysing and clarifying ideas, building on others ideas and admitting weaknesses (Stein, Silver & Smith, 1998).

To briefly conclude this discussion, it suffices to mention that the discussion on teacher educators' qualitative ways of seeing MTE, and their thoughts on desirable development, as well as their knowledge and skills sharing strategies show a reasonable connection between teacher educators' ideas about what they think and do and the theoretical framework. However, this discussion will be extended a little further in Chapter 7 as part of concluding thoughts.

7 Concluding thoughts

To reach the climax of a research process such as this one is an inspiring course of action. Naturally, the research process requires serious reflection on the results and their implication for teacher education. The results of this study follow from a situation of searching questions about MTE, for which no satisfactory answers existed at the beginning. This in turn allowed for the development of methodological solutions which could guide the process of investigation to identify teacher educators' conception of MTE, development-oriented thoughts, and knowledge sharing strategies of the same. I associate this process with a critical path over which I always kept tight control in order to remain within the study research questions. It now makes sense to look back and reflect on the study as part of concluding thoughts.

I begin with a reminder of the research questions in Chapter 3, together with the study motives in mind. Out of this stems the intention of the study. I argue that this study has found a variety of ways of thinking about the process of becoming a mathematics teacher, and of course a set of thoughts on MTE development. Finally, I highlight the implication of the study for teacher education. From the very beginning I had a special interest in the two-sided debate among teacher educators on whether mathematics content has been compromised by an emphasis on methods to an unacceptable level (TIE, 2007). In a number of teacher education forums that I have attended, teacher educators, curriculum developers, inspectors and university lecturers have questioned the over-emphasis on methods, given the weak background of many student teachers in terms of subject content. On this basis, some of the higher learning institutions have attempted to close off university entry on what one would say is subjective grounds for lack of informed decision. The attempt to close off entry was questionable as experience indicates some made it through given the high demand of prospective candidates for university education. Conceding to one part of the debate is possibly a rethinking of the subject matter vs. pedagogy debate. It was on the basis of this problematic situation of teacher education narrowed to MTE, the lack of research-based knowledge in the area and my personal experience that I developed an interest in investigating teacher educators' conceptions of mathematics teacher education, and based on this providing a description of the conceptual variation.

In order to position the study intentions, it was important to review the relevant literature for the purpose of an informed theoretical framework, for which the themes of choice and justification include the changing focuses of MTE in Tanzania in order to unearth the meaning assigned to MTE in a given timeframe. This was intended to shed light on MTE and act as a point of departure in the discussion of current MTE. Within the theoretical framework there is a discussion of perspectives in MTE intended to make sense out of it what? Emerging perspectives as a result of literature review illuminate MTE as a composite of many influences, a process of becoming a mathematics teacher and as a blend between subject matter and pedagogical knowledge. Finally, MTE can be viewed as learning about teaching. Working on the assumption that teacher educators are 'forward looking'; I made an attempt to explore 'thoughts' about the development of MTE. This was based on the assumption that mathematics teachers are learners after qualification. It is a continued deepening of knowledge from pre-service teacher education, through induction, to a series of planned in-service teacher education sessions. I emphasise planned in-service or professional development for the purpose of professional growth. Within the area of teacher educators´ professional growth, the rationale, purpose and modes of assessments are discussed.

Two research questions guided this investigation with the aim of identifying teacher educators' conceptions of MTE, their thoughts concerning further development and finally a description of their variation. To that effect, the research questions of interest were a) what are teacher educators' conceptions of mathematics teacher education? And b) what are teacher educators' thoughts on the development of mathematics teacher education? The first research question focused on teacher educators' conceptions, which in the actual interview responses could be seen in terms of perceptions, ideas, views, understanding, and impressions associated with MTE. The second question sought teacher educators´ thoughts, and in the open-ended questionnaire could be seen in terms of ideas, considerations, reflections, notions, and understanding. The notion 'conception' and 'thought' are closely related, but the notion 'thought' has been used to add dynamism and a 'forward looking' perspective in the study.

On the basis of these fundamental research questions, this study has found the existence of first, a variety of ways of thinking about MTE; second, a variety of ways of thinking about what to emphasise in the event of development of MTE; and third, a variety of ways of thinking about knowledge-sharing strategies. Within each of the three category systems lie different categories of descriptions of MTE, development thoughts and knowledge-sharing strategies. In view of Pang (2003), the differences in seeing the same object have evolved as a result of differences in focus on the critical aspects, which in a strict sense define the categories of descriptions. It is the tendency of teacher educators to focus differently on the aspects of each category of description which creates the variation within the same category system. On the basis of the findings in terms of categories of descriptions (see Figure19), I was able to draw conclusions which are presented in Chapter five as findings.

Important conclusions from research question one are that teacher educators experience sharp conceptual variation of MTE in qualitatively different ways (see Figure 19), and in a specific way, it has been confirmed that some teacher educators conceive MTE as school mathematics. Seen in this way, failure to differentiate between the centrality of MTE and school mathematics raises questions about their role as mathematics teacher educators. In research question two, the following conclusions are drawn: teacher educators' thoughts on development of MTE vary and gravitate around pedagogical knowledge and skills. Equally interesting, teacher educators' thoughts on professional development are sometimes influenced or sensitive to emerging shifts of thinking: for example, from teaching as telling to learning through investigation, MTE as a source of inspiration, and from traditional ways of assessment to portfolio assessment. In addition, educators' ideas about development in MTE indicate the need to consider building relationships between educators and student teachers as a way of addressing stigmatisation, phobias, low self-esteem and apathy. What's more, teacher educators' pedagogical thinking is challenged on many fronts. As a way of putting into action what they consider MTE development ideas, the study findings indicate distinct variations in thinking about knowledge-sharing strategies. There are thoughts built around neighbourhood learning groups as one option, and through distance collaboration and networking as another. Finally, the recognition of MTE as a 'tool' for research potentially exists, but has little recognition.

As a conclusion, I would like to propose a framework which may help to clarify and pull together the different ideas in this study. This framework takes into account teacher educators' conceptions of MTE, thoughts on development, and knowledge-sharing strategies of the same. The proposed framework has been developed from the research results and the literature reviewed. It pulls together the three category systems and their corresponding categories of descriptions to form a comprehensive picture of teacher educators' qualitative ways of viewing MTE. Figure19 is a summary presentation of this framework and is elaborated on further. The framework consists of the three conceptual foci (1, 2, and 3) of MTE by teacher educators, which are described one at a time. Conceptual focus 1 refers to teacher educators' conception of MTE. I name them conceptual focuses because within each category of description teacher educators focused differently, influenced by aspects. In the same order, conceptual focus 2 is a presentation of their thoughts on the present development of MTE. Conceptual focus 3 is a presentation of teacher educators' thoughts on knowledge and skills-sharing strategies. This is a generic product of research question two, which asked for practical ideas on how thoughts on development could be put into practice. Conceptual focus 3 has a special relationship with both conceptual foci 1 and 2 in the sense of carrying forward ideas from conceptual foci 1 and 2; that is, putting the ideas in focus 1 and 2 into action. The relationship between conceptual focus 3 and conceptual foci 1 and 2 is shown in dotted lines to indicate this generic relationship.

7.1 Teacher educators' qualitative ways of seeing MTE

Conceptual focus 1: Teacher educators' conceptions of MTE

The notion of conceptual foci refers to pulling together the different conceptions of MTE. Conceptual focus 1 represents teacher educators' qualitative ways of viewing mathematics teacher education (see Figure 19). That is to say, MTE is seen in different conceptual foci or categories of description. The conceptual foci under discussion are MTE as mathematical investigation, inspiration, problem-solving, pedagogical knowledge and skills, subject didactics, subject matter, and its integration with pedagogy. It is pertinent here to pose a question regarding the contents of the conceptual foci. Given the conceptual foci, do mathematics teacher educators act in unison? The results do not support this in view of the range of options. It is thus challenging to arrive at shared conceptions of MTE in the context of this study. In addition, the phenomenon taken as a whole represents an inherent conceptual dilemma.

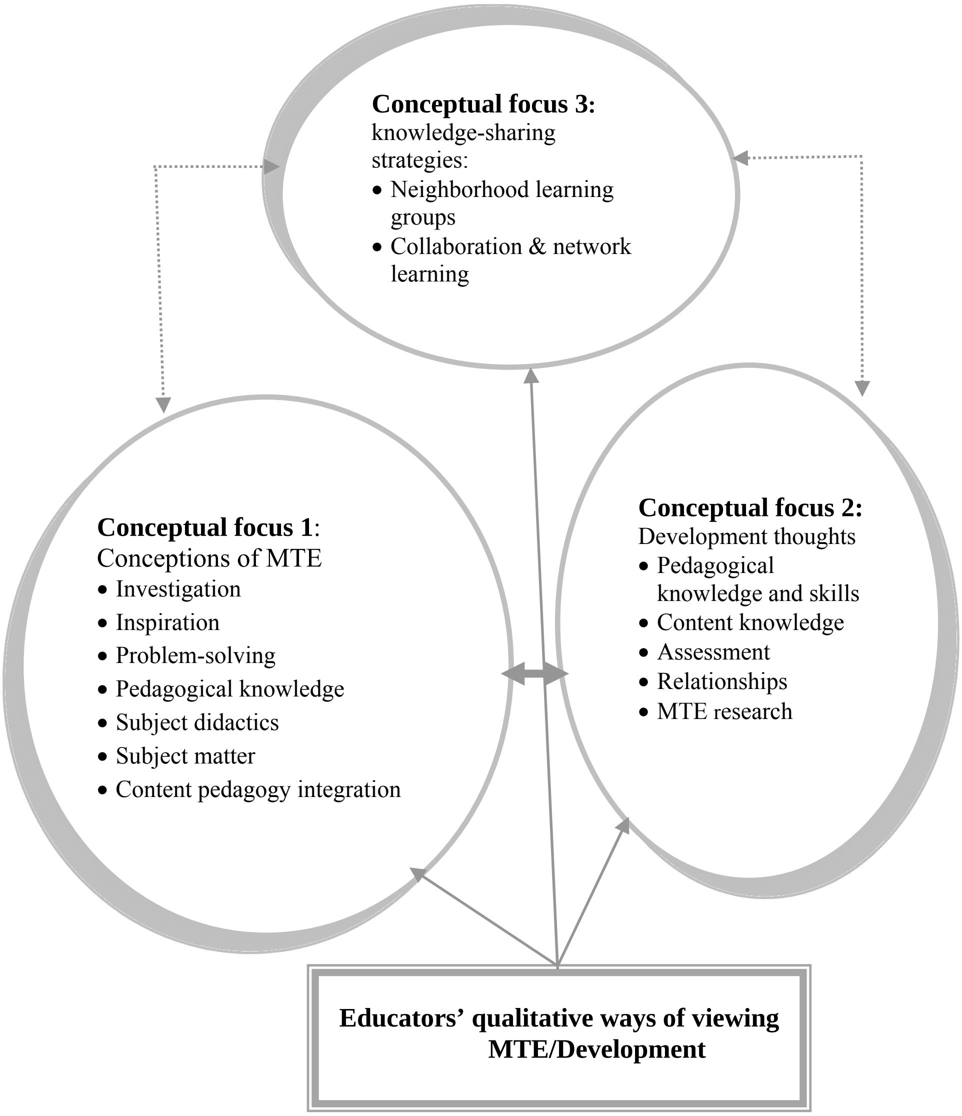

Figure 19. Framework showing teacher educators' qualitative ways of viewing MTE

The seven options of understanding MTE, for example, may pose a challenge for an outsider to select with precision the exact meaning of MTE, as well as for teacher educators, who have shown distinct conceptions. Expressed in a different way, teacher educators make pedagogical choices based on these conceptions. Furthermore, they even take up positions based on the conceptions. This situation is an inherent dilemma among mathematics teacher educators. However, I am aware that this situation is not unique to MTE. We may be aware of similar dilemmas facing teachers in mathematics and mathematics education. For example, assessment is a tool to support learning on the one side, and on the other assessment is educational power (the power to provide the relationship between ambitions and abilities, and a comparison of abilities between individuals). Some students can learn mathematics, and some cannot. An even more complex situation is how research and teaching is viewed by teachers. In any case, teacher educators find themselves in a position of making choices between options. This situation does not negate the possibility of taking positions on what to emphasize. Taking positions contributes to what I may call a pedagogical dilemma. It is also a conceptual dilemma because of the thinking process before a choice is made.

In both the beginning of the study and in the theoretical framework, I attempted to establish the relationship between MTE and mathematics education. It was pointed out that, despite their differences in focus, both have an underlying common denominator. Both are processes and deal with teaching of mathematics and the learning of mathematics, and both are fields of research. They are again different in important ways, as MTE is grounded in the process of developing a mathematics teacher. As an example, one of the mathematics teacher educators voiced an idea on this process of developing a mathematics teacher:

> "*It is knowledge and skills on how to teach and learn mathematics, seeking solutions in the teaching and learning procedures. It is about enabling student teachers to develop strategies on how to facilitate teaching and learning mathematics. It is a process of teacher preparation..".* (Safari, December, 2007)

This statement makes a case for considering MTE as a process to model or develop a mathematics teacher. Therefore, this way of viewing MTE provides a platform to think more about conceptual focus 2.

Conceptual focus 2: teacher educators' thoughts on the development of MTE

Teacher educators' conceptions of MTE originate from their experience in the work they do as mathematics teacher educators (see Figure 19) Development thoughts in focus 2 originate from the same, but perhaps more from the tasks they do in mathematics development activities. In practice this involves tasks in mathematics seminars, meetings, workshops, professional associations, and conferences. The array of development thoughts generated from educators' experience as categories of descriptions are pedagogical knowledge and skills, content knowledge, assessment, building relationships, and research in MTE. The comments by Lunenberg and Willimse (2006) on research and the professional development of 'teachers who teach teachers' seem appropriate. The two researchers attempt to raise questions on how many of the 'teachers who teach teachers' have ever found themselves in classrooms of teacher educator preparation. They argue that many who are serving today have entered the field from two sources. First, they have been good teachers, and second, they were experts in some particular area. This situation has not been the case in Tanzania because of the existence of a teacher educator programme developed since 1997. This potential project for the development of 'teachers of teachers' was a collaborative initiative between MoEVT and The Stockholm Institute of Education. It was based at Morogoro Teacher College and, now in 2010 is under MoEVT and, accreditation is done by The Open University of Tanzania. Despite some of its achievements regarding the development of teacher educators, it is not without challenges for example experts in the field, appropriate curriculum to mention a few.

It is important to note some of the challenges in relation to the whole notion of development of MTE. First, MTE is regarded by a number of teacher educators as a 'once-and-for-all' learning activity. This view, likely to be regarded as a non-starter, is what researchers have been struggling to have in the right perspective, and Loughran (2006) is one of them through a discussion on the pedagogy of teacher education. Completion of a set teacher education course in mathematics is viewed as an end in itself rather than an ongoing process for years to come. That is, it has a beginning and an end. This may be erroneous in many ways because at no time do teachers stop learning. Secondly, it is difficult for many to come to a point and realise that the task of MTE is to teach and learn about mathematics teaching. In my view, this is clearly beyond the act of teaching the school mathematics we know. A statement by one of the teacher educators provides further evidence:

> *"Development of mathematics teacher education therefore refers to the professional development of mathematics teacher educators. In professional development teaching/learning strategies are a norm. I think also teacher educators need to know more than the subject matter and strategies for the student teachers they support to become teachers. Teacher educators themselves need to improve their mathematics teaching and learning knowledge".* (Violet, December 2007)

This statement made by a teacher educator has a lot of implications for mathematics teacher educator learning. Further reflection on it seems to reveal that a natural solution is to improve mathematics teacher educator practice. Specifically, I support the position of Loughran (2006) on the possibility of developing the pedagogy of teacher education, which then can bring to light the need for professional development for 'teachers who teach teachers'. What does that really mean? Experience tells us that teacher educators have two important tasks. They start with facilitating how to teach to mathematics student teachers, and then learning about teaching, that is they are also learners. Put it in better way:

> *"Becoming a teacher educator (or teacher of teachers) has the potential (not always realised) to generate a second level thought about teaching, one that focuses not on content but on how to teach …This new perspective constitutes making the 'pedagogy turn', thinking long and hard about how we teach and messages conveyed by how we teach… I have come to believe that learning to teach is far more complex than we have ever acknowledged".* (Loughran & Russel, 1997, p. 44).

The point of emphasis in connection with teacher educators' ways of understanding MTE is on this important task of teaching about teaching rather than an overemphasis on subject content knowledge for any reason. I think another important task for teacher educators is their own study, and to guide student teachers to learn about teaching with an appropriate combination of emphasising subject matter and mathematics knowledge for teaching.

Conceptual focus 3: Teacher educators' thoughts on knowledge and skills sharing strategies in MTE

In conceptual focus 3 teacher educators indicate their thoughts on strategies for sharing knowledge and skills. In a real sense, it concerns teacher educator development as a process towards professionalism. The conceived strategies are neighbourhood learning groups and collaboration and network. Neighbourhood learning groups are more convenient for teachers and teacher educators in close vicinity to each other. It is not all about going to seminars, or workshops. It also means taking advantage of the neighbours' expertise. Collaboration and networking as a way of supporting each other may take the form of distance learning. It has frequently been described by mathematics teacher educators as a potential strategy for professional development, sometimes associating it with seminars and workshops. Elsewhere outside Tanzania, traditional seminars and workshops of a top-down nature have appeared not to work, as they are fragmented, disconnected and irrelevant to real classroom practice (Dalgarno & Colgan, 2007; Lieberman & Mace, 2008;). Again, as one of the teacher educators stated:

> *"I expect to share experiences in mathematics development ideas with staff and colleagues from neighbouring colleges by visiting different colleges to see how they deal with challenges facing teaching and learning. I also share ideas by discussing difficult topics in mathematics and how to solve problems with my colleagues in our college or from other nearby colleges. Lastly, I could introduce a capacity-building programme in my college with a focus on challenges during teaching and learning mathematics, regarding difficult topics, solutions involving preparation of teaching and learning materials, and selection and use of relevant teaching and learning methods".* (Shyrose, December 2007)

The key message being voiced by mathematics teacher educators concerns opportunities for sharing knowledge and skills. They can take the form of face-to-face sessions when teacher educators come together. This is an example of the much advocated 'community of learners'. The opportunities are wide open and make it possible for them to share experiences through other professional models like lesson study, coaching, and modelling, as well as through associations and clubs dealing with mathematics.

It is relevant to discuss briefly how the three conceptual foci relate to each other. As stated at the beginning, the conceptual foci 1, 2, and 3 represent the category systems. Teacher educators' qualitative ways of seeing MTE evolve as a result of the differences in focusing on the critical aspects. Above all, they constitute part of what teacher educators think and do in MTE. Secondly, the conceptual foci seem to follow a logical sequence from the meanings they assign to MTE, development thoughts and knowledge-sharing strategies. On these grounds, it sounds reasonable to assert that the conceptions of MTE form the basis of teacher educators' pedagogical decision making.

7.2 Research and MTE

In both conceptual focus 2 and conceptual focus 3, mathematics teacher educators were able to revealed potential thoughts which otherwise would have remained on the periphery. This is about teacher educators who recognise MTE as a research field. In this study, it is plausible to state that there is wide recognition, for example, that MTE constitutes teaching methods rather than being a field of study. In Chapter 5, I tried to bring to light the reasons given in support of research-based teacher education (Jakku-Sihvonen & Niemi, 2006; Kynaslahti, Kansanen, Jyrhama, Krokfors, Maaranen & Toom, 2006; Westbury, Hansen, Kansanen & Bjorkvist, 2005). I have nothing really serious against research-based teacher education, except for being wary about thinking of it as 'one-size-fits' all. I am of the view that research-based teacher education is context-specific and it even has influence on the vision of teacher education, as well as MTE. Otherwise it makes a lot of sense that research-based teacher education introduces the value of research and very likely helps teachers to make better pedagogical decisions. In the same way, Carrilo (2001) views research-based teacher education as a new way of seeing how teacher education could be made better. It is known that teaching as telling is no longer appropriate for the knowledge society that needs students who are prepared to solve problems, are adaptable critical thinkers, and digitally literate (Lieberman & Mace, 2008).

Conducting teacher education using practical wisdom may not take us far. Consider, for example, very common activities which frequently happen without much thought about research in Tanzania: reviewing mathematics curricula, preparing teaching and learning materials, and orientation of teacher educators. Engaging in such processes does not guarantee a better teaching and learning of mathematics. To do it better and in a sustainable way is to give research its due weight. In making mathematics research-based, for example, immediate questions would be why do we want teachers to do research? How can we make teachers reflect if that is the essence of working as a teacher and studying at the same time? Sometimes there are no clear-cut solutions to these questions. Moreover, I feel obliged to reflect, and pose better questions in order to make research-based MTE a successful reality.

From many of the studies cited in the previous paragraph, it is possible to see that research in MTE creates a sense of reflection among teacher educators. Research by its very nature make teacher educators to continually reflect on their work. Research-based teacher education implies that the researcher and the teacher is the same person. Further, teaching is essentially a pedagogical decision-making process and research provides the ingredients for better decision making. It is also argued by Carrillo (2001) that processes that aim to integrate theory (research) and practice (teaching) contributes to forming a connection between initial teacher education and in-service training. In addition, they offer a frame of reference for future teachers to work and do research at the same time. In this way they serve as role models to colleagues. Now, if the argument is to place research in MTE at the front and centre, what is the focus of research in MTE? What are the aims and ways of reporting? These questions need specific answers, not necessarily by reinventing the wheel. Mathematics education has already cleared away some of these doubts, and indicated the route to follow (Sierpinska & Kilpatrick, 1998). Perhaps enhancement is what is needed, but this may warrant a study of its own. Some suggestions are given below.

7.3 Research results, implications and addressing the knowledge gaps

Implications for teacher education

The results of this study have several implications for teacher education. To begin with, the findings indicate that current practice in MTE rests on the content versus pedagogy divide. To some extent, this is a reflection of the diverse mathematical background carried by mathematics teacher educators rather than on mathematics knowledge for teaching. To address this sharp distinction, the MTE curriculum needs to be redesigned to take into account a more integrated approach if it is to make an impact. The second implication concerns teacher educators still practising school mathematics in the name of MTE demonstrating a serious gap in the process of teacher preparation. It suggests a need for modelling and coaching them in the form of professional development in order to address this gap. The third implication points to the building of relationships. This is a concern and is a potential task for curriculum developers in mathematics teacher education. Behind phrases like mathematics avoidance-syndrome (shrinking interest), low self-esteem, and stigmatisation of mathematics lie voices calling for the building of relationships as a possible solution. I would also suggest that aestheticism plays a part. This issue is linked with MKT and reflects the curriculum in use. The MTE curriculum is an appropriate entry point in addressing shortcomings concerning mathematics-avoidance syndrome, negative attitudes, low self-esteem and stigmatisation. Mathematics has a profound aesthetic aspect, recognised by many. The argument by Hudson et al (1999), for example, is thought-provoking. The aesthetic nature of mathematics in the eyes of the learner, being hated as well as loved, awful and beautiful, clear and unclear, frustrating and positively challenging, is without doubt interesting. I hold this view and suggest that this could be just one point of entry to examine the various mixed opinions about mathematics. If this is done, then MTE is likely to move towards its rightful place.

The fourth implication concerns the use of mathematics practices which have been proved to work. I have in mind the use of evidence-based practices or research-based approaches in MTE. This has implications for teacher education in important ways. For example, if the decision to implement a competency-based or problem-solving curriculum is proposed, then a new vision of teacher education becomes inevitable. More important is the issue of opening up opportunities for learning mathematics. MTE (pre-service teacher education), mathematics teacher development (in-service) or changes need to be instituted with an emphasis on practices which take into account teacher educators' needs originating from their own hands. The writing of articles and demonstration of best practices through professional learning, collaboration and networking among educators should be options to seriously consider.

Addressing the knowledge gap in MTE

In this study I did not have an opportunity to visit live lessons in MTE. Findings and conclusions are based on coding and analysis of the research materials. Furthermore, I subjected research results to validity and reliability criteria for judgement. Let me use Adler et al´s (2005) argument that at times it is difficult to take a sceptical stance towards the work in spite of all these cautionary measures. There are chances of omission of important questions. To solve this, there is a need to invite external eyes to carry forward some of the emerging research tasks found in the course of discussion.

In view of the assertion above, research in MTE has gradually changed focus from curricula in the 1970s to learners in the 1980s, and now there is an increasing focus on teacher education (Adler, Ball, Krainer, Lin & Novotna, 2005). Viewed from this point of view, and in the context of this study, it has been confirmed that some teacher educators conceive MTE as school mathematics. Yet others view mathematics teacher preparation as a 'once-and-for-all learning activity', rather than a process of continuing to learn during and after qualification. These issues are now seen at practice level. Conventional solutions would very often be to train teacher educators, sometimes without knowing their needs first. I suggest a study to find out what teacher educators need to know as part of MTE (teach about teaching? learn about teaching?). What kinds of strategies would meet their needs? What is the purpose of the knowledge they want to gain? Such a study, if conducted, would likely take note of what is happening in MTE classrooms or find out what teacher educators do. This would augment the findings of this study which did not penetrate into classroom actions.

Finally, as I prepared to submit this research report, I received an official yet personal letter from teacher educators raising the alarm about the professional risks recently taken in teacher education in Tanzania. It makes sense to share the content of the letter written to me by science/mathematics teacher educators as a way of illuminating the size of the problem situation regarding science/mathematics teacher education as raised in the very beginning. Though this letter makes reference to the present science/mathematics curriculum, it has a lot of relevance to the previous curriculum, and at the same time serves as a starting point for future studies. Part of the professional letter affirmed:

> *"..there is a serious mismatch between the present curriculum and both the needs and ability of the students... we are convinced that it would be possible to produce better classroom teachers if we were able to focus on the craft of teaching rather than on issues of little or no relevance to teachers after qualification... we are not saying that academics should not be included. Quite the contrary, but we would insist academic content relate to the actual needs of the teacher. For instance, most students have never understood the fundamentals of the O-level course they will be teaching. They never had the chance to think about it for themselves, to ask questions. Yet the syllabus contains topics more appropriate to a*

> *university course. In the end of year examinations many of our first year students, on straight-forward questions, have scored no more than 1/10 on the academic questions. In our view, this is not a measure of their effort, or ability, but more so represents a score of 9/10 for a mismatch between the curriculum and their real needs".* (Science/mathematics teacher educators, 2009*)*

This statement highlights the problematic situation of science/mathematics teacher education. It adds to the motives of the study pointed out in Chapter 1. In circumstances like this, it is motivating to see people at the implementation level showing a deep concern about a similar problem I have been trying to investigate. In a way, some of the results in this study answer the concerns raised. However, further research would provide an opportunity to investigate in more depth.

Summary of the study

Introduction

This summary provides an overview of research related to teacher educators' conceptions of mathematics teacher education (MTE) conducted in teacher colleges in Tanzania. The study was triggered by the tension between content-oriented ideas as opposed to emphasis on pedagogy among teacher educators, teachers and curriculum developers in Tanzania. A conceptual difference among teacher educators has continued to be challenging, and the associated tension has been translated into a number of concerns worth pointing out.

The first concern of maths teacher educators, school maths teachers and curriculum developers is that of compromising mathematics content at the expense of pedagogical knowledge. There is a criticism that maths content has been compromised by methods of teaching to an unacceptable level under the pretext of enhancing classroom interactions. This demands a fresh look at 'what is' and 'what it means' to become teacher (Wort, Hardman & Mmbando, 2008), and, specifically, a mathematics teacher. Then there is concern over dissatisfaction with students' failure in mathematics at elementary and secondary school level, resulting in mathematics examinations often being in the spotlight (NECTA, 2008).

There is also the issue of the mismatch between the intentions of teacher education and the nature of assessment, as argued in a recent report on assessing the new diploma in the education syllabus (TIE, 2008). The assessment system in MTE did not meet the expectations of an educational assessment relevant to MTE; an excessive school mathematics-related evaluation approach was also of concern. Ideally, assessment is expected to support the 'process of one becoming a teacher'. This is often not the case, as assessment of MTE is subject-matter laden. Of course, as argued elsewhere, content testing is part of the process of making a teacher, because strong pedagogical approaches need to be based on sound subject matter knowledge (Wu, 2005).

Finally, a more striking concern is the decline in the number of student teachers who opt to train as mathematics teachers - from 7,960 (2003) to 3,001 (2006) (NECTA, 2007). Experience indicates that the problem does not end with declining admissions as very often there are signs of stigmatisation, and a mathematics-avoidance syndrome or shrinking interest in teacher education. The key message in this case is that declining admissions may be connected to how MTE is seen by student teachers.

At the global level, views about MTE seem to raise similar questions, with only differences in focus. Adler and Davis (2006) and Roschelle et al (2008), for example, respectively worked on two similar themes regarding how much, and what kind of, mathematics for teaching is appropriate for teachers, and what aspects of mathematics are worth knowing as a foundation for success in mathematics education. I find this is an ideal example of the complex nature not only of mathematics education but also of MTE. Against this view, MTE is regarded as a complex and layered domain, with distinct sites like pre-service, in-service, primary and secondary (Adler, 2005). Being so broad, it is even more challenging in terms of it being a newly emerging field of study and close to being a 'black box'. I think this is taken to mean very little is known in terms of research in the area. On this ground, the domain is in need of a better theoretical framework of teacher learning. In addition, research in this area is dominated by teacher educators studying their own contexts without capacity building in research as part of the teacher education process (Adler, Ball, Krainer, Lin & Novotna, 2005; Lerman, 2001).

In view of local concerns about individual teacher educators, together with the global picture about MTE as a domain of practice, I am of the view that different ideas representing teacher educators' experiences of MTE exist. They seem to have different interpretations, views, impressions and perspectives about 'what is' and 'what it means' to become a mathematics teacher through MTE. The current state of affairs of MTE in Tanzania gives only a glimpse of the differences in understanding, given the evidence and concerns brought to light in the previous discussion. It is these concerns, especially at teacher educator level, about the different ways of seeing MTE and related issues which motivated me to investigate more, as explained in the following section.

Motives for the study

The first motive points to the problematic views of MTE, and in particular the content-pedagogy divide, which is experienced by teacher educators in Tanzania. However, none of the reports and studies cited from within Tanzania indicated the main features of what was debated on this important phenomenon. For example, what does emphasising subject matter or pedagogical content knowledge mean? The background has indicated different interpretations and variations in focus, as well as contradicting views and impressions of each notion, all of which remind me of a related argument by Shulman (1986), who indicated a concern that the teaching of content has rarely been given serious attention. Westbury, Hopmann and Riquarts (2000) took this debate a stage further by indicating that if the teaching of content that is expected to bind everything together is not given attention, the likely result would be the drifting apart of subject matter and pedagogy, and the two would become separate fields.

The second motive is connected to lack of action concerning the attainment of research-based knowledge regarding MTE. Many terms have been used by researchers to indicate under-researching, and MTE has been labelled differently from 'blank sheet' to 'black box' notions (Adler & Davis 2006; Lerman, 2001; Roschelle, Singleton, Sabelli, Pea, & Bransford, 2008). I found this to be an ideal opportunity to reflect more on MTE through research, with the purpose of being an informed practitioner.

The third motive is connected to the desire to reflect more on my personal experience and that of teacher educators, with whom I have been working for more than two decades now. I started as a lower secondary mathematics teacher in 1976, after which I became a mathematics teacher educator at certificate and diploma level for about ten years before taking on other responsibilities, but without losing touch with MTE. It has been natural for me to look for an opportunity to critically reflect on my experience through this study situated in the field of pedagogy, and more specifically on subject didactics concerned with the selection of content, how to teach the selected content, and reasons behind the processes. The desire to reflect more about my involvement in teacher educator development in relation to MTE warrants an examination in the form of a study like this one.

General aim of the study

In view of the background and motives, the overall aim was to identify what teacher educators conceive as MTE, and their thoughts on what they do in the education of teachers of mathematics. In view of the aims, it is expected that the results of this study on conceptions of MTE will provide a deep understanding of the challenges and the various means of teaching and learning mathematics in elementary, secondary and teacher education. There are possibilities to inform other beneficiaries, for example teacher educators, student teachers and curriculum developers. Furthermore, the results of this study might be useful in designing pre-service and in-service courses for primary and secondary school teachers. In important ways, the results of the study will hopefully shed light on possible strategies for solving problems related to, 'low self-esteem' and 'mathematics-avoidance syndrome' - shrinking interest in mathematics. Finally, apart from the practical applications, the study has the potential to contribute to the theoretical knowledge base for the enhancement of practice in teacher education.

Theoretical framework

Inspired by the debate among teacher educators in teacher colleges in Tanzania about what the balance should be between subject matter and methods of teaching mathematics, it was important to look at the theoretical faces of MTE. To achieve this it was important to review the changing focus of MTE within the local context. In so doing, it has also been possible to provide a brief discussion of the teacher education set-up in Tanzania, a treatment of the perspectives of MTE, and professional development and assessment in MTE. In order to have a detailed discussion of what is conceived as MTE, and what teacher educators consider as desirable development, the following tasks were undertaken as part of the theoretical framework. First is a discussion of the trends and shifts of thinking in MTE from the 1960s to the time of the study, and by reflecting critically on the processes, it was possible to see how the present has evolved from the past. This helped to shed light on how the meaning of MTE has been negotiated along the timeframe within Tanzania.

The second task was to investigate the perspectives of MTE, showing it to be a complex and layered domain (Adler, 2005; Adler, Ball, Krainer, Lin & Novotna, 2005). In the process, four perspectives of MTE have been discussed. The perspectives are not to be seen as standpoints of researchers who hold a certain line of thinking, nor is there a one-to-one relationship between researchers and the perspectives. But, before this task was done, it seemed natural to first briefly discuss the relationship between mathematics education and MTE and research in the respective domains. My ambition to discuss the terms under 'one roof' made it necessary to review a number of studies in the two domains (Adler, Ball, Krainer, Lin & Novotna, 2005; Niss, 2007; Ball; 2009; Lin & Cooney, 2001; Lerman, 2001). I also consulted Sierpinska and Kilpatrick (1998) and then pulled together the main features of the two domains. To achieve this it was necessary to develop some criteria which could serve the purpose. The criteria used are: first, features associated with making sense of mathematics education and MTE; second, features related to making sense of research in the two domains, what appears to be common; and finally, challenges regarding the two domains.

In view of the wide range of literature cited above, mathematics education, unlike MTE, is viewed as a domain of research characterised by what scholarly groups do, and craft in order to enhance teaching and learning mathematics. On the other hand, MTE is seen as a domain of practice which is complex and layered in the preparation and development of teachers of mathematics. In the case of research in mathematics education, the object of the research is better defined and easy to see, mostly falling within the teaching of mathematics, learning of mathematics, and related settings. In MTE, one has to work hard to have a vision or a definition of the object of research given the complex nature of MTE. My interpretation of work done by Adler et al (2005), Niss (2007), Ball (2009), Lin and Cooney (2001), Lerman (2001), as well as Sierpinska and Kilpatrick (1998) seems to support that the pragmatic implication of research in the two domains is what binds them together. Of course, mathematics education research goes beyond this to cover theoretical perspectives. Regarding the main challenges in mathematics education as a research domain, there are still differences in the conceptions of the domain concerning the objects and purposes of research questions, especially when mathematicians are involved. On the other hand, MTE is seen as a newly emerging field of study, yet to scale up, and even what has been done is in the context of teacher education, and for this reason it lacks better theoretical frameworks. To compound this, research in MTE is dominated by teacher educators studying their own contexts at the middle of limited research capacity. Without going into detail, I now present one perspective at a time.

In the first and most general of perspectives, MTE is viewed as a composite of many factors, meaning that a combination of factors influences this subject of interest. In a discussion on making sense of MTE from an international perspective, Fou-Lai Lin and Cooney (2001), Jaworski (1998), and Jaworski, Wood and Dawson (1999) are of the view that there is growing interest in research in this domain. The complex and layered nature of MTE, as discussed by Adler (2005), means that attempts to analyse this domain must consider a host of other factors, which include curriculum-making, mathematics education, and the broad field of teacher education and its traditional linkage with mathematics. In addition, and by the same researcher, it also makes sense to think of MTE as a field of distinct sites, which include pre-service and in-service teacher education to respectively match the notions commonly applied as 'preparation' and 'professional development'.

In the second perspective, MTE is viewed as 'the process of becoming' a teacher, which in this study means becoming a mathematics teacher. To substantiate this perspective, Garcia et al (2006) argue that the process of becoming a primary school teacher, for example, may be understood as the process of being introduced into the community of practising teachers. In this way, learning to teach is seen as the beginning of the use of conceptual and technical tools in carrying out professional tasks, whereby the term 'conceptual tools' refers to concepts and constructs which have been generated from research in teaching mathematics. Along the same lines, the term 'technical tools' refers to tools used in 'practice' and may include teaching materials, software, techniques for managing discussions, procedures and answers to problems. I find this to be quite a different way of understanding not only teacher education but also MTE.

In the third perspective, which is one of the most common*, MTE is viewed as a combination of pedagogical knowledge and subject matter knowledge* (Attorps, 2006; Bass, 2005; Bullough Jr., 2001; Lester & Lambdin, 1999; Niss, 2007; Shulman, 1986; Shulman, 1987). In a discussion about competencies of mathematics teachers and how they should be developed, Niss (2007) reminds us of the longstanding conception of teachers of mathematics as persons who know concepts, facts, results, rules, and methods on the one hand (theory), and how to put the lesson across on the other (practice). The necessary competencies of a teacher are thus a demonstration of subject matter knowledge and general pedagogy. Shulman (1983), for example, argues for pedagogical content knowledge (PCK) in the sense that there is a particular form of content which embodies the aspects most relevant to teaching, and the methods relevant to teaching a specific subject. Shulman (1987) expanded this argument in an attempt to make a case for teacher professionalism, whereas Bass (2005) makes a case for 'mathematical knowledge for teaching'. In the same way, Lester and Lambdin (1999) argue for a combination of content, pedagogical knowledge and some school-based practice, and view MTE as a practical way of guiding teachers' actions, which leads to the level of subject didactics, where the focus is on what content to teach, how and why.

*In the fourth perspective, MTE is viewed as learning about teaching***.** That is, mathematics teacher educators have two main tasks, one is to teach about teaching mathematics and the second is learning about teaching mathematics. While the first task may be regarded as exclusively the work of teacher educators, the second task also engages mathematics student teachers. Thus, they are in the process of learning about teaching, and this continues even after qualification. In addition, Loughran (2006) emphasised that this is an important view yet to be explored and the research knowledge gained to be put to good use. Though its main focus is on what teacher educators do, it also reflects a lot on the process of learning how to teach mathematics.

Methodological solutions

This part of the study deals with the methods of inquiry. It covers the research questions, research design, subjects of the study, data collection techniques, coding process and data analysis. It is important to note that some researchers may be interested in finding out what reality is like and why, while others may want to focus on what kind of conceptions individuals have of a given object (Marton, Beaty, & Dall'Alba, 1993). This study sought to identify teacher educators' conceptions of MTE and what they consider as desirable development and provides a description of the patterns of ideas. To achieve this, two questions were asked, to which no clear solutions satisfactorily existed. First, what are teacher educators' conceptions of MTE, and second, what are teacher educators' thoughts on the development of MTE? The nature of the two research questions invites impressions of MTE from a second order perspective. This is what led to the choice of phenomenography as the methodological approach. Further, the guiding questions demands responses which are largely qualitative.

The process of data collection involved interviewing 27 maths teacher educators, whose statements were coded and analysed. Further, from the same teacher educators (five additional teacher educators responded, leading to a total of 32 participants), written statements from an open-ended questionnaire intended to identify desirable development of MTE were also coded and analysed. The data was collected between August, 2006 and March, 2008. Coding and data analysis were done in two phases in the order of the research questions. The entire process generated qualitatively different ways of seeing MTE, and what teacher educators consider as desirable development of MTE. The results are briefly described. Unless otherwise stated, the numbers in brackets represent the proportion (variations) of teacher educators who belong to the respective category of description.

Results

In answer to *research question one*, a detailed analysis revealed seven qualitatively different conceptions of MTE. The first category of description relates to MTE as a process of learning via investigation (4 of 27). Key aspects used to explain or illuminate the category of description as revealed by teacher educators' responses were creativity, discovery, activity-based learning, and inquiry into teaching and learning mathematics. The second category of description refers to MTE as a process of inspiration in learning (2 of 27). Inspiration as a category stands on its own because its ultimate aim is to cultivate interest and motivate student teachers in MTE through stimulation, amusement, enterprising activities, as well as use of puzzles. It is thought to be a solution to address issues of phobias, negative attitudes, low self-esteem, and mathematics-avoidance syndrome or ‘shrinking interest’. The third category of description strongly pointed to MTE as a process of learning with a focus on problem-solving (3 of 27). It is distinguishable from others when aspects which illuminate it are considered, and these are the application of mathematics knowledge to practical situations and solving real life problems.

The fourth category of description emanates from MTE having a focus on pedagogical knowledge and skills (6 of 27). This conception is rooted in methods/strategies of teaching and learning among teacher educators. Furthermore, MTE is seen as a didactical process of teaching and learning mathematics (1 of 27). Some aspects which have been used to illuminate the didactics of mathematics, and therefore qualify it to stand on its own, are what to teach, instruction, study about teaching and learning and the actions which teacher educators take (how) in the course of teaching. In addition, MTE was also seen as a process of teaching and learning with emphasis on subject matter (2 of 27), and it is exemplified by aspects like subject matter knowledge and solid mathematics. Finally, MTE is viewed as a process of learning with emphasis on integration of subject matter, and pedagogical knowledge and skills (9 of 27). In a strict sense, MTE is understood as an organised combination between MKT (mathematics knowledge for teaching) and subject matter knowledge.

Research question two generated two category systems, with qualitatively different ways of seeing the development of MTE. In the first place, development of MTE is seen as as a process with an emphasis on pedagogical knowledge and skills (15 of 32 conceived it this way). A few of the aspects to illuminate this category of description are classroom interactions, preparation and use of teaching and learning materials, team teaching, and pedagogical reflection. The second category of description refers to the development of MTE with a focus on subject matter knowledge and skills (5 of 32). In this category, teacher educators conceived MTE development as attaching special importance to subject matter knowledge and skills. For them, knowledge of MTE development refers to conceptual and procedural knowledge, and in addition they thought that it is natural first to think of what content to teach than what method to use. The third category of description relates development of MTE as a process integrating the results of assessment (3 of 32). In this category, teacher educators associated the development of mathematics teacher education with the need to take into account assessment in order to support learning. Significantly, the development of MTE was also seen as a matter of building relationships (7 of 32). In this category, teacher educators expressed ideas on developing MTE which involved building relationships among student teachers. Other aspects which make this category of description distinguishable were interactions between student-tutor, also going beyond human relations to cover learning materials (textbooks), relationships within topics and between disciplines. Finally, MTE was also seen as the studying of teaching and learning contexts (2 of 32). In this, teacher educators thought that the development of MTE involves emphasis on research on teaching and learning situations. Aspects which illuminate this category are the technique of identifying problem areas, concerns about teaching and learning mathematics, motives, and conducting surveys.

The question on how teacher educators shared knowledge and skills revealed patterns of interest. First, it was conceived that knowledge and skills in MTE were shared through neighbourhood learning groups (21 of 32 indicated this view). In this category, teacher educators conceived neighbourhood learning groups as a nucleus of professional development sessions and a sustainable strategy for sharing ideas. The aspects to illuminate the category of description include neighbourhood learning, ranging from seminars, workshops, and in-house capacity building, to mentoring and team-teaching. Second, sharing knowledge and skills through collaboration and networking was identified as another category of description (11 of 32). The central issue is how do teacher educators take forward and share development ideas in mathematics? In a strict sense, it is a discussion about professional development and strategies for making it happen. The use of ICT as a tool for teaching and learning, article newsletters and module writing further explained the category of description.

Discussion of research method and results

The choice of methodological approach is argued for first because of its centrality in the study. Phenomenography was chosen as the methodological solution. In addition, a self-assessment of the trustworthiness of the research results is made by looking at how the issues of validity (credibility) and reliability (dependability) have been addressed. Finally, I draw some conclusions and implication for teacher education.

The main principle of choosing phenomenography as a research approach is determined by the need for it to reflect the research questions. This, in turn, means conducting the investigation in terms of how a phenomenon is conceived from a second-order perspective. This also follows from the argument that it is the concrete research problem or aim rather than a fixed position of either being qualitative or quantitative which determines the study approach (Niglas, 2004). This is what was kept in mind.

Regarding phenomenography as a research approach and its associated notion 'conception', I find myself obliged to point out some of the criticisms pointed at phenomenography. Säljö (1994) for instance, strongly criticised phenomenography as a research approach in neglecting the participants' context. Because of the interest in conceptions, phenomenographers have been reducing what is stated by 'participants' to isolated statements, and 'siphoning off' conceptions, without taking into account the contexts in which they have been constructed. The proponents of phenomenography, specifically Marton (1995), refuted this, and argued that *learning and thinking* are context-based. I find Marton's argument strong, and I cannot personally think of a learning situation which is devoid of context. In this study, I took into consideration the natural setting of the interview to avoid the exercise being reduced to teacher educator assessment. This made it possible for the interviewees to express themselves in an environment they had good control over, including the use of mathematics teaching and learning materials posted on their office walls. Further, Åkerlind (2005) reminds us that in phenomenography the aim and the outcome is to explore a range of meanings within a sample group, as a group, not a range of meanings for each individual within the group, which means that no interview statements, for example, can be understood in isolation from others. This appeared to me as a fundamental principle very often misunderstood by the critics.

The second criticism casts doubt on whether the conceptions or categories of description really reflect the content of the interviews (Francis, 1993). Of course, this is a validity issue, and to address it one may ask, for example, how much interpretation of teacher educators' collected statements needs to be done to avoid distortion. There are two views on data treatment (Strauss & Corbin, 1990). One view is to let the data speak for itself without any reduction or rephrasing. Other researchers are concerned with accurate descriptions, but given the difficulties in using all the interview material, it becomes necessary to reduce it, which actually involves selection and interpretation. This is at the heart of qualitative studies and what guides the principle of reduction. My immediate reaction is what kind of scientific investigation that has no interest in precision? I am of the view that generating conceptions, and handling interview words and the research material can be managed skilfully to minimise some of the doubts and contradictions being claimed.

The third criticism is related to the challenge that most researchers using phenomenography work individually and are confined during data analysis (Burns, 1994; Åkerlind, 2005). They argue that there is an opportunity to open up very late towards the end of the study. The researchers argue for bringing in additional researchers in order to make the data collected open to challenge from the beginning rather than waiting until the end. Their second reason is the potential to have an even better outcome space because of the greater open-mindedness and awareness of alternative perspectives. It is further argued that though an individual researcher can make a substantial contribution to the understanding of a phenomenon, group research might have the potential to take that understanding further. Again, it is not only an issue of researcher competence in making a valid interpretation, but also in making an unbiased interpretation. To address this challenge, I engaged former practising mathematics teacher educators (research assistants) in the process of developing draft and agreed categories of descriptions.

In the first research question, the results indicate that teacher educators conceived MTE and what they consider as desirable development in qualitatively different ways. How they came to conceive MTE and related development in different qualitative ways is linked to their own diverse background and emphasis. One possible explanation for seeing MTE in qualitatively different ways can be associated with differences in focus of the critical aspects (Pang, 2003). Various studies link prior knowledge and learning, and the longstanding pedagogy by Ausubel (1978) is one source in which there is a strong argument that pre-conceptions influence present learning. Teacher educators who were asked to reveal their conceptions about MTE and thoughts on development are in the final analysis learners, and therefore not unique in this particular case. Evidence to support pre-conceptions includes statements from teacher educators like '*I learnt portfolio assessment from institutions outside Tanzania*' or '*I attended a seminar on inspiration mathematics*'. This is a clear indication of carrying personal previous experience in the form of ideas into MTE, and perhaps a better response to critics who argue that phenomenography is interested in siphoning-off conceptions (Säljö, 1994) with a total disregard for context. In this case, the respondents revealed the source and context of conceptions and thoughts on the development of MTE, as a clear indication of situated practice.

The second comment on the results is based on the permanence of the conceptions of MTE, desirable thoughts on development and strategies for sharing knowledge and skills. These qualitative ways of seeing the phenomenon reflect what temporarily dominated during the material time of the study. I am of the view that conceptions can be context and time sensitive. A number of studies have been done to bring this argument to light. Marton's (1995) view on learning as a situated practice is a logical argument. For this reason, it may not be plausible to generalise much about the research results, as the findings are situational.

The third comment concerns the status of teacher educators' conceptions of MTE. The responses indicate differences in views, not strictly as 'narrow' or the ability to 'dig deep', but as elaborative or compact. The ambition from the start was not to test how knowledgeable they are, but rather to study mathematics teacher educators' conceptions through their eyes and be able to provide a description of the variations. This has been achieved, and I have realised that they are at different levels of reflection on what MTE is.

The fourth comment reflects the connection between the results and the existing theoretical framework. The results indicate that one has to work hard in order to establish shared conceptions of MTE, and this has been observed earlier about the complex nature of MTE (Adler, 2005). The same situation exists for mathematics education (Bass, 2005; Ernest, 1998; Lerman, 2001; Lester & Lambdin, 1999; Mura, 1998; Niss, 1998; Wittmann, 1998). Following these broad comments on the research results, I still find it necessary to conduct a brief discussion on the issue of validity and reliability in terms of credibility and dependability respectively.

Credibility (internal validity) can be addressed by using different sources of data collection techniques (*triangulation*), as argued by Denzin (1970). Specific to this study, I recorded interviews, taking notes and focusing on key aspects of the research question to crosscheck the two ways of taking statements. Equally important, the open-ended questionnaire for research question two was supported by an immediate follow-up interview with five (5) teacher educators in order to crosscheck the authenticity of the data and clarification of some areas. The questionnaires allowed teacher educators to state their ideas in ways not pre-selected by any interested person and their use revealed hidden ideas or unsuspected answers, as well as enabling the respondents to challenge some ideas normally taken for granted. Secondly, I engaged three research assistants who independently coded half of the teacher educator statements on their understanding of MTE, discussed them in order to *agree* on the categories of descriptions, and then I continued the process of completing the rest of the statements. The purpose was to make the data open to challenge and minimise bias (Ary, Jacob & Razanieh, 2002). The second level involved the use of a peer in order to critically examine and validate the categories, creatively putting the categories in a more distinct way, rather than right or wrong matching as is often the case of a co-judge in phenomenography. Why was this necessary? The coding and categorisation process had been a long and demanding exercise. Some factors might have intervened during the writing process. Within this strategy it was also important to constantly reflect on my own actions in order to avoid bias as a way of addressing internal validity, for example, by selecting educators as respondents from different geographical locations.

In relation to reliability, one concern is to explain how one could build confidence on what has been found. First, there is a complete description of the study conduct from Phase I of testing the instruments in August 2006, to data collection for research question one in December of the same year. This was followed by pilot-testing of the instrument for research question two in March 2007, and data collection in December 2007. This sequence of research tasks, combined with triangulation of data collection procedures, is the first stage in ascertaining the reliability of the research results. Secondly, the reliability of the research results was addressed by using participants from different geographical locations to allow for variability of ideas. To achieve this, I collected data from a heterogeneous sample of teacher educators working in certificate and diploma teacher colleges, as these are staffed by teacher educators of different qualification (diploma, university graduates).

Beyond validity and reliability, it is natural sometimes to think of generalisation. The issue of generalisation is problematic because in important ways the results are situational. There are questions regarding the 'same participants', and 'same context' for human behaviour, which is difficult to attain. As mentioned previously, the purposeful selection of subjects of the study was not for generalisation. Purposeful selection of respondents in this case means teacher educators who met the intentions of the study and were chosen on the basis of principles. The representativeness is, however, satisfactory for using the results of this study to learn about other situations in Tanzania. Generalisation (transferability) of the research findings outside Tanzania has been addressed by a careful description of the national and local conditions, and by the quotes that exemplify the conceptions and thoughts in Tanzania, and some might correspond to situations in other countries with similar problems in MTE.

Concluding thoughts

To reach the climax of a research process like this one is an inspiration. Naturally, the research process conveys a message for the need to seriously reflect on the results and their implications for teacher education. The results of this study follow from the research questions about MTE, for which no answers existed at the beginning, which allowed for the development of methodological solutions to guide the investigation on teacher educators' conception of MTE, thoughts on desirable development, and knowledge-sharing strategies of the same. I associate this process with a critical path over which I always kept a tight control in order to remain within the stream of the study of the research questions. It now makes sense to look back and reflect on the study by way of concluding thoughts.

In order to position the intentions of this stduy, it was important to review relevant literature to produce a theoretical framework, for which the criteria of choice and justification include the changing focuses of MTE in Tanzania in order to unearth the meanings assigned to MTE in a given timeframe. This was intended to shed light on MTE and as a point of departure for the discussion of the present MTE. Within the theoretical framework several faces or perspectives of MTE have been brought to light and discussed. The main ones are MTE viewed as a composite of many influences, a process of one becoming a mathematics teacher and as a blend between subject matter and pedagogical knowledge. Finally, MTE can be viewed as learning about teaching. Working on the assumption that teacher educators are 'forward looking'; I made an attempt to explore 'thoughts' about the development of MTE. This was based on the assumption that mathematics teachers are learners after qualification.

A comprehensive picture (framework) is thus proposed to take into account teacher educators' conceptions of MTE, desirable development, and knowledge-sharing strategies of the same. The proposed comprehensive representation has been developed from the research results and the theoretical framework. It pulls together the three category systems and their corresponding categories of descriptions to form a comprehensive picture of teacher educators' qualitative ways of viewing MTE. Further reflection of the research question is represented in a picture of the inbuilt dynamism of the research process. One may be able to see the flow of teacher educators' ideas from their formation (conceptions of MTE), to how to put them into action (desirable development of MTE), and finally, how to share knowledge (strategies) with colleagues.

In view of the preceding paragraph, it was plausible to draw a few conclusions, as follows: teacher educators exhibit sharp conceptual variations concerning MTE in qualitatively different ways as a result of their diverse historical backgrounds; the sharp conceptual variations are possible grounds for differences in making pedagogical decisions, while at the same time telling about MTE as it is; and some mathematics teacher educators see mathematics as an unquestionable field of knowledge. Further, the integration of subject matter and pedagogical knowledge is the dominant conception of MTE, and teacher educators' thoughts on professional development gravitate mostly around pedagogical knowledge and skills. Teacher educators' thoughts on how to share knowledge and skills in MTE indicate various strategies in use, and are grounded in college-based neighbourhood learning groups, and in distance collaboration and networking.

In this study I did not have the opportunity to visit live lessons in MTE to see, for example, what is called learning mathematics through investigation. Findings and conclusions are based on coding and analysis of the research material. To reach this stage is an investment in terms of critical thinking and time as a resource, as well as other forms of resources. Let me use the argument of Adler et al (2005) that at times it is difficult to take a sceptical stance towards the work in spite of all the precautionary measures taken. There are chances of omitting important questions. To solve this, there is a need to invite external help to carry forward some of the emerging research tasks found in the course of discussion. I suggest a study to find out what teacher educators need to know as part of MTE. What is the purpose of the knowledge they want to gain? What kinds of strategies would meet their needs? Such a study, if conducted, would likely take note of what is happening in MTE classrooms, or simply find out what teacher educators do. This would augment the findings of this study, which did not penetrate into classroom action.

Svensk sammanfattning

Inledning

Avhandlingens titel lyder på svenska ”Uppfattningar av utbildningen av matematiklärare. Tankar bland lärarutbildare i Tanzania.” Denna sammanfattning ger en översikt över forskning som berör lärarutbildares uppfattningar av utbildning av matematiklärare och som bedrivits vid lärarhögskolor i Tanzania. Studien har sitt ursprung i en spänning som förekommit mellan innehållsorienterade uppfattningar och betoning av pedagogik bland lärarutbildare, lärare och läroplansutvecklare i Tanzania. Skillnaderna i uppfattningar bland lärarutbildarna har inneburit en kontinuerlig utmaning, och den spänning som kan förknippas med dem har här uttolkats i ett antal angelägna problem som är värda uppmärksamhet.

Det första bekymret för lärarutbildare i matematik, lärare i skolor och läroplansutvecklare gäller avkall på det matematiska innehållet till förmån för pedagogisk kunskap. Det har riktats kritik mot att man har kompromissat med det matematiska innehållet till en oacceptabel nivå under förevändningen att man stärker växelverkan i klassen. Detta kräver en förnyad granskning av ”vad det är” och ”vad det betyder” att bli lärare (Wort, Hardman & Mmbando, 2008), och närmare bestämt matematiklärare. Sedan bekymrar man sig över ett utbrett missnöje med att elever misslyckas i matematik i skolan på första och andra utbildningsstadiet, vilket resulterat i att examensprov i matematik ofta befinner sig i rampljuset (NECTA, 2008).

Det är också en fråga om missanpassning mellan lärarutbildningens intentioner och utvärderingens natur, såsom det hävdas i en nyutkommen rapport om utvärderingen inom läroplanen för det nya lärarutbildningsdiplomet (TIE, 2008). Utvärderingssystemet i matematiklärarutbildningen (här används förkortningen MTE för ”Mathematics teacher education”) kunde inte möta förväntningarna på en pedagogisk utvärdering som är relevant för MTE; en överdriven utvärdering med tonvikt på skolmatematik var också ett bekymmer. I en idealsituation förväntas utvärderingen stöda ”processen som leder till att man blir lärare”. Detta är ofta inte fallet, då utvärderingen inom MTE har sin tyngdpunkt på ämnesinnehåll. Naturligtvis är ämnesprov, som det hävdas i andra sammanhang, en del av processen genom vilken man blir lärare, eftersom starka pedagogiska angreppssätt måste basera sig på säker ämnesmässig kunskap (Wu, 2005).

Slutligen utgörs ett mera slående bekymmer av nedgången i antalet lärarstuderande som väljer att utbilda sig till matematiklärare – från 7960 (år 2003) till 3001 (år 2006) (NECTA, 2007). Erfarenheten visar att problemet inte tar slut i och med att antalet antagna går ned, eftersom det ofta finns tecken på stigmatisering och ett syndrom med undvikande av matematik och avtagande intresse för lärarutbildning. Det centrala budskapet är i detta fall att nedgång i antagningen kan vara förknippad med hur MTE uppfattas av lärarstuderande.

På en global nivå verkar uppfattningar av MTE ha liknande drag, med skillnader endast i fråga om fokus. Adler och Davis (2006) respektive Roschelle et al. (2008) arbetade till exempel med två snarlika teman som gällde hur mycket och vilket slag av matematik för undervisning som är lämpligt för lärare, och vilka aspekter av matematik som är värda att känna till som en grund för framgång inom matematikutbildning. Jag ser detta som ett idealiskt exempel på hur komplex inte bara matematikutbildning utan också MTE är. Mot denna bakgrund är MTE ett område som är komplext, består av olika lager och hör hemma på olika ställen, såsom grundutbildning av lärare, lärarfortbildning och stadieinriktning mot första eller andra stadiet (Adler, 2005). Så brett som det är, är det ännu mer en utmaning genom att det är ett nyligen framvuxet forskningsområde och inte långt ifrån en ”svart låda”. Enligt mig avses med detta att ganska litet är känt i fråga om forskning på området. Mot den bakgrunden är området i behov av bättre teoretiska ramar för hur lärare lär sig. Det kan tilläggas att forskningen på området domineras av lärarutbildare som studerar sin egen kontext utan att det byggs upp forskningskapacitet som en del av lärarutbildningen (Adler, Ball, Krainer, Lin & Novotna, 2005; Lerman, 2001).

Med tanke på de lokala bekymmer som gäller individuella lärarutbildare, tillsammans med den globala bilden av MTE som en praktikdomän, existerar det enligt mig olika uppfattningar som representerar lärarutbildares erfarenheter av MTE. De förefaller ha olika tolkningar, synsätt, intryck och perspektiv i fråga om ”vad det är” och ”vad det betyder” att bli matematiklärare genom MTE. Det nuvarande tillståndet när det gäller MTE i Tanzania ger bara en liten inblick i olikheterna i fråga om förståelse, med beaktande av den evidens och de bekymmer som ovan anförts. Det är dessa bekymmer, särskilt de som lärarutbildarna själva har, i fråga om de olika sätten att se på MTE och närliggande frågor, som motiverade mig till ytterligare undersökningar.

Motiv för studien

Det första motivet pekar i riktning mot de problematiska uppfattningarna av MTE och spccifikt skiljelinjen mellan ämnesinnehåll och pedagogik, som lärarutbildare i Tanzania upplever. Emellertid anger ingen av de rapporter och undersökningar som man hänvisar till i Tanzania vilka huvuddragen är i det som debatteras om detta viktiga fenomen. Vad betyder det till exempel att betona ämnesinnehåll respektive pedagogisk ämneskunskap? Tecknandet av bakgrunden har påvisat vissa tolkningar och variationer i fråga om fokus, liksom även motsägelsefulla uppfattningar och intryck av vartdera begreppet, vilket sammantaget påminner mig om ett liknande argument av Shulman (1986), som uppgav sig vara bekymrad över att ämnesundervisningen sällan har uppmärksammats ordentligt. Westbury, Hopmann och Riquarts (2000) förde denna debatt ett steg vidare genom att antyda att om man inte uppmärksammar undervisandet av sådant ämnesinnehåll som håller allting samman så är det sannolika resultatet att ämnesinnehållet och pedagogiken driver ifrån varandra, och de två blir separata områden.

Det andra motivet är sammankopplat med handlingsförlamningen när det gäller uppnående av forskningsbaserad kunskap om MTE. Forskare har uttryckt sig på många olika sätt när det gäller bristen på forskning, och MTE har karakteriserats på olika sätt från ”ett oskrivet blad” till ”en svart låda” (Adler & Davis 2006;

Lerman, 2001; Roschelle, Singleton, Sabelli, Pea, & Bransford, 2008). Jag fann det vara ett idealiskt läge att via forskning reflektera mera över MTE med syftet att vara en välinformerad praktiker.

Det tredje motivet har att göra med min önskan att reflektera mera över min personliga erfarenhet och erfarenheterna som sådana lärarutbildare har med vilka jag samarbetat i mera än två decennier. Jag började min bana år 1976 som lärare i det andra stadiets lägre årskurser, varefter jag blev lärarutbildare i matematik på certifikat- och diplomnivån i ungefär tio år innan jag tog över annat ansvar, men utan att jag förlorade kontakten med MTE. Det har varit naturligt för mig att söka ett tillfälle att kritiskt reflektera över min erfarenhet genom denna studie inom det pedagogiska fältet, och närmare bestämt över ämnesdidaktik som handlar om urvalet av ämnesinnehåll, hur man undervisar det utvalda ämnesinnehållet och de orsaker som ligger bakom processerna. Som jag ser det förutsätter min önskan att reflektera mera över mitt engagemang i utvecklandet av lärarutbildningen i förhållande till MTE att jag gör en undersökning i form av en studie som denna.

Studiens allmänna syfte

Med beaktande av bakgrunden och motiven var det övergripande syftet med studien att identifiera vad lärarutbildare uppfattar som MTE och deras tankar om vad de gör i samband med utbildningen av matematiklärare. Med hänvisning till syftena är det min förhoppning att resultaten av denna studie av uppfattningar av MTE ska ge en djup förståelse av de utmaningar som finns och de metoder som finns till hands när det gäller undervisning och lärande av matematik på första och andra stadiet och inom lärarutbildningen. Förhoppningen är att andra ska kunna dra nytta av informationen, till exempel lärarutbildare, lärarstuderande och läroplansutvecklare. Vidare kan resultaten av studien vara nyttiga vid design av kurser inom grundutbildningen och vid fortbildningen av lärare för första och andra stadiet. Resultaten av studien kommer förhoppningsvis också att på viktiga sätt belysa möjliga strategier att lösa sådana problem som hänför sig till ”låg självkänsla” och ”undvik-matematiken-syndromet” – minskande intresse för matematik. Slutligen har studien vid sidan av sina praktiska tillämpningsmöjligheter en potential att bidra till den teoretiska kunskapsbasen för stärkande av praktiken i lärarutbildningen.

Teoretiska ramar

Med inspiration från debatten bland lärarutbildare vid lärarhögskolorna i Tanzania om vilken balans som behövs mellan ämnesinnehåll och metoder för undervisning i matematik var det viktigt för mig att se vilket teoretiskt ansikte MTE visar utåt. För att åstadkomma detta var det viktigt att överblicka hur fokus för MTE har förändrats i den lokala kontexten. Därvid har det också varit möjligt att tillhandahålla en kort beskrivning av hur lärarutbildningen är anordnad i Tanzania samt en behandling av de teoretiska perspektiven i MTE och av professionell utveckling och utvärdering inom MTE. För att åstadkomma en detaljerad diskussion av vad som uppfattas som MTE och vad lärarutbildare anser vara behövlig utveckling åtog jag mig följande uppgifter som en del av de teoretiska ramarna. För det första diskuterade jag trenderna och förskjutningarna i tänkesättet inom MTE från 1960-talet fram till den tid då undersökningen företogs, och genom kritisk reflektion över processerna var det möjligt att se hur det närvarande hade utvecklats ur det förflutna. Detta bidrog till att belysa hur man inom denna tidsram hade förhandlat om betydelsen av MTE i Tanzania.

Den andra uppgiften var att undersöka perspektiv inom MTE, varvid MTE visade sig vara en komplex domän som bestod av olika lager (Adler, 2005; Adler, Ball, Krainer, Lin & Novotna, 2005). Under denna process diskuterades fyra perspektiv inom MTE. Dessa perspektiv ska inte ses som ståndpunkter för forskare som ansluter sig till ett bestämt tänkesätt, och det finns inte heller en ett-till-ett-relation mellan forskarna och perspektiven. Men innan denna uppgift var utförd tedde det sig naturligt att diskutera relationen mellan matematikutbildning och MTE samt forskning inom de bägge domänerna. Min ambition att diskutera termerna under ”samma tak” gjorde det nödvändigt att granska ett antal studier inom de båda domänerna (Adler, Ball, Krainer, Lin & Novotna, 2005; Niss, 2007; Ball; 2009; Lin & Cooney, 2001; Lerman, 2001). Jag konsulterade också Sierpinska och Kilpatrick (1998) och sammanställde sedan de viktigaste dragen hos de båda domänerna. För att åstadkomma detta var det nödvändigt att utveckla några ändamålsenliga kriterier. De som användes är, för det första, sådana drag som kunde förknippas med att göra MTE begriplig; för det andra, sådana drag som hade att göra med att göra forskning på de två områdena begriplig, vad som verkar vara gemensamt; och slutligen, utmaningar beträffande de båda områdena.

Med beaktande av den bredden i den litteratur som hänvisas till ovan ses matematikutbildning, i motsats till MTE, som en forskningsdomän som kännetecknas av vad forskare gör, och som en konst som ska stärka undervisningen och lärandet av matematik. Å andra sidan ses MTE som en praktikdomän som är komplex och består av olika lager inom matematiklärares utbildning och utveckling. När det gäller forskning om matematikutbildning är forskningsobjektet lättare att se, och det faller ofta inom undervisning och lärande av matematik. I fråga om MTE måste man anstränga sig för att få en klar bild eller en definition av vad forskningsobjektet är, om den komplexa naturen hos MTE tas för given. Min tolkning av arbete som utförts av Adler et al. (2005), Niss (2007), Ball (2009), Lin och Cooney (2001), Lerman (2001) samt Sierpinska och Kilpatrick (1998) verkar stöda den pragmatiska implikationen att det är forskning inom de båda områdena som binder dem samman. Naturligtvis är forskning om matematikutbildning bredare än så och omfattar teoretiska perspektiv. Vad beträffar de viktigaste utmaningarna för utbildningen i matematik som ett forskningsområde, finns det fortfarande delade uppfattningar om forskningsobjekt och forskningsfrågornas syften, särskilt när matematiker är inblandade. Å andra sidan ses forskning om MTE som ett nytillkommet forskningsområde, som ännu inte hunnit växa till sig, och det som har åstadkommits har utförts inom ramen för lärarutbildning och saknar därför bättre teoretiska ramar. Därtill domineras forskningen om MTE av lärarutbildare som med begränsad forskningskapacitet studerar sin egen kontext. Utan att gå in på detaljer, presenterar jag här ett perspektiv i taget.

I det första och mest allmänna perspektivet ses MTE som sammansatt av många faktorer, vilket betyder att en kombination av faktorer påverkar det ämne man intresserar sig för. I en diskussion om hur man kan göra MTE begriplig ur ett internationellt perspektiv hävdar Fou-Lai Lin och Cooney (2001), Jaworski (1998), och Jaworski, Wood och Dawson (1999) att intresset för forskningsområdet håller på att växa. Att MTE enligt Adler (2005) är komplext och består av lager betyder att försöken att analysera domänen måste beakta en mängd andra faktorer, vilka inbegriper uppgörande av läroplaner, matematikutbildning samt det breda fältet lärarutbildning med dess traditionella länkar till matematiken. Ytterligare är det enligt samma författare förnuftigt att se på MTE som förlagd till situationer som skiljer sig från varandra. Dessa inkluderar grundutbildning och fortbildning av lärare, vilka motsvarar ”förberedelse” och ”professionell utveckling”.

I det andra perspektivet ses MTE som ”processen att bli” lärare, vilket i denna studie betyder att bli matematiklärare. Detta perspektiv kan ges substans – Garcia et al. (2006) hävdar att processen att bli till exempel lågstadielärare kan förstås som processen att introduceras i de utövande lärarnas samfund. Att man lär sig undervisa uppfattas därmed så att man börjar använda begreppsliga och tekniska redskap då man utför yrkesuppgifter. Termen ”begreppsliga redskap” hänvisar till begrepp som genererats ur eller specialkonstruerats inom forskning om undervisning i matematik. I enlighet med detta hänvisar ”tekniska redskap” till redskap som används inom ”praktiken” och de kan inkludera undervisningsmaterial, mjukvara, tekniker för att leda diskussioner, olika tillvägagångssätt och svar på problem. Jag uppfattar detta som ett helt annorlunda sätt att se på inte bara lärarutbildning utan också MTE.

I det tredje perspektivet, som är ett av de vanligaste, *ses MTE som en kombination av pedagogisk kunskap och ämneskunskap* (Attorps, 2006; Bass, 2005; Bullough Jr., 2001; Lester & Lambdin, 1999; Niss, 2007; Shulman, 1986; Shulman, 1987). I en diskussion om kompetenser hos matematiklärare och hur de borde utvecklas påminner oss Niss (2007) om den väletablerade uppfattningen av matematiklärare som personer som behärskar begrepp, fakta, resultat, regler och metoder å ena sidan (teori) och hur man får en lektion att gå fram å den andra (praktik). De nödvändiga lärarkompetenserna visar därmed på ämneskunskap och allmän pedagogik. Till exempel Shulman (1983) argumenterar för pedagogisk ämneskunskap i den betydelsen att det finns ett särskilt innehåll som förkroppsligar de aspekter som är mest relevanta för undervisning och de metoder som är relevanta för undervisning av ett visst tema. Shulman (1987) utvidgade argumentationen i ett försök att tala för professionalism inom läraryrket, medan Bass (2005) förespråkade ”matematikkunskaper för undervisning”. På samma sätt argumenterar Lester och Lambdin (1999) för en kombination av innehåll, pedagogisk kunskap och ett visst mått skolbaserad praktik och ser MTE som ett praktiskt sätt att styra vad lärare gör, vilket leder till nivån ämnesdidaktik där fokus är på vilket innehåll som ska undervisas, samt hur och varför.

I det fjärde perspektivet uppfattas MTE gälla att lära sig om undervisning. Detta innebär att lärarutbildare i matematik har två huvuduppgifter. Den ena är att undervisa om hur man undervisar i matematik och den andra är att lära sig om hur man undervisar i matematik. Medan den första uppgiften kan ses uteslutande som lärarutbildarnas uppgift, engagerar den andra uppgiften också dem som studerar för att bli lärare. Dessa studerande befinner sig alltså i en process där man lär sig om undervisning, och den fortsätter även efter det att man uppnått sin lärarbehörighet. Loughran (2006) betonade att detta är ett viktigt perspektiv inom vilket man borde forska vidare och sätta forskningsresultaten i användning. Även om huvudfokus är på vad lärarutbildare gör, återspeglar det också mycket av den process varigenom man lär sig undervisa i matematik.

Metodologiska lösningar

Denna del av studien handlar om undersökningsmetoderna. Den omfattar forskningsfrågorna, forskningens design, de undersökta personerna, tekniken för datainsamlingen, kodningsprocessen och dataanalysen. Det är viktigt att lägga märke till att vissa forskare kan vara intresserade av hurdan verkligheten är, och varför den är sådan, medan andra kan vilja lägga fokus på vilka slag av uppfattningar individer har om ett givet objekt (Marton, Beaty, & Dall'Alba, 1993). Genom min studie eftersträvade jag att identifiera lärarutbildares uppfattningar av MTE och vad de ansåg vara önskvärt i fråga om utvecklingen av MTE. Den för fram en beskrivning av idémönstren. För att åstadkomma detta ställde jag två frågor på vilka inga tydliga svar fanns. För det första, vilka är lärarutbildarnas uppfattningar av MTE, och för det andra, vilka är lärarutbildarnas tankar om utvecklingen av MTE? De två forskningsfrågornas natur inbjuder till intryck av MTE från ett andra ordningens perspektiv. Det var detta som ledde till fenomenografi som metodologisk forskningsansats. Vidare kräver de ledande frågorna svar som väsentligen är kvalitativa.

Datainsamlingsprocessen gällde intervjuer med 27 lärarutbildare i matematik, vilkas utsagor kodades och analyserades. För samma grupp lärarutbildare (ytterligare 5 lärarutbildare svarade, vilket ledde till 32 deltagare) kodades och analyserades också skriftliga utsagor på ett öppet frågeformulär genom vilket jag ville identifiera vad de ansåg vara önskvärt i fråga om utvecklingen av MTE. Data insamlades mellan augusti 2006 och mars 2008. Kodandet och analysen utfördes i två skeden i forskningsfrågornas ordningsföljd. Hela processen genererade kvalitativt olika sätt att se på MTE och tankar om utvecklingen av MTE. Resultaten beskrivs i korthet. Om inte annat anges, representerar talen inom parentes proportionen (variationen) av lärarutbildare som tillhör den ifrågavarande beskrivningskategorin.

Resultat

Som svar på forskningsfråga ett avslöjade en noggrann analys sju kvalitativt olika uppfattningar av MTE. Den första beskrivningskategorin relaterar till MTE som en lärandeprocess via undersökning (4 av 27). Några nyckelaspekter som förklarar eller belyser beskrivningskategorin utgående från lärarutbildarnas svar är kreativitet, upptäckt, aktivitetsbaserat lärande samt utforskning av vad det är att undervisa och lära sig matematik. Den andra beskrivningskategorin hänvisar till MTE som en inspirationsprocess i fråga om lärande (2 av 27). Inspiration som kategori står för sig själv eftersom dess yttersta syfte är att odla intresse och motivera matematiklärarstuderande genom stimulans, förströelser, företagsamhet samt användning av huvudbry. Det är tänkt att vara en lösning på fobier, negativa attityder, låg självkänsla och undvik-matematiken-syndromet eller ”avtagande intresse”. Den tredje beskrivningskategorin pekade starkt på MTE som en lärandeprocess med fokus på problemlösning (3 av 27). Den kan särskiljas från andra i fråga om de aspekter som belyser den, och dessa är tillämpning av matematik på praktiska situationer samt lösande av problem som härrör ur verkliga livet.

Den fjärde beskrivningskategorin har sitt ursprung i att MTE har fokus på pedagogisk kunskap och skicklighet (6 av 27). Denna uppfattning har sina rötter i metoder/strategier för undervisning och lärande bland lärarutbildare. Ytterligare ses MTE som en didaktisk undervisnings- och lärandeprocess som gäller matematik (1 av 27). Några aspekter som använts för att belysa matematikdidaktiken, och som därför motiverar att den står för sig själv, är vad man ska undervisa, undervisningen, studier av undervisning och lärande samt de åtgärder som lärare vidtar (och hur de gör det) i samband med undervisning. Därtill sågs MTE också som en undervisnings- och lärandeprocess med tonvikt på ämnesinnehåll (2 av 27) och det exemplifieras av aspekter såsom ämneskunskap och gedigen matematik. Slutligen ses MTE som en lärandeprocess med tonvikt på integration av ämnesinnehåll med pedagogisk kunskap och skicklighet (9 av 27). I sträng mening uppfattas MTE som en organiserad kombination av MKT (matematikkunskaper för undervisning) och ämneskunskaper.

Forskningsfråga två genererade två kategorisystem, med kvalitativt olika sätt att se på utvecklingen av MTE. Först och främst sågs MTE som en process med tonvikt på pedagogisk kunskap och skicklighet (15 av 32 uppfattade MTE på detta sätt). Några av de aspekter som belyser denna beskrivningskategori är växelverkan i klassen, framställning och användning av material för undervisning och lärande, undervisning i lärarlag och pedagogisk reflektion. Den andra beskrivningskategorin hänför sig till utveckling av MTE med fokus på kunskaper och skicklighet i ämnet (5 av 32). I denna kategori ansåg lärarutbildarna att utvecklingen av lärarutbildningen har att göra med den speciella vikt man lägger vid kunskaper och skicklighet i ämnet. För dem refererar kunskaper i samband med utvecklingen av MTE till begrepps- och procedurkunskap, och därtill ansåg de att det är naturligt att först tänka på vilket innehåll man ska undervisa snarare än vilken metod man ska använda. Den tredje beskrivningskategorin skildrar utvecklingen av MTE som en process där man integrerar resultat av utvärdering (3 av 32). I denna kategori förknippar lärarutbildare utvecklingen av utbildningen av matematiklärare med behovet att ta i beaktande utvärdering för att stöda lärandet. MTE sågs också tydligt som en fråga om uppbyggande av relationer (7 av 32). I denna kategori uttryckte lärarutbildare idéer för utveckling av MTE som inbegrep uppbyggande av samhörighet mellan lärarstuderande. Andra aspekter som får denna kategori att framträda är växelverkan mellan studerande och handledare, vilket också går längre än mänskliga relationer så att det gäller material för lärande (läroböcker), relationer mellan teman och relationer mellan läroämnen. Slutligen sågs MTE också som studiet av kontexter för undervisning och lärande (2 av 32). Därvid ansåg lärarutbildare att utvecklandet av MTE innefattade tonvikt på forskning om situationer för undervisning och lärande. Aspekter som belyser denna kategori är tekniker för att identifiera problemområden, bekymmer i fråga om undervisning och lärande av matematik, motiv och genomförande av kartläggningar.

Frågan om hur lärarutbildare delade med sig av kunskaper och skicklighet avslöjade intressanta mönster. Först och främst uppfattades det så att den kunskap och skicklighet som hänför sig till MTE förmedlades via lokala grupper (21 av 32 angav denna uppfattning). Inom denna kategori uppfattade lärarutbildare lokala studiegrupper som kärnelement i fråga om professionell utveckling och som en hållbar strategi för att dela med sig av idéer. De aspekter som belyser denna beskrivningskategori inkluderar lokalt lärande i former som sträcker sig från seminarier, workshoppar och lokal kompetensutveckling till mentorskap och undervisning i lärarlag. Som en annan beskrivningskategori (11 av 32) kunde man identifiera att man delar med sig av kunskaper och skicklighet via samarbete och nätverk. Den centrala frågan är hur lärarutbildare för framåt och delar med sig av utvecklingsidéer i matematik. Strängt taget är det en diskussion om professionell utveckling och strategier för att få en sådan utveckling att äga rum. Användning av IKT för undervisning och lärande, nyhetsbrev i artikelform och författande av moduler förklarade denna kategori ytterligare.

Diskussion av forskningsmetoden och resultaten

Valet av metodologisk forskningsansats berörs först på grund av dess centrala roll i studien. Fenomenografi valdes som metodologisk lösning. Därtill görs en egen utvärdering av forskningens giltighet genom att jag granskar hur frågor om validitet (trovärdighet) och reliabilitet (tillförlitlighet) har hanterats. Slutligen dras några slutsatser med implikationer för lärarutbildning.

Den huvudsakliga principen vid valet av fenomenografi som forskningsansats är behovet av anpassning till forskningsfrågorna. Detta betyder i sin tur att undersökningen utförs utgående från hur ett fenomen uppfattas från ett andra ordningens perspektiv. Det följer också från argumentet att det är forskningsfrågan eller syftet snarare än en fastslagen position som antingen kvantitativ eller kvalitativ som bestämmer ansatsen (Niglas, 2004). Det här har noga beaktats.

I fråga om fenomenografi som forskningsansats och det anknutna begreppet ”uppfattning” ser jag mig nödsakad att beröra en del av den kritik som har riktats mot fenomenografi. Till exempel Säljö (1994) riktade skarp kritik mot fenomenografin som forskningsansats eftersom den försummar deltagarnas kontext. På grund av sitt intresse för uppfattningar har fenomenografer reducerat det som uttalas av ”deltagarna” till lösryckta utsagor och ”sugit ut” uppfattningar utan hänsyn till de kontexter i vilka de har konstruerats. De som fört fram fenomenografin, specifikt Marton (1995), vederlade detta och hävdade att *lärande och tänkande* är kontextbaserade. Jag ser Martons argument som starkt och jag kan personligen inte tänka mig en lärandesituation som är kontextfri. I denna studie tog jag intervjuns naturliga inramning i beaktande för att undvika att övningen reducerades till en utvärdering av lärarutbildare. Detta gjorde det möjligt för de intervjuade att uttrycka sig i en miljö som de hade god kontroll över, och det inkluderade användning av material för undervisning och lärande av matematik som hängde på väggarna i deras arbetsrum. Vidare påminner oss Åkerlind (2005) om att syftet med och utfallet av fenomenografi är att utforska spridningen av innebörder som innehas av en sampelgrupp i egenskap av grupp, inte spridningen av innebörder för varje individ inom gruppen. Detta betyder till exempel att inga intervjuutsagor kan förstås isolerade från andra. Det här har för mig framstått som en fundamental princip som ofta missförstås av kritikerna.

Den andra typen av kritik uttrycker tvivel om att det sätt på vilket man uppfattar kategorierna av uppfattningar verkligen återspeglar innehållet i intervjuerna (Francis, 1993). Det här är naturligtvis en validitetsfråga, och för att tackla den kan man till exempel fråga sig hur mycket man måste tolka lärarutbildares samlade utsagor för att undvika snedvridning. Det finns två vägar i fråga om behandlingen av data (Strauss & Corbin, 1990). Ett synsätt är att låta data tala för sig själva utan någon reduktion eller omformulering. Andra forskare är måna om exakta beskrivningar, men om man accepterar svårigheterna med att använda allt intervjumaterial blir det nödvändigt att reducera det, och detta innefattar i själva verket urval och tolkning. Det här är centralt för kvalitativa studier och något som styr reduktionsprincipen. Min omedelbara reaktion är: vilket slag av vetenskaplig undersökning är det som inte intresserar sig för precision? Min åsikt är att genererandet av nya uppfattningar, liksom även behandlingen av orden i intervjuerna och forskningsmaterialet kan skötas så skickligt att det minimerar det tvivel och de motsägelser som uppges förekomma.

Den tredje typen av kritik har samband med utmaningen att de flesta forskare som använder fenomenografi arbetar individuellt och i slutna rum under dataanalysen (Burns, 1994; Åkerlind, 2005). De hävdar att det finns möjlighet att ta med flera forskare för att göra insamlade data tillgängliga för utmaningarna från början, snarare än att man väntar till slutet. Deras andra orsak har att göra med potentialen att ha ett ännu bättre utfallsrum tack vare ökad fördomsfrihet och medvetenhet om alternativa perspektiv. Det argumenteras vidare att även om en individuell forskare kan ge ett substantiellt bidrag till förståelsen av ett fenomen, så kan en forskargrupp ha potential att föra denna förståelse vidare. Det är igen inte enbart fråga om forskares kompetens att göra trovärdiga tolkningar, utan också att göra tolkningar utan fördomar. För att tackla denna utmaning anlitade jag forskarassistenter som tidigare hade arbetat som lärarutbildare i matematik i den process som ledde till första versioner och sedan till de versioner av beskrivningskategorierna som man omfattade gemensamt.

Resultaten på den första forskningsfrågan indikerar att lärarutbildarna uppfattar MTE och önskvärd utveckling av MTE på kvalitativt olika sätt. Hur de har kommit till en viss uppfattning återspeglar deras egen personliga bakgrund och prioriteringar. En möjlig förklaring till varför MTE uppfattas på kvalitativt olika sätt kan vara skillnaderna i fokus när det gäller de kritiska aspekterna (Pang, 2003). Olika studier länkar ihop tidigare kunskaper med lärande, och den etablerade pedagogik som tillskrivs Ausubel (1978) är en källa med ett starkt argument att föruppfattningar påverkar det lärande som pågår. Lärarutbildare som blev ombedda att avslöja sina uppfattningar om MTE och vad de uppfattade vara önskvärd utveckling av MTE är i en slutlig analys också personer som lär sig och därför inte unika i detta speciella fall. Betydelsen av föruppfattningar stöds av evidens i form av utsagor av typen ”Jag lärde mig utvärdering med portfolio från institutioner utanför Tanzania” eller ”Jag deltog i ett seminarium om inspirationsmatematik”. Det här är klar indikation på att man bär med sig personliga tidigare erfarenheter i form av idéer till MTE och är kanske ett bättre svar till kritiker som menar att fenomenografin suger ut uppfattningarna utan hänsyn till kontext (Säljö, 1994). I detta fall avslöjade respondenterna källa och kontext för uppfattningarna om MTE och tankarna om dess önskvärda utveckling – en klar indikation på situerad praktik.

Den andra kommentaren om resultaten utgår från varaktigheten i uppfattningarna av MTE samt i tankarna om en önskvärd utveckling av MTE och strategierna för att dela med sig av kunskap och skicklighet. Dessa kvalitativa sätt att se fenomenet återspeglar vad som temporärt dominerade under studiens materiella tid. Jag ser det så att uppfattningarna kan vara känsliga för både kontext och tid. Det har utförts ett antal studier som belyser detta argument. Martons (1995) åsikt om lärande som situerad praktik är ett logiskt argument. Av den här anledningen är det inte nödvändigtvis trovärdigt att ge forskningsresultaten långtgående generaliseringar – det som erhållits är situationsbaserat.

Den tredje kommentaren gäller vilken status lärarutbildarnas uppfattningar av MTE har. Svaren visar på olikheter i uppfattningar, inte enbart som ”smala” eller ”djuplodande”, utan också som utvecklade eller kompakta. Ambitionen från starten var inte att testa hur kunniga de var, utan snarare att studera lärarutbildares uppfattningar genom deras egna ögon och att komma med en beskrivning av variationerna. Detta har åstadkommits, och jag har konstaterat att lärarutbildarna har olika reflektionsnivåer i fråga om vad MTE är.

Den fjärde kommentaren återspeglar sambandet mellan resultaten och de antagna teoretiska ramarna för studien. Resultaten indikerar att man måste anstränga sig för att säkerställa vilka delade uppfattningar av MTE som förekommer, och det här har konstaterats tidigare i fråga om dess komplexa natur (Adler, 2005). Samma situation gäller för utbildning i matematik (Bass, 2005; Ernest, 1998; Lerman, 2001; Lester & Lambdin, 1999; Mura, 1998; Niss, 1998; Wittmann, 1998). Efter dessa övergripande kommentarer om forskningsresultaten finner jag det fortfarande nödvändigt att kort diskutera validitets- och reliabilitetsfrågan i termer av trovärdighet respektive pålitlighet.

Trovärdighet (inre validitet) är något som man kan inrikta sig på genom användning av olika tekniker för datainsamling (*triangulering*) enligt Denzin (1970). I denna studie spelade jag in intervjuer, förde anteckningar och fokuserade mig på centrala aspekter av forskningsfrågorna för att hålla kontroll på de båda sätten att registrera uttalanden. Lika viktigt var att det öppna frågeformuläret i forskningsfråga två stöddes av en omedelbar uppföljande intervju med fem lärarutbildare för kontroll av äktheten i data och klarläggande av några områden. Frågeformulären gjorde det möjligt för lärarutbildarna att uttrycka sina tankar på sätt som inte hade valts ut på förhand av någon intresserad person, och användningen av dem avslöjade dolda uppfattningar och gav oväntade svar. Det blev också möjligt för respondenterna att utmana några uppfattningar som normalt accepteras utan vidare. För det andra anlitade jag tre forskningsassistenter som oberoende av varandra kodade hälften av lärarutbildarnas utsagor i fråga om deras uppfattning av MTE och som diskuterade dem i syfte att *komma överens* om beskrivningskategorierna, varefter jag fullföljde med resten av utsagorna. Det som eftersträvades var att data skulle vara öppna för att kunna ifrågasättas och att eventuella snedvridningar skulle minimeras (Ary, Jacob & Razanieh, 2002). Den andra nivån innebar att en kollega anlitades för att kritiskt granska och validera kategorierna och att på ett kreativt sätt placera kategorierna mera åskådligt, snarare än att matcha i termer av rätt och fel, vilket ofta är en medbedömares uppgift då fenomenografi används. Varför var detta nödvändigt? Kodandet och kategoriseringen hade varit lång och prövande. Några störande faktorer kunde ha kommit emellan under skrivprocessen. Inom ramen för denna strategi var det också viktigt för mig att hela tiden reflektera över mina egna handlingar. För att inrikta mig på den interna validiteten måste jag till exempel undvika snedvridning och välja respondenter i form av lärarutbildare från olika geografiska områden.

Med avseende på reliabilitet är det angeläget för mig att reda ut hur man kan förlita sig på resultaten. Först och främst finns det en fullständig beskrivning av hur studien genomförts från och med den första fasen då instrumenten testades i augusti 2006 till datainsamling för forskningsfråga ett i december samma år. Detta efterföljdes av pilottestning av instrumentet för forskningsfråga två i mars 2007 samt datainsamling i december 2007. Denna sekvens av forskningsuppgifter i kombination med triangulering av datainsamlingsproceduren utgjorde det första stadiet för säkerställande av forskningsresultatens reliabilitet. För det andra inriktade jag mig på reliabla forskningsresultat genom att använda deltagare från olika geografiska områden vilket kunde bidra till variabilitet i uppfattningarna. För att åstadkomma detta samlade jag in data från ett heterogent sampel av lärarutbildare som arbetade med lärarutbildning för certifikat respektive diplom, eftersom dessa har personal med olika meriter (diplom, universitetsutbildning).

Utöver validitet och reliabilitet är det ibland naturligt att tänka på generaliseringsmöjligheter. Frågan om generalisering är problematisk eftersom resultaten i viktiga avseenden är situationsbundna. Det går att ifrågasätta ”samma deltagare” och ”samma kontext” när det gäller mänskligt beteende, och likartade situationer kan vara svåra att åstadkomma. Som tidigare nämnts gjordes det avsiktliga urvalet av respondenter inte med tanke på generalisering. Avsiktligt urval innebär i detta fall val av lärarutbildare som motsvarade studiens syfte utgående från speciella principer. Representativiteten är emellertid tillräcklig för att resultaten av studien ska kunna användas för att lära sig något om andra situationer i Tanzania. Generalisering (möjlighet till transfer) av forskningsresultaten utanför Tanzania har understötts genom noggrann beskrivning av de nationella och lokala omständigheterna och genom citat som exemplifierar uppfattningar och tankar i Tanzania, och en del kan motsvara situationer i andra länder med motsvarande problem i MTE.

Avslutande tankar

Att nå höjdpunkten av en forskningsprocess som denna är inspirerande. Naturligtvis förmedlar forskningsprocessen ett budskap om behovet att allvarligt reflektera över resultaten och deras implikationer för lärarutbildningen. Studiens resultat följer av forskningsfrågorna om MTE, för vilka inga svar existerade i början. Detta möjliggjorde utvecklandet av metodologiska lösningar som styrde undersökningen av lärarutbildares uppfattningar av MTE, tankar om önskvärd utveckling av MTE samt strategier att dela med sig av kunskaper inom detta område. Jag förliknar denna process vid en kritisk stig som jag hela tiden höll noggrann kontroll över för att hålla mig till forskningsfrågorna. Det är nu meningsfullt att se tillbaka och reflektera över studien i form av några avslutande tankar.

För att placera in mina forskningsavsikter var det viktigt för mig att granska relevant litteratur och sammanställa teoretiska ramar, som utvaldes och rättfärdigades på basis av förändringarna i fokus för MTE i Tanzania. Det här gjorde jag i syfte att bringa i dagen de betydelser som tillskrivits MTE inom en given tidsram. Det skulle belysa MTE och utgöra en utgångspunkt för diskussionen om MTE idag. Inom de teoretiska ramarna har flera olika fasader eller perspektiv hos MTE lyfts upp i ljuset och diskuterats. De huvudsakliga utgörs av MTE som en sammansättning av flera inflytanden, som en process genom vilken man blir matematiklärare samt som en blandning av ämnesinnehåll och pedagogisk kunskap. Slutligen kan MTE uppfattas som lärande om undervisning. Eftersom jag arbetade under antagandet att lärare ”ser framåt” gjorde jag ett försök att undersöka ”tankar” om utvecklingen av MTE. Detta baserade sig på antagandet att lärare lär sig även efter det att de blivit behöriga.

En sammanfattande bild (ett ramverk) föreslås sålunda ta i beaktande lärarutbildares uppfattningar av MTE, deras tankar ifråga om en önskvärd utveckling av MTE och de strategier genom vilka de delar med sig av sina kunskaper på området. Den föreslagna sammanfattande representationen har utvecklats ur forskningsresultaten och de teoretiska ramarna. Den sammanfattar de tre kategorisystemen och deras motsvarande beskrivningskategorier i en enda bild av lärarutbildares kvalitativa sätt att se på MTE. Ytterligare reflektion över forskningsfrågorna finns representerad i en bild av den inbyggda dynamiken i forskningsprocessen. Man kan se flödet av lärarutbildares idéer från det att de uppstår (uppfattningar av MTE) till hur de förverkligas (önskvärd utveckling av MTE) och slutligen hur man delar med sig av kunskap (strategier) till kollegor.

Med hänvisning till detta var det plausibelt att dra några slutsatser enligt följande: lärarutbildare uppvisar tydliga begreppsliga variationer i fråga om MTE på kvalitativt olika sätt som ett resultat av olikheter i deras historiska bakgrund; de tydliga begreppsliga variationerna utgör möjliga grundvalar för pedagogiska beslut samtidigt som de beskriver MTE som den är; och en del lärarutbildare ser matematiken som ett kunskapsområde av odiskutabelt slag. Vidare är integration av ämnesinnehåll och pedagogisk kunskap den dominanta uppfattningen av MTE, och lärarutbildares tankar i fråga om professionell utveckling rör sig mestadels om pedagogisk kunskap och skicklighet.

Lärarutbildares tankar om hur man kan dela med sig av kunskaper och skicklighet pekar på olika strategier som är i användning, och de grundar sig på lokala lärandegrupper inom lärarhögskolorna samt på distanssamarbete och nätverk.

I samband med denna studie var jag inte i tillfälle att närvara vid verkliga MTE-lektioner för att se till exempel vad som kallas att lära sig matematik genom undersökningar. Mina resultat och slutsatser baserar sig på kodandet och analysen av forskningsmaterialet.

Att nå detta stadium är en investering i fråga om kritiskt tänkande och tid som resurser, liksom även andra resurser. Låt mig använda ett argument av Adler et al. (2005) att det vissa gånger är svårt att ha en skeptisk inställning till sitt eget arbete, trots alla försiktighetsåtgärder som vidtagits. Det finns risker att man utelämnar viktiga frågor. För att råda bot på detta finns det behov att inbjuda extra hjälp som för vidare vissa forskningsuppgifter som framträtt som en följd av diskussionen. Jag föreställer mig en studie av vad lärarutbildare behöver känna till som en del av MTE. Vad är syftet med den kunskap de vill uppnå? Vilka slags strategier skulle tillgodose deras behov? Om en sådan studie förverkligades, är det sannolikt att den skulle beakta vad som verkligen sker i MTE-klassrum, eller helt enkelt reda ut vad lärarutbildare gör. Det här skulle utgöra tillägg till resultaten i min studie, som inte nådde in till aktiviteterna i klassrummen.

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Appendices

Appendix I. Letter of introduction to teachers' college principals

THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION AND VOCATIONAL TRAINING

Cable: "ELIMU" **DAR ES SALAAM**
Telex: 41742 Elimu Tz
Tel: 2110146/2120403/2120412-21
Fax: 255-51-113271 Elimu Tz
Website: www/moe.go.tz

P.O. BOX 9121
DAR ES SALAAM

Ref: No: B/1512

DATE: October 26th, 2006

To:
Principals
Teachers' College Principals.
TANZANIA MAINLAND.

Ref: INTRODUCTION OF MR. A. L. BINDE

Mr. A. L. Binde is a student of Abo Akademi University. He is currently conducting a study on mathematic education. The study focuses on teacher educators as respondents to a planned interview. The title of the study is *'Teacher educators' conception of mathematics teacher education'*. From a very general point of view, the study seems to be a relevant undertaking in this changing times and changing purposes of teacher education. The study may contribute to the changes taking place. With this in mind, please help him to realize his ambition. He is likely to interview mathematics teacher educators and also request them to submit written statements in the interview process.

Assist within limits.

Bakari G. Issa
For; PERMANENT SECRETARY

Appendix II. Interview guide (establishing rapport)

Topic: Conceptions of mathematics teacher education:

Thoughts among teacher educators in Tanzania

1. Introduction

a) Establishing rapport and time for interview with mathematics teacher educator.

b) Self introduction to the mathematics teacher educator.

c) Explaining the purpose of interview and use of the open-ended written expressions from mathematics teacher educator.

d) Structure of the interview.

e) Ensuring ethical issues.

f) How study results will be used and possibility of sharing.

2. Knowing the interview respondent (subject)

a) Name of the mathematics teacher educator.

b) Work experience of mathematics teacher educator/teacher, educational background, training opportunities attended as a mathematics teacher educator, what influenced him/her to the career paths?

c) Any other relevant information?

Tasks of interest to the study

A: Mathematics, mathematics teacher education

1. What comes to your mind when you think of?

   a) Mathematics?

   b) Mathematics teacher education? Mathematics education?

2. If now I ask you what is mathematics teacher education, what would you say?

3. What constitutes mathematics teacher education? (Focus on understanding, aspects, features (characteristics), and major parts)

4. Can you describe some of the present teaching and learning strategies which you have used to teach mathematics? Say what you actually do rather than what you think you could do. What have you done as an individual to contribute to the development of mathematics teacher education within your college or elsewhere? Not necessarily as a result of national directives. Do you have any personal initiative in mathematics teaching and learning to be proud of?

5. a) What do count as your most important goal in the development of present mathematics teacher education?

   b) What are the most interesting things (strengths) in the implementation of the mathematics teacher education curriculum?

   c) What do you count as development challenges or unresolved issues in the course of you teaching mathematics teacher education?

d) What is your understanding of subject matter content in mathematics teacher education? b) What does pedagogical content knowledge mean to you? c) What should be emphasized between academic content and mathematics teaching knowledge or pedagogy? Can you consider a compromise between these two lines?

e) What are your general views on present mathematics teacher education? Any more ideas to support the development of present mathematics teacher education?

THANK YOU FOR YOUR RESPONSE

Appendix III: Open-ended questionnaire

Thoughts on development of mathematics teacher education

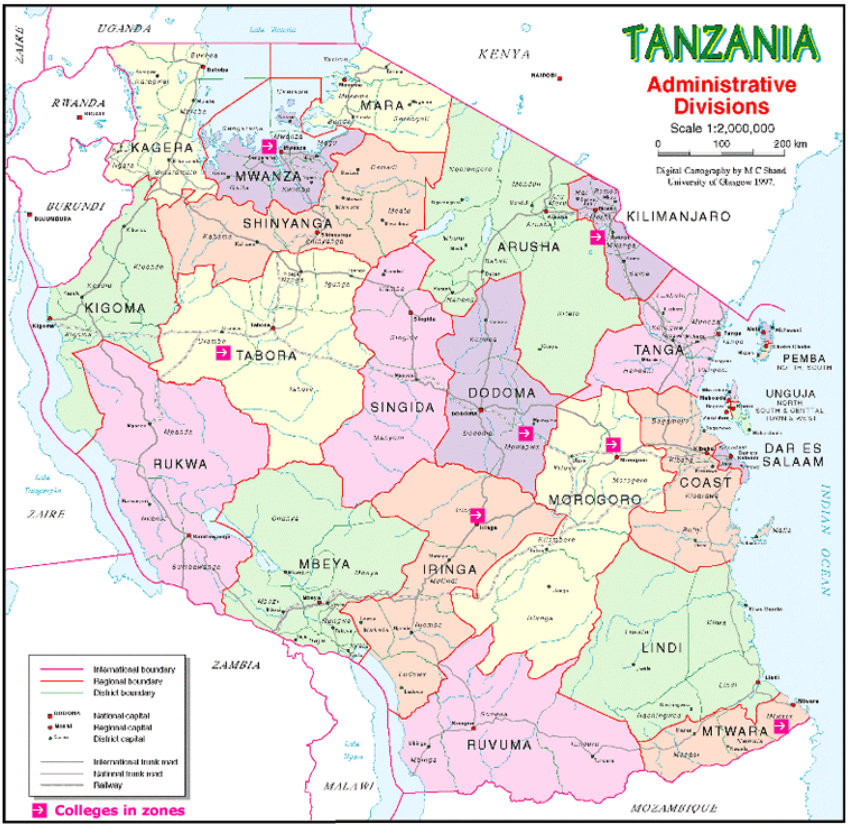
1. Think of some mathematics teacher education development ideas you have really implemented well as an individual teacher educator or as a team. Describe them, and say how you prepared yourself and actually put them into practice. (*written statements requested*)
2. In what ways do you share your development ideas in mathematics teacher education with colleagues? (*written statements requested*)

Appendix IV. Background data of research subjects captured between August 2006 and March2007.

Given name	Sex Male/Female		Highest educational level		Teaching experience (years)		Work station/zone
	M	F	Diploma	Bachelor masters	Less or 5 years	More than 5 years	
1. Arthur	M			B. Ed		8	Northern
2. Kahe	M		Diploma			12	Eastern
3. Mbilia		F	Diploma		5		Lake
4. Asha		F		B. Sc		6	Western
5. Miraji	M			B. Ed		7	Southern
6. Iddi	M			B. Ed		9	Northern
7. George	M			B. Ed	5		Western
8. Aden	M		Diploma			11	Southern
9. Anna		F	Diploma		5		Western
10. Salum	M			M. Ed		14	Lake
11. Yambo		F		B. Ed	4		Central
12. Shyrose		F	Diploma			10	northern
13. Omari	M			M. Ed		16	Eastern
14. Safari	M		Diploma			8	S/highlands
15. Freddy	M			B. Sc		10	Southern
16. Hamida		F		B. Ed		6	Eastern
17. Faith		F		B. Ed		6	Lake
18. Nesse		F	Diploma		4		Southern
19. Fadhili	M		Diploma		5		Lake
20. Yusufu	M			B.Sc.		16	S/highlands
21. Kezzy	M			M. Ed		10	Northern
22. Kia	M			B. Ed		13	Central
23. Danny	M			M. Ed		13	Eastern
24. David	M			M. Ed		11	Northern
25. Lea		F	Diploma			12	central
26. Elias	M			B. Ed		10	Lake

27. Kally	M		Diploma			16	S/highlands
28. Bahati		F	Diploma			9	Southern
29. Ndella	M		Diploma			14	S/highlands
30. Gama		F	Diploma		5		S/highlands
31. Besta	M			M. Ed		12	Northern
32. Violet		F	Diploma			6	S/highlands
Total							

Appendix V: Geographical locations of research subjects according to educational zones



Note: Tanzania has regions for administrative purposes, but also the Ministry of Education and vocational training divides the country into zones for functional convenience. The research participants came from the different zones: not necessarily from the main town of the zone but from colleges within the zones. The seven (7) zones at the time of the study, with their head office in brackets, are Eastern (Morogoro), Western (Tabora), Northern (Moshi), Southern (Mtwara), Southern Highlands (Iringa), Lake (Mwanza), and Central (Dodoma). Teacher colleges are located within these seven zones (See the shaded dots on the map of Tanzania for the zone headquarters.

Appendix VI: Diploma and certificate mathematics teacher education syllabus (summary)

Objectives of mathematics teaching methods syllabus for Diploma in Education: 2000-2007.

At the end of the two year Diploma course, the teacher trainees should be able to: a) Apply interactive and participatory teaching methods and strategies in the teaching of different topics of mathematics; b) Plan mathematics teaching lessons where the learners are actively involved throughout the teaching- learning process; c) Design, develop and use different teaching –learning materials in collaboration with the teacher trainees; and d) Acquire and develop a positive attitude in teacher trainees towards the teaching of mathematics.

Objectives, teaching and learning strategies of Diploma in Education syllabus

Topics	Objectives	Teaching and learning strategies
Teaching Methods *What is a teaching method? participatory and non-participatory methods*	The trainee should be able to mention methods of teaching maths and their characteristics, advantages and disadvantages; mention strategies used in the teaching of maths, their characteristics, differences, advantages and disadvantages.	Discussion on characteristics, advantages and disadvantages of various methods and strategies, categorize teaching methods as participatory and non-participatory. **Teaching /learning resources:** Various references according to tutors' preferences and availability.
Analysis of curriculum materials *Syllabus, textbooks,* *teacher's guide*	The trainee should be able to explain, state the characteristics of and categorize various mathematics curricula, state the uses of different curriculum materials. The trainee should be able to Identify and analyze the qualities and features of curriculum materials.	Discuss the use and importance of curriculum materials such as syllabus, textbook and teacher's guide. Discuss the main features and selection criteria of the curriculum; discuss the use of curriculum materials. **Teaching /learning resources:** Syllabus and textbook for secondary schools (forms 1-IV), Teacher's guide; Syllabus and textbook for secondary schools (forms 1-IV).
Subject Content All topics at ordinary level secondary education	The trainee should be able to analyse, develop schemes of work, and develop lesson plans. Make presentations in the classroom effectively and interactively on topics covered in ordinary level secondary school syllabus (form 1-IV).	Tutor and trainees using various methods and teaching learning strategies, microteaching, Block Teaching Practice. Develop methods and strategies for teaching the following topics: word problems, operations on real numbers, operation on matrices, statistics, probability, set, algebraic expressions, solving equations up to simultaneous equations, linear programming, exponents and logarithms, relations and functions, geometrical measurement, geometrical shapes and figures, geometrical proofs, congruence and similarity theorems.

Topics	Objectives	Teaching and learning strategies
		Teaching /learning resources: Syllabus for secondary schools forms 1-IV, textbook and teacher's guide.
Preparation to teach *Importance of preparation, schemes of work. lesson plans* *lesson notes.*	The trainee should be able to state the need for lesson preparation before classroom sessions, outline steps for lesson preparation, identify components and use the given format to prepare schemes of work and lesson plans.	The tutor and trainees, using discussion, and presentation, discuss the advantages of preparation before teaching, problems of teaching without preparation, and how to involve learners in lesson preparation. The tutor and trainees, using discussion, and demonstrations, state the importance of lesson plans and schemes of work, practice preparation of schemes of work and lesson plans. **Teaching /learning resources:** Various references according to teachers' preferences and availability
Teaching/learning materials *What are teaching aids? Types of teaching aids.* *Preparation of teaching aids, importance of teaching aids.*	The trainee should be able to describe and illustrate various types of teaching /learning materials for teaching in secondary school forms 1-IV. Prepare or improvise teaching materials for various topics in secondary school forms 1-IV	Tutor and trainees discuss ways of selecting teaching /learning material, types of teaching /learning material for secondary school forms 1-IV. Tutor and trainees discuss the meaning of improvisation and preparation of teaching /learning materials for secondary school forms 1-IV The tutor and trainees prepare or improvise various teaching and learning materials for secondary school forms 1-IV using locally available materials **Teaching /learning resources:** Various references according to tutors' preferences and availability. Syllabus for secondary schools forms 1-IV , textbook and teacher's guide
Lesson evaluation *What is evaluation?* *When to evaluate.* *Types of evaluation.* *Why evaluations*	The trainee should be able to explain the importance of evaluating the teaching/learning process, explain procedures and tools used in evaluation, describe types of exercises/test questions, construct evaluation tools and tests, mark exercises and	Tutor and trainees discuss the importance of evaluating the teaching/learning of mathematics. Tutor and trainees discuss the tools (exercises, tests, observation of students behaviour as a whole, interviews) used in evaluation. Tutor and trainees discuss the strengths and weakness of each evaluation tool or procedure.

Topics	Objectives	Teaching and learning strategies
	tests.	Tutor guide trainees to construct various evaluation tools and mark various types of questions. Trainees practice constructing and using evaluation tools. **Teaching /learning resources:** Syllabuses for secondary schools forms 1-IV. Textbook and teacher's guide. Various references according to tutors' preferences and availability

Objectives of elementary mathematics teaching methods syllabus (2000-2007)

At the end of the two years of the certificate course in teacher education (Grade IIIA), teacher trainees should be able to: a) Use interactive , participatory teaching methods and strategies in the teaching of different topics of mathematics in primary schools; b) Plan lessons of mathematics where the learners are actively involved throughout the teaching- learning process; c) Design, develop and use different teaching –learning materials in collaboration with the teacher trainees; and d) Inculcate and develop a positive attitude in teacher trainees towards the teaching of mathematics.

Objectives, teaching and learning strategies of Certificate in Education course for elementary teachers.

Topics	Objectives	Teaching and learning strategies
Teaching methods *What is a teaching method? participatory and non-participatory methods*	The trainee should be able to mention methods of teaching maths and their characteristics, advantages and disadvantages. Mention strategies used in the teaching of maths and their characteristics, differences, advantages and disadvantages.	Discussion on characteristics, advantages and disadvantages of various methods and strategies (games, role play experiments, question and answers, etc). Categorize teaching methods as participatory and non-participatory. **Teaching /learning resources:** Various references according to teachers´ preferences and availability
Analysis of curriculum materials *Syllabus, textbooks,* *teacher's guide*	The trainee should be able to explain various curricula, state their characteristics, categorize differences and state the uses of different curriculum materials. The trainee should be able to identify and analyze the qualities and features of curriculum materials.	Discuss about the use and importance of curriculum materials, such as syllabus, textbook and teacher's guide Discuss the main features and selection criteria of the curriculum. Discuss the use of curriculum materials. **Teaching /learning resources:** syllabus and textbook for primary schools, teachers' guide.

Topics	Objectives	Teaching and learning strategies
Subject Content All topics in primary schools (std 1-7)	The trainee should be able to analyse, develop schemes of work, lesson plans and make presentations in the classroom effectively and interactively on topics covered at primary school level (std 1-7).	Tutor and trainees using various methods and teaching learning strategies, microteaching, Block Teaching Practice, develop methods and strategies for teaching the following topics: word problems, operation on real numbers, operation with fractions, percentages, roots and square roots, coordinate geometry (allocating a point on x-y plane), algebraic expressions in one variable, geometrical measurements (perimeter, area and volumes), graphs (line, bar and histograms and pie charts), etc. **Teaching /learning resources:** Syllabus and textbook for primary schools, Teacher's guide**.**
Preparation to teach *Importance of preparation.* *Scheme of work.* *Lesson plan* *Lesson notes.*	The trainee should be able to state the need for lesson preparation before classroom sessions, outline steps for lesson preparation, identify the components of lesson plans and use the given format to prepare schemes of work and lesson plans.	The tutor and trainees, using discussion, and presentation, discuss the advantages of preparation before teaching, problems of teaching without preparation, and how to involve learners in lesson preparation. Tutor and trainees, using discussion, and demonstrations, state the importance of the lesson plan and scheme of work, practice preparation of schemes of work and lesson plans. **Teaching /learning resources:** Selected examples of schemes of work, lesson plans. Teacher's guide
Teaching/learning materials *What is teaching material or teaching aids?* *Types of teaching aids.* *Preparation of teaching aids.* *Importance of teaching aids*	The trainee should be able to describe and illustrate various types of teaching /learning materials for teaching in primary schools (std 1-7). Make or improvise teaching materials for various topics at primary school level (std 1-7), keep a collection of teaching materials for schools	The tutor and trainees discuss ways of selecting teaching /learning material, types of teaching /learning material for primary schools (std 1-7). The tutor and trainees discuss the meaning of improvisation and preparation of teaching /learning materials for primary schools (std 1-7). The tutor and trainees prepare or improvise various teaching and learning materials for primary schools (std 1-7). Using locally available materials. **Teaching /learning resources:** References according to tutors' preferences and availability. Syllabus and textbook for primary schools teacher's guide.
Lesson evaluation *What is evaluation?*	The trainee should be able to explain the importance of evaluating the	Tutor and trainees discuss the importance of evaluating the teaching/learning of mathematics and trainees discuss the tools

Topics	Objectives	Teaching and learning strategies
When to evaluate. *Types of evaluation.* *Why evaluations*	teaching/learning process, Explain procedures and tools used in evaluation. Describe types of exercises, test questions Construct evaluation tools and tests. Mark excises and tests	(exercises, tests, observation of students´ behaviour as a whole, interviews) used in evaluation. Tutor and trainees discuss the strengths and weakness of each evaluation tool or procedure. Tutors guide trainees to construct various evaluation tools and mark various types of questions and how to report the results. Trainees practice constructing and using evaluation tools. **Teaching /learning resources:** Syllabus and textbook for primary schools, teacher's guide. Various references according to tutors' preferences and availability

Important to note:

- Evaluation does not feature among the objectives of the certificate and diploma teacher education syllabus.
- Objectives for elementary teacher education (grade IIIA) are similar to the diploma level except for the level intended and English as the language of instruction.
- The Diploma in Education mathematics teacher education syllabus was reviewed in 2007 and a new one is currently under implementation with emphasis on subject matter).
- The Certificate in Education mathematics teacher education syllabus has been reviewed and implementation started in July, 2009, again with emphasis on content.

What are teacher educators' conceptions of mathematics teacher education (MTE) in teacher colleges in Tanzania? What do they consider as desirable development of their field? What are their strategies for sharing knowledge and skills?

In answers to these questions, it has been found that there are sharp conceptual variations concerning MTE among teacher educators in qualitatively different ways. These differences are rooted in their diverse backgrounds in teaching and learning mathematics, and they are, in turn, possible grounds for differences in making pedagogical decisions. Some teacher educators see mathematics as an unquestionable field of knowledge, while their thoughts on professional development gravitate mostly around pedagogical knowledge and skills. Integration of subject matter and pedagogical knowledge is the dominant conception of MTE, but it is also conceived as a process of becoming a mathematics teacher via mathematical investigation, inspiration, or problem solving. The options for sharing knowledge and skills in MTE focus on face-to-face sessions, as well as distance collaboration and networking. In both strategies the use of ICT as a mediating tool is emphasised.

It is expected that the results of this study could find their way into professional development programmes, especially when designing pre-service and in-service courses for primary and secondary school teachers. The study comes at a critical time when common voices are heard across countries, sending messages of problems related to shrinking interest and a 'mathematics-avoidance syndrome', as well as low student self-esteem with respect to mathematics learning.

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